



Nucleon resonances in Compton scattering

Gernot Eichmann

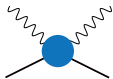
IST Lisboa, Portugal

Emergent mass and its consequences in the Standard Model

ECT*, Trento, Italy

September 18, 2018

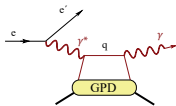
Compton scattering



**Structure functions
& PDFs in forward limit**

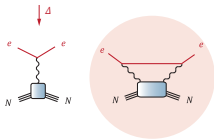
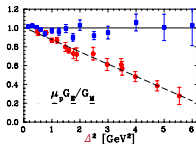
$$\text{Diagram} = \sum \text{Diagrams} \sim \left| \text{Diagram} \right|^2$$

**Handbag dominance
& GPDs in DVCS**



TPE corrections to form factors

Guichon, Vanderhaeghen, PRL 91 (2003)

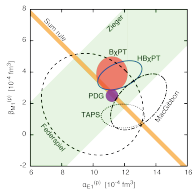


Proton radius puzzle?

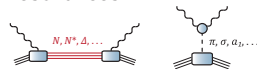
Antonigni et al., 2013, Pohl et al. 2013,
Birse, McGovern 2012, Carlson 2015

Nucleon polarizabilities

Hagelstein, Miskimen, Pascalutsa,
Prog. Part. Nucl. Phys. 88 (2016)



Resonances!



Motivation

Hadron spectrum:



mesons



baryons



glueballs?



hybrids?



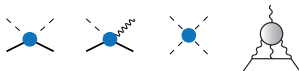
tetraquarks?



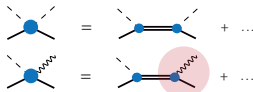
pentaquarks??

Form factors: resonance transition FFs,
spacelike vs. timelike properties

Hadron structure & scattering amplitudes



Extraction of resonances?



Σ^+	Σ^-	Σ^+	Σ^-	Σ^+	Σ^-	Σ^+
$N(939)$	$N(1535)$	$N(1720)$	$N(1520)$	$N(1680)$	$N(1675)$	$N(1900)$
$N(1440)$	$N(1650)$	$N(1900)$	$N(1700)$	$N(1860)$	$N(1860)$	$N(1860)$
$N(1710)$	$N(1895)$		$N(1875)$	$N(2000)$		
$N(1880)$						
$\Delta(1910)$	$\Delta(1620)$	$\Delta(1232)$	$\Delta(1700)$	$\Delta(1905)$	$\Delta(1980)$	$\Delta(1950)$
	$\Delta(1900)$	$\Delta(1800)$	$\Delta(1940)$	$\Delta(2000)$		
		$\Delta(1920)$				
$\Lambda(1115)$	$\Lambda(1405)$	$\Lambda(1890)$	$\Lambda(1520)$	$\Lambda(1820)$	$\Lambda(1830)$	
$\Lambda(1800)$	$\Lambda(1670)$		$\Lambda(1690)$			
$\Lambda(1810)$	$\Lambda(1800)$					
$\Sigma(1189)$	$\Sigma(1760)$	$\Sigma(1385)$	$\Sigma(1670)$	$\Sigma(1915)$	$\Sigma(1775)$	
$\Sigma(1690)$			$\Sigma(1940)$			
$\Sigma(1880)$						
$\Xi(1315)$		$\Xi(1530)$	$\Xi(1820)$			
		$\Omega(1672)$				

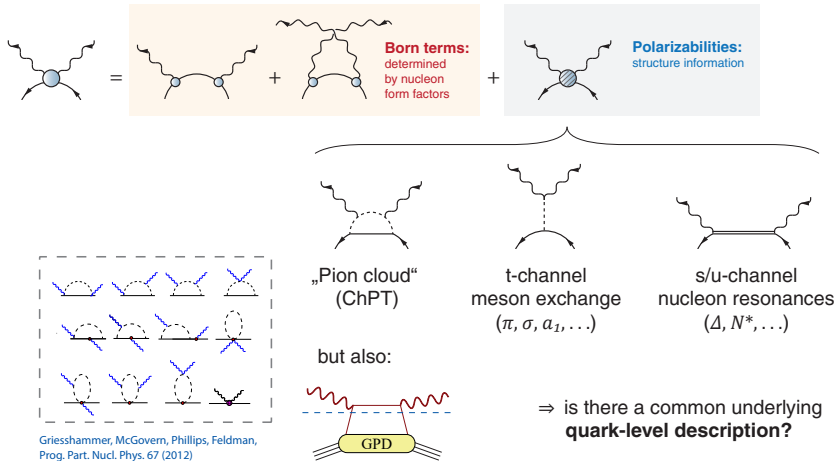


Outline

- **Introduction**
- **DSEs, BSEs:**
From quarks and gluons to baryon resonances
- **Nucleon resonances in Compton scattering,**
transition form factors
[GE, Ramalho, 1806.04579](#)

Compton scattering

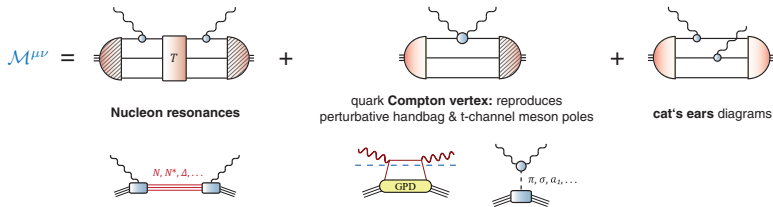
Compton amplitude = sum of **Born terms** + 1PI structure part:



Griesshammer, McGovern, Phillips, Feldman, Prog. Part. Nucl. Phys. 67 (2012)

Compton scattering

Scattering amplitude: [GE, Fischer, PRD 85 \(2012\) & PRD 87 \(2013\)](#)

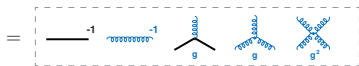


- **Poincaré covariance** and **crossing symmetry** automatic
- **em. gauge invariance** and **chiral symmetry** automatic
as long as all ingredients calculated from symmetry-preserving kernel
- **perturbative processes** included
- **s, t, u channel poles** dynamically generated,
no need for “offshell hadrons”

DSEs & BSEs

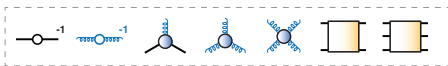
QCD's classical action:

$$S = \int d^4x \left[\bar{\psi} (\not{\partial} + ig\mathbf{A} + m) \psi + \frac{1}{4} F_{\mu\nu}^a F_a^{\mu\nu} \right]$$



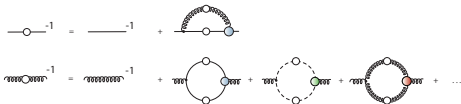
Quantum "effective action":

$$\int \mathcal{D}[\psi, \bar{\psi}, A] e^{-S} = e^{-\Gamma}$$

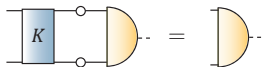


DSEs = quantum equations of motion:

derived from path integral, relate n-point functions



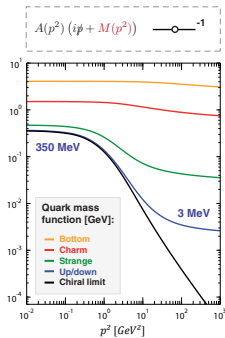
Bethe-Salpeter equations for hadronic bound states:



- Poincaré covariance
- Chiral symmetry
- EM gauge invariance
- Only quark & gluon d.o.f., hadron poles generated dynamically
- multiscale problems feasible
- gauge-fixed
- **truncations:** neglect higher n-point functions to obtain closed system

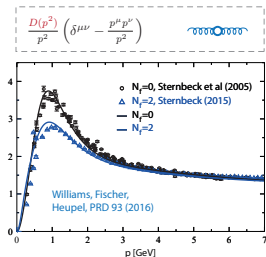
QCD's n-point functions

• Quark propagator



Dynamical chiral symmetry breaking generates 'constituent-quark masses'

• Gluon propagator



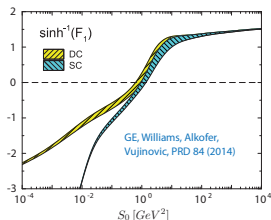
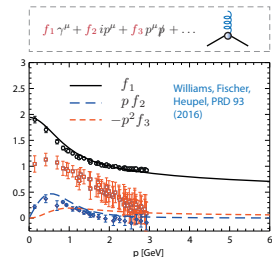
• Three-gluon vertex

$F_1 [\delta^{\mu\nu} (p_1 - p_2)^\rho + \delta^{\nu\rho} (p_2 - p_3)^\mu + \delta^{\rho\mu} (p_3 - p_1)^\nu] + \dots$

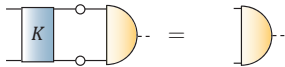
Agreement between lattice, DSE & FRG within reach

Huber, EPJ C77 (2017),
Cyrol, Mitter, Pawłowski, PRD 97 (2018), ...

• Quark-gluon vertex

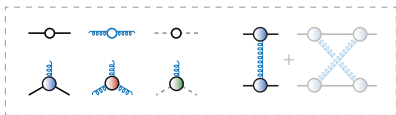


Truncations

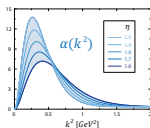
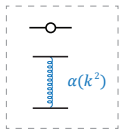


- **3PI system:** all 2 & 3-point functions calculated

Williams, Fischer, Heupel, PRD 93 (2016)



- **Rainbow-ladder:** quark propagator calculated, kernel = effective gluon exchange



$$\alpha(k^2) = \alpha_{\text{IR}}\left(\frac{k^2}{\Lambda^2}, \eta\right) + \alpha_{\text{UV}}(k^2)$$

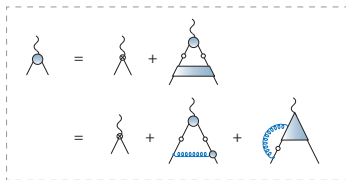
adjust scale Λ to observable,
keep width η as parameter

Maris, Tandy, PRC 60 (1999),
Qin et al., PRC 84 (2011)

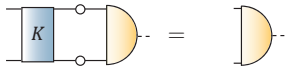
- **Beyond rainbow-ladder** using symmetries and quark-gluon vertex ansätze

Chang, Roberts, PRL 103 (2009),

Binosi, Chang, Papavassiliou, Qin, Roberts, PRD 93 (2016)

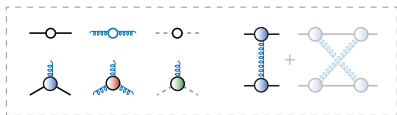


Truncations

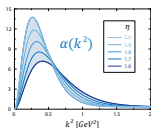
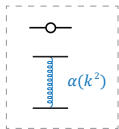


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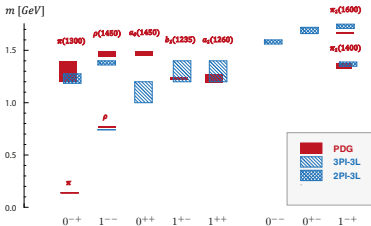
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Maris, Tandy, PRC 60 (1999),

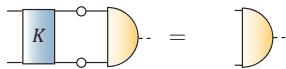
Qin et al., PRC 84 (2011)

Light meson spectrum beyond rainbow-ladder:



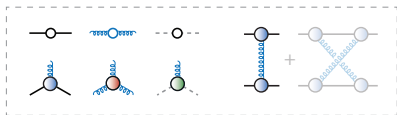
GE, Sanchis-Alepuz, Williams,
Alkofer, Fischer, PPNP 91 (2016)

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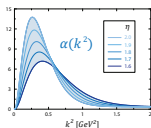
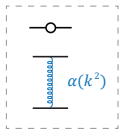


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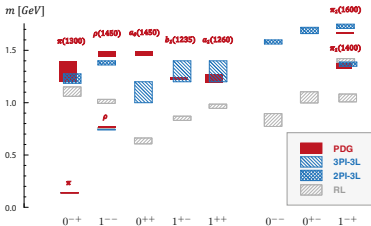
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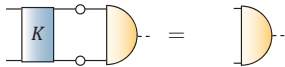
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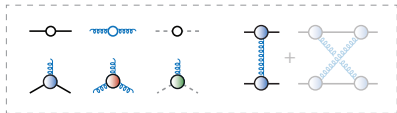


GE, Sanchis-Alepuz, Williams,
Alkofer, Fischer, PPNP 91 (2016)

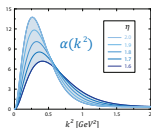
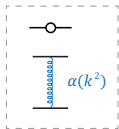
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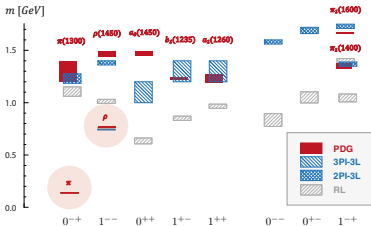


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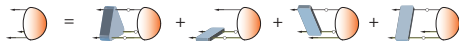


GE, Sanchis-Alepuz, Williams,
Alkofer, Fischer, PPNP 91 (2016)

Baryons

Covariant Faddeev equation for baryons:

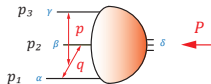
GE, Alkofer, Krassnigg, Nicmorus, PRL 104 (2010)



- 3-gluon diagram vanishes \Rightarrow **3-body effects small?**

Sanchis-Alepuz, Williams, PLB 749 (2015)

- 2-body kernels same as for mesons, no further approximations:

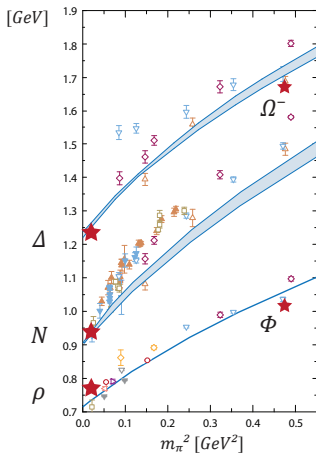


$$\Psi_{\alpha\beta\gamma\delta}(p, q, P) = \sum_i f_i(p^2, q^2, p \cdot q, p \cdot P, q \cdot P) \tau_i(p, q, P)_{\alpha\beta\gamma\delta}$$

Lorentz-invariant
dressing functions

Dirac-Lorentz
tensors carry
OAM: s, p, d,...

Review: GE, Sanchis-Alepuz, Williams, Alkofer, Fischer,
PPNP 91 (2016), 1606.09602

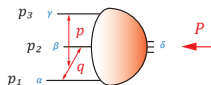


The role of diquarks

Three-body equation knows nothing of **diquarks**, but dynamically generates them in iteration

Group Lorentz invariants into **multiplets of permutation group S3**:

GE, Fischer, Heupel, PRD 92 (2015), GE, Sanchis-Alepuz, in preparation



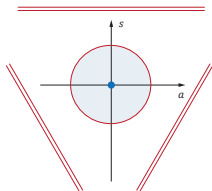
- **Singlet:**

symmetric variable,
carries overall scale:

$$S_0 \sim p_1^2 + p_2^2 + p_3^2 + \frac{M^2}{3}$$

- **Doublet:**

$$\mathcal{D}_0 \sim \frac{1}{S_0} \begin{bmatrix} -\sqrt{3}(\delta x + 2\delta\omega) \\ x + 2\omega \end{bmatrix}$$



- Second **doublet:**

$$\mathcal{D}_1 \sim \frac{1}{\sqrt{S_0}} \begin{bmatrix} -\sqrt{3}(\delta x - \delta\omega) \\ x - \omega \end{bmatrix}$$

Mandelstam plane,
outside: **diquark poles!**

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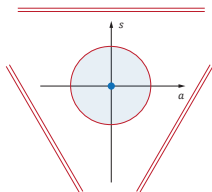
GE, Fischer, Heupel, PRD 92 (2015), GE, Sanchis-Alepuz, in preparation

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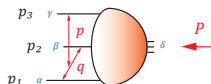
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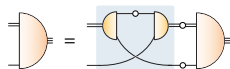


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⇒ Simplify 3-body equation to **quark-diquark BSE**

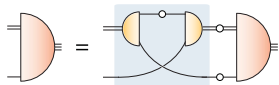
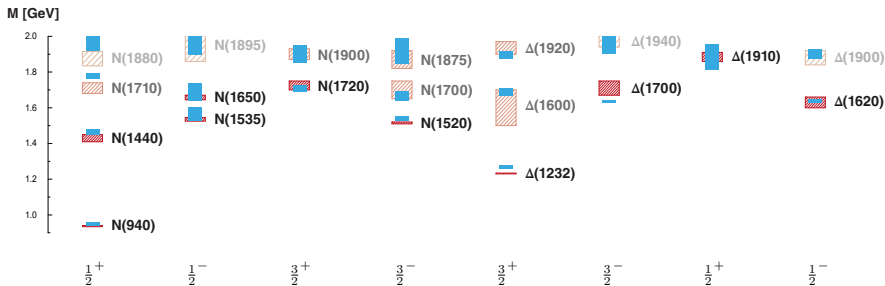


Oettel, Alkofer, Hellstern Reinhardt, PRC 58 (1998),
Cloet, GE, El-Bennich, Klähn, Roberts, FBS 46 (2009)
GE, Krassnigg, Schwinzerl, Alkofer,
Ann. Phys. 323 (2008)
Segovia, El-Bennich, Rojas, Cloet, Roberts,
Xu, Zong, PRL 115 (2015)

...

Baryon spectrum

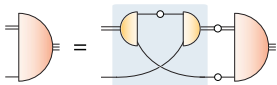
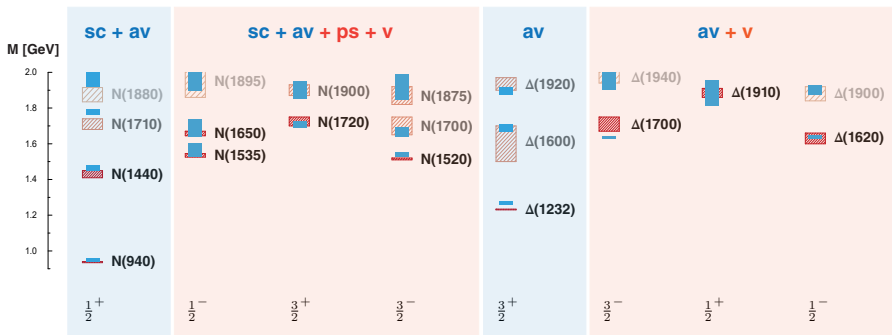
Quark-diquark with reduced pseudoscalar + vector diquarks: [GE, Fischer, Sanchis-Alepuz, PRD 94 \(2016\)](#)



- Scale Λ set by f_π
- Current-quark mass m_q set by m_π
- c adjusted to ρ - a_1 splitting
- η doesn't change much

Baryon spectrum

Quark-diquark with reduced pseudoscalar + vector diquarks: [GE, Fischer, Sanchis-Alepuz, PRD 94 \(2016\)](#)

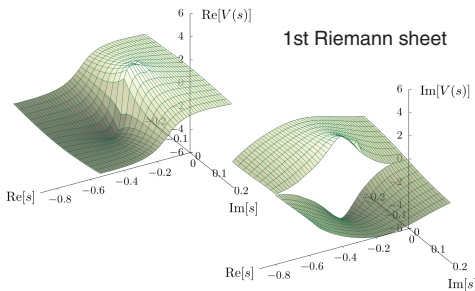
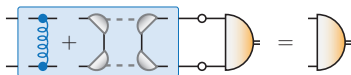


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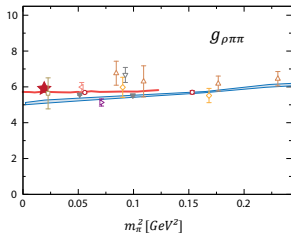
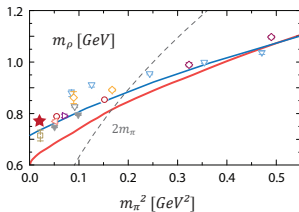
Resonances!

ρ meson as a dynamical resonance

Williams, 1804.11161



— RL
— RL + $\pi\pi$

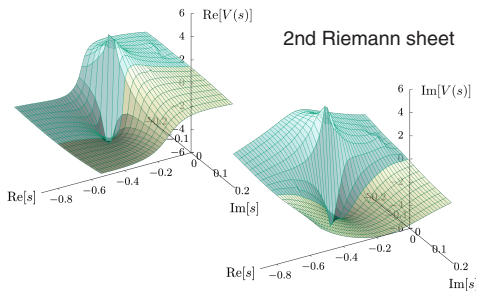
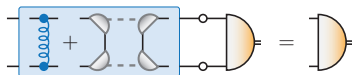


Lattice references: GE et al.,
PPNP 91 (2016) 1606.09602

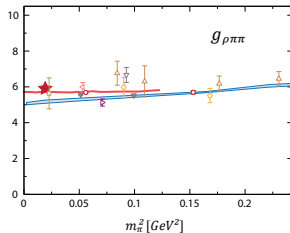
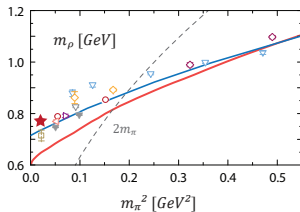
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— RL
— RL + $\pi\pi$

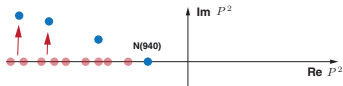


Lattice references: GE et al.,
PNP 91 (2016) 1606.09602

Resonances!

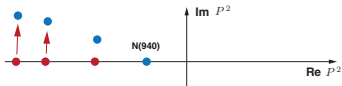
Lattice:

Proper treatment of resonances essential

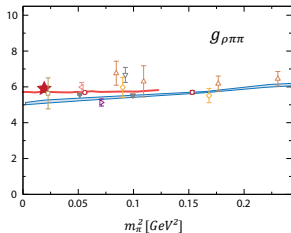
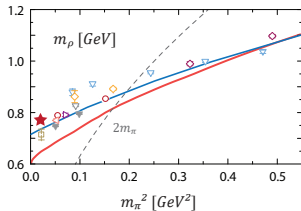


DSE / BSE:

Resonance dynamics
“on top of” quark-gluon dynamics

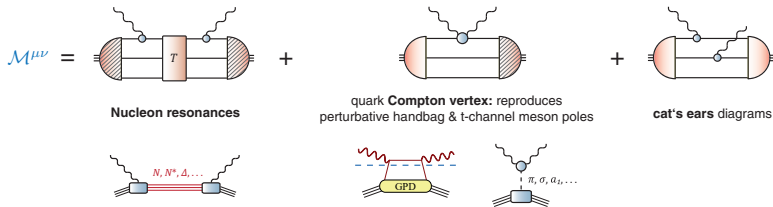


— RL
— RL + $\pi\pi$



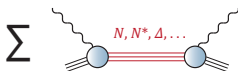
Compton scattering

Scattering amplitude: [GE, Fischer, PRD 85 \(2012\) & PRD 87 \(2013\)](#)



- **Poincaré covariance** and **crossing symmetry** automatic
- **em. gauge invariance** and **chiral symmetry** automatic
as long as all ingredients calculated from symmetry-preserving kernel
- **perturbative processes** included
- **s, t, u channel poles** dynamically generated,
no need for “offshell hadrons”

Nucleon resonances



$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940) N(1440) N(1710) N(1880)	N(1720) N(1900)	N(1535) N(1650) N(1895)	N(1520) N(1700) N(1875)
Δ(1910)	Δ(1232) Δ(1600) Δ(1920)	Δ(1620) Δ(1900)	Δ(1700) Δ(1940)

Need em. transition FFs

But vertices are half offshell:
need 'consistent couplings'

Pascalutsa, Timmermans, PRC 60 (1999)

- **em gauge invariance:** $Q^\mu \Gamma^{\alpha\mu} = 0$
- **spin-3/2 gauge invariance:** $k^\alpha \Gamma^{\alpha\mu} = 0$
- invariance under **point transformations:** $\gamma^\alpha \Gamma^{\alpha\mu} = 0$
- no kinematic dependencies, **"minimal" basis**

E.g. Jones-Scadron current
cannot be used offshell:

$$\Gamma^{\alpha\mu} \sim \bar{u}^\alpha(k) \left[m^2 \lambda_- (G_M^* - G_E^*) \varepsilon_{kQ}^{\alpha\mu} - G_E^* \varepsilon_{kQ}^{\alpha\beta} \varepsilon_{kQ}^{\beta\mu} - \frac{1}{2} G_C^* Q^\alpha k^\beta t_{QQ}^{\beta\mu} \right] u(k')$$

$$t_{AB}^{\alpha\beta} = A \cdot B \delta^{\alpha\beta} - B^\alpha A^\beta$$

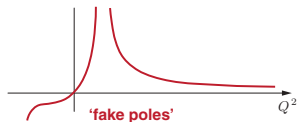
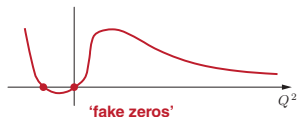
$$\varepsilon_{AB}^{\alpha\beta} = \gamma_5 \varepsilon^{\alpha\beta\gamma\delta} A^\gamma B^\delta$$

Minimal tensor bases

$$\Gamma^{\mu\nu} = \sum_i c_i K_i^{\mu\nu} = \underbrace{\sum_i g_i G_i^{\mu\nu}}_G + \underbrace{\sum_j f_j X_j^{\mu\nu}}_T$$

Minimal basis: neither g_i, f_j
nor G_i, X_j become singular

Without minimal basis:

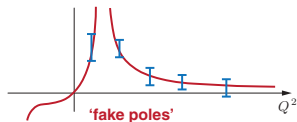
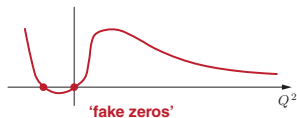


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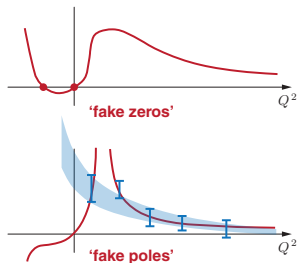


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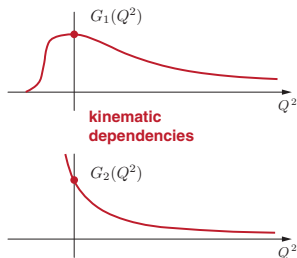


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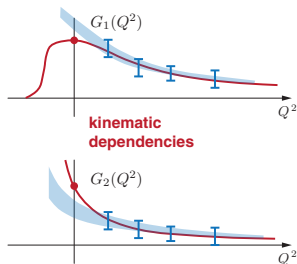


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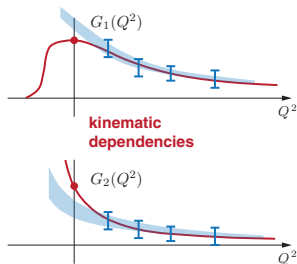


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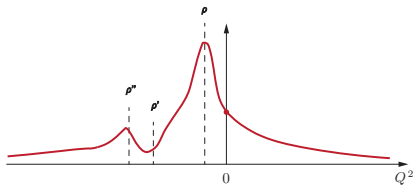
Minimal basis: neither g_i, f_j nor G_i, X_j become singular

Without minimal basis:



With minimal basis:

no kinematic dependencies,
only 'physical' poles and cuts!



Minimal tensor bases

$$\Gamma^{\mu\nu} = \sum_i c_i K_i^{\mu\nu} = \underbrace{\sum_i g_i G_i^{\mu\nu}}_G + \underbrace{\sum_j f_j X_j^{\mu\nu}}_T$$

Minimal basis: neither g_i, f_j nor G_i, X_j become singular

Transversality constraints:

$$\begin{aligned} Q'^{\mu} \Gamma^{\mu\nu} &= 0 \\ Q^{\nu} \Gamma^{\mu\nu} &= 0 \end{aligned} \Rightarrow \begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = 0$$

Minimal tensor bases

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\Rightarrow

$$\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix} \begin{bmatrix} c_1 \\ \vdots \\ c_n \end{bmatrix} = 0$$

Row-reduced echelon form:

$$\begin{array}{c|cccccccc} & \dim G & & & \dim T & & & & \\ \hline & \underbrace{1} & 0 & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & 0 & \underbrace{1} & 0 & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ & 0 & 0 & \underbrace{1} & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \\ \hline & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = 0$$

Minimal tensor bases

$$\Gamma^{\mu\nu} = \sum_i c_i K_i^{\mu\nu} = \underbrace{\sum_i \mathbf{g}_i G_i^{\mu\nu}}_G + \underbrace{\sum_j \mathbf{f}_j X_j^{\mu\nu}}_T$$

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A **minimal basis** exists, if

- by swapping columns
(= renaming basis tensors)
- adding / subtracting rows,
multiplying rows with scalars
(Gauss-Jordan elimination)

one can find a **row-reduced echelon form**

where $\begin{bmatrix} \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot & \cdot \end{bmatrix}$ is nonsingular in any kinematic limit

$$\begin{bmatrix} \overbrace{1 \ 0 \ 0}^{\dim G} & \overbrace{\cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot}^{\dim T} \\ 0 \ 1 \ 0 & \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \\ 0 \ 0 \ 1 & \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \ \cdot \\ \hline 0 \ 0 \ 0 & 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \\ 0 \ 0 \ 0 & 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \ 0 \end{bmatrix} \begin{bmatrix} \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \\ \cdot \end{bmatrix} = 0$$

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Minimal basis: neither g_i, f_j nor G_i, X_j become singular

Prerequisites:

- K_i must be **linearly** and **kinematically** independent
- symmetries should be exploited beforehand \Rightarrow arrange K_i in **singlets**

e.g. $K_1 \dots K_5$, but $k \cdot Q K_3 = Q^2 K_4 + k^2 K_5$?

A **minimal basis** exists, if

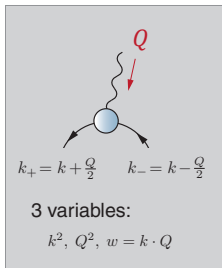
- by swapping columns (= renaming basis tensors)
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one can find a **row-reduced echelon form**

where $\begin{matrix} \bullet & \bullet & \bullet & \bullet & \bullet \\ \bullet & \bullet & \bullet & \bullet & \bullet \end{matrix}$ is nonsingular in any kinematic limit

$$\left[\begin{array}{ccc|ccccc} \text{dim G} & & & \text{dim T} & & & & & & & \\ \hline 1 & 0 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 1 & 0 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ 0 & 0 & 1 & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet & \bullet \\ \hline 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{array} \right] \begin{bmatrix} \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \\ \bullet \end{bmatrix} = 0$$

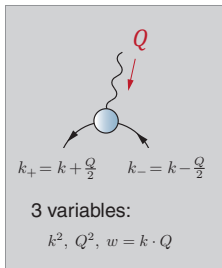
An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 Q^\mu$$

$$\bar{\Gamma}^\mu(k, Q) := \Gamma^\mu(-k, -Q) \stackrel{!}{=} -\Gamma^\mu(k, -Q) \quad (\text{charge conjugation})$$

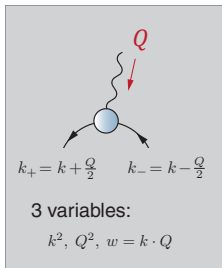
An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 (k \cdot Q) Q^\mu$$

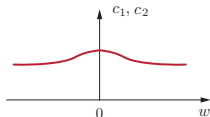
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An example

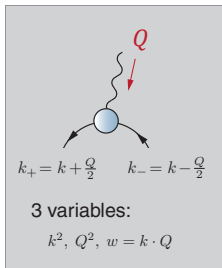


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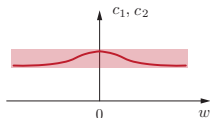


An example

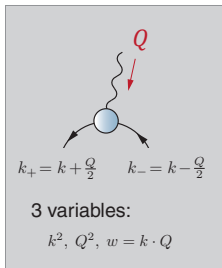


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An example

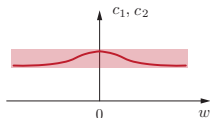


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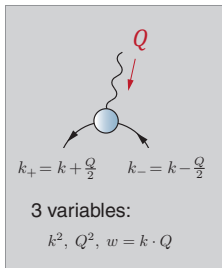
$$\bar{\Gamma}^\mu(k, Q) := \Gamma^\mu(-k, -Q) \stackrel{!}{=} -\Gamma^\mu(k, -Q) \quad (\text{charge conjugation})$$

Transversality:

$$Q^\mu \Gamma_\mu = c_1 w + c_2 w Q^2 = 0$$



An example



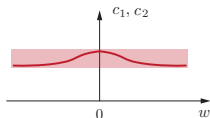
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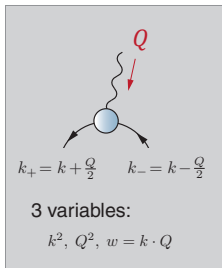
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$$\Rightarrow \begin{bmatrix} w & w Q^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$



An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 (k \cdot Q) Q^\mu$$

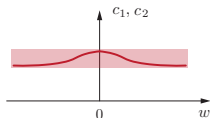
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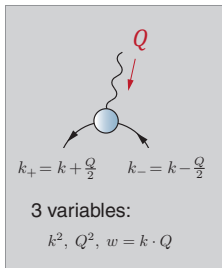
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$$\Rightarrow \begin{bmatrix} w & w Q^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$

$$\Rightarrow \begin{bmatrix} 1 & Q^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$



An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 (k \cdot Q) Q^\mu$$

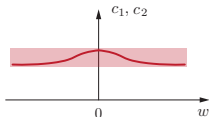
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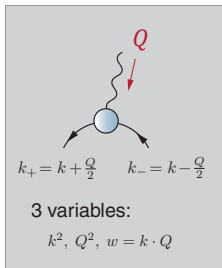
$$\Rightarrow \begin{bmatrix} 1 & Q^2 \end{bmatrix} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix} = 0$$



but **not**

$$\begin{bmatrix} 1 & \frac{1}{Q^2} \end{bmatrix} \begin{bmatrix} c_2 \\ c_1 \end{bmatrix} = 0 \quad !!$$

An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 (k \cdot Q) Q^\mu$$

$$\bar{\Gamma}^\mu(k, Q) := \Gamma^\mu(-k, -Q) \stackrel{!}{=} -\Gamma^\mu(k, -Q) \quad (\text{charge conjugation})$$

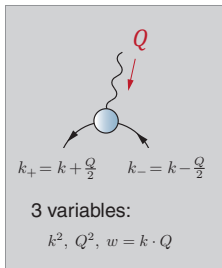
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$$Q^\mu \Gamma_\mu = c_1 w + c_2 w Q^2 = 0$$

$$\begin{aligned} c_1 = -c_2 Q^2 &\Rightarrow \Gamma^\mu = -c_2 (Q^2 k^\mu - w Q^\mu) \\ &= -c_2 (Q^2 \delta^{\mu\nu} - Q^\mu Q^\nu) k^\nu \\ &= -c_2 t_{QQ}^{\mu\nu} k^\nu \end{aligned}$$

$$\Rightarrow \Gamma^\mu(k, Q) = \underbrace{g_1}_{\mathbf{G}} k^\mu + \underbrace{f_1}_{\mathbf{T}} t_{QQ}^{\mu\nu} k^\nu$$

An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 (k \cdot Q) Q^\mu$$

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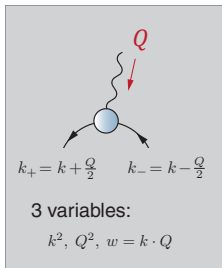
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Ward-Takahashi identity only affects **G**:

$$\begin{aligned} Q^\mu \Gamma_\mu &= D(k_+)^{-1} - D(k_-)^{-1} = g_1 w &\Rightarrow \Gamma^\mu(k, Q) &= \underbrace{g_1 k^\mu}_{\mathbf{G}} + \underbrace{f_1 t_{QQ}^{\mu\nu} k^\nu}_{\mathbf{T}} \\ \Rightarrow g_1 &= 2 \frac{D(k_+)^{-1} - D(k_-)^{-1}}{k_+^2 - k_-^2} = 2\Delta \end{aligned}$$

An example



$$\Gamma^\mu(k, Q) = c_1 k^\mu + c_2 (k \cdot Q) Q^\mu$$

$$\bar{\Gamma}^\mu(k, Q) := \Gamma^\mu(-k, -Q) \stackrel{!}{=} -\Gamma^\mu(k, -Q) \quad (\text{charge conjugation})$$

Transversality:

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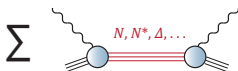
Transverse-longitudinal separation?

$$\Gamma^\mu(k, Q) = \tilde{g}_1 w Q^\mu + \tilde{f}_1 t_{QQ}^{\mu\nu} k^\nu \quad \Rightarrow \quad \Gamma^\mu(k, Q) = \underbrace{g_1 k^\mu}_{\mathbf{G}} + \underbrace{f_1 t_{QQ}^{\mu\nu} k^\nu}_{\mathbf{T}}$$

$$Q^\mu \Gamma_\mu = D(k_+)^{-1} - D(k_-)^{-1} = \tilde{g}_1 w Q^2$$

$$\Rightarrow \tilde{g}_1 = \frac{2\Delta}{Q^2} \quad \Rightarrow \quad \tilde{f}_1 = f_1 + \frac{2\Delta}{Q^2} \quad \Rightarrow \quad \text{both kinematically dependent and singular!}$$

Nucleon resonances



$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940) N(1440) N(1710) N(1880)	N(1720) N(1900)	N(1535) N(1650) N(1895)	N(1520) N(1700) N(1875)
Δ(1910)	Δ(1232) Δ(1600) Δ(1920)	Δ(1620) Δ(1900)	Δ(1700) Δ(1940)

Need em. transition FFs

But vertices are half offshell:
need 'consistent couplings'

[Pascalutsa, Timmermans, PRC 60 \(1999\)](#)

- **em gauge invariance:** $Q^\mu \Gamma^{\alpha\mu} = 0$
- **spin-3/2 gauge invariance:** $k^\alpha \Gamma^{\alpha\mu} = 0$
- invariance under **point transformations:** $\gamma^\alpha \Gamma^{\alpha\mu} = 0$
- no kinematic dependencies, **"minimal" basis**

Most general offshell vertices

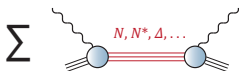
satisfying these constraints:

[GE, Ramalho, 1806.04579](#)

$$\frac{1}{2}^+ \rightarrow \frac{1}{2}^\pm : \Gamma^\mu = \begin{bmatrix} 1 \\ \gamma_5 \end{bmatrix} \sum_{i=1}^8 F_i T_i^\mu \left\{ \begin{array}{l} t_{QQ}^{\mu\nu} \gamma^\nu \\ [\gamma^\mu, \not{Q}] \\ \dots \end{array} \right.$$

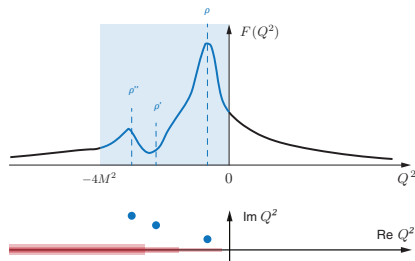
$$\frac{1}{2}^+ \rightarrow \frac{3}{2}^\pm : \Gamma^{\alpha\mu} = \begin{bmatrix} \gamma_5 \\ 1 \end{bmatrix} \sum_{i=1}^{12} F_i T_i^{\alpha\mu} \left\{ \begin{array}{l} \epsilon_{kQ}^{\alpha\mu} \\ t_{kQ}^{\alpha\mu} \\ it_{k\gamma}^{\alpha\beta} t_{Q\mu}^{\beta\mu} \\ \dots \end{array} \right.$$

Nucleon resonances

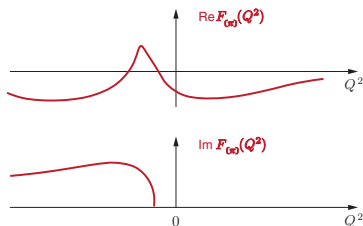


Constraint-free transition FFs:
only physical poles and cuts

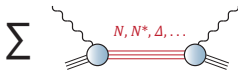
- ρ **poles** \sim monotonous behavior
(+ zero crossings for excited states)



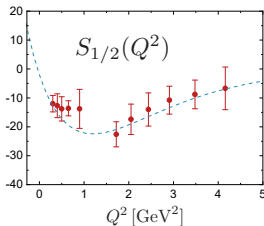
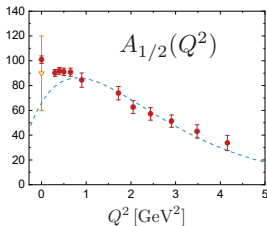
- Non-monotonicity at low Q^2
 \sim signature for cuts ($\rho \rightarrow \pi\pi$, etc.):
meson cloud



Nucleon resonances



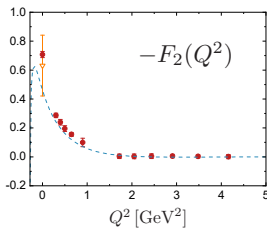
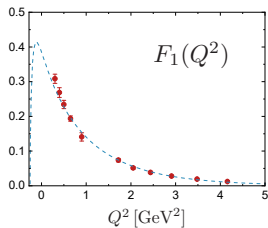
$J^P = \frac{1}{2}^+$	$\frac{3}{2}^+$	$\frac{1}{2}^-$	$\frac{3}{2}^-$
N(940) N(1440) N(1710) N(1880)	N(1720) N(1900)	N(1535) N(1650) N(1895)	N(1520) N(1700) N(1875)
Δ(1910)	Δ(1232) Δ(1600) Δ(1920)	Δ(1620) Δ(1900)	Δ(1700) Δ(1940)



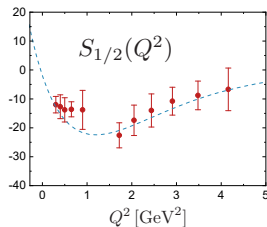
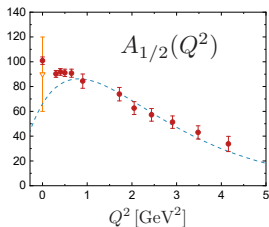
Example:
N(1535) helicity amplitudes

- PDG
- CLAS data
userweb.jlab.org/~mokeev/resonance_electrocouplings
- MAID
Tiator, Drechsel, Kamalov, Vanderhaeghen, EPJ 198 (2011)

Nucleon resonances



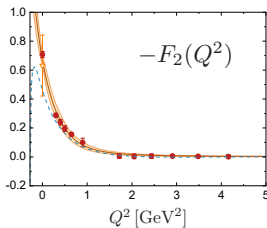
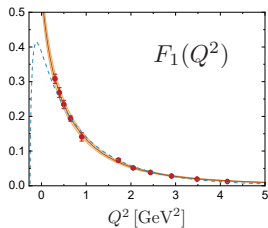
N(1535) transition FFs:
no kinematic constraints



Example:
N(1535) helicity amplitudes

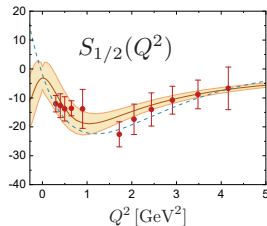
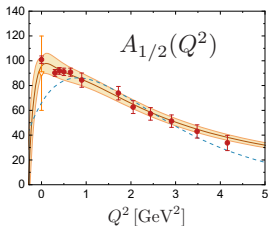
- PDG
- CLAS data
userweb.jlab.org/~mokeev/resonance_electrocouplings
- MAID
Tiator, Drechsel, Kamalov, Vanderhaeghen, EPJ 198 (2011)

Nucleon resonances



N(1535) transition FFs:
no kinematic constraints

Fit



Example:
N(1535) helicity amplitudes

PDG

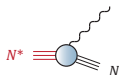
CLAS data

userweb.jlab.org/~mokeev/resonance_electrocouplings

MAID

Tiator, Drechsel, Kamalov, Vanderhaeghen, EPJ 198 (2011)

Nucleon resonances



$$J^P = \frac{1}{2}^+$$

$$\frac{3}{2}^+$$

$$\frac{1}{2}^-$$

$$\frac{3}{2}^-$$

N(940)

N(1720)

N(1535)

N(1520)

N(1440)

N(1900)

N(1650)

N(1700)

N(1710)

N(1895)

N(1875)

N(1880)

$\Delta(1910)$

$\Delta(1232)$

$\Delta(1620)$

$\Delta(1700)$

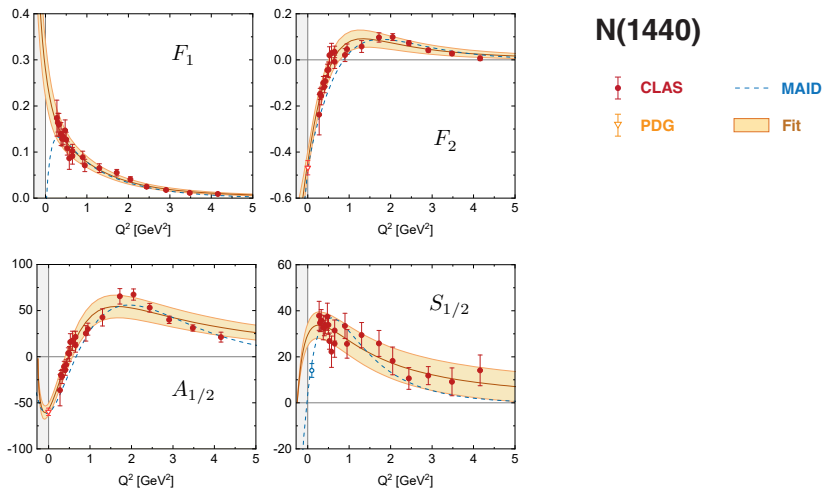
$\Delta(1600)$

$\Delta(1900)$

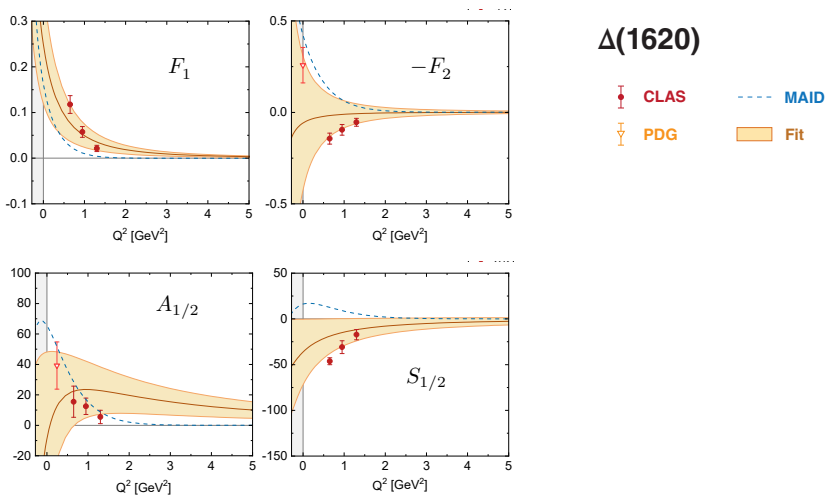
$\Delta(1940)$

$\Delta(1920)$

Nucleon resonances



Nucleon resonances



Nucleon resonances

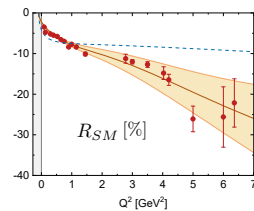
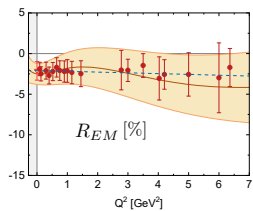
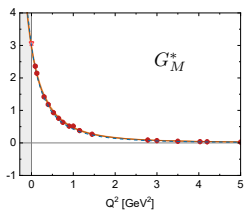
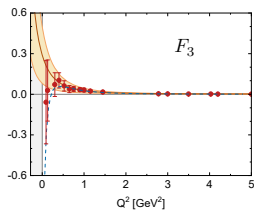
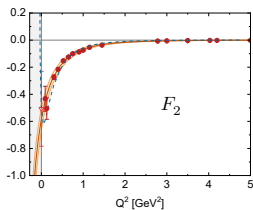
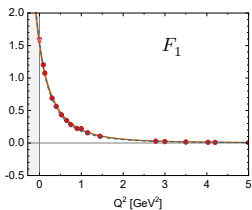
$\Delta(1232)$

CLAS

PDG

MAID

Fit



Nucleon resonances

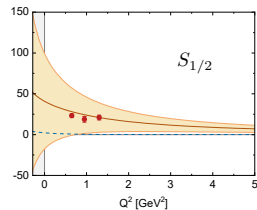
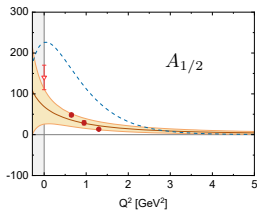
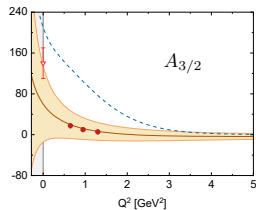
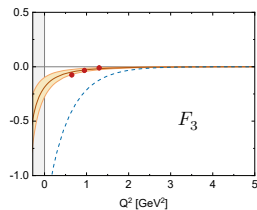
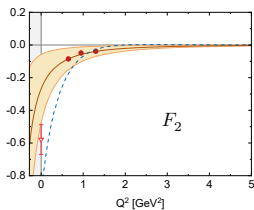
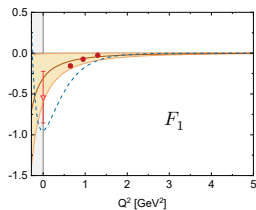
$\Delta(1700)$

CLAS

PDG

MAID

Fit



Nucleon resonances

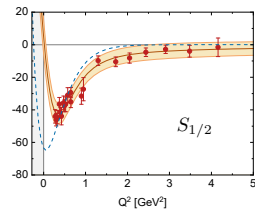
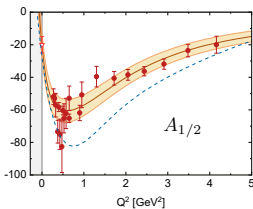
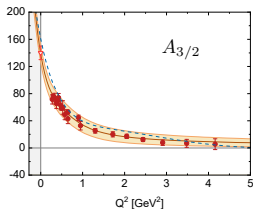
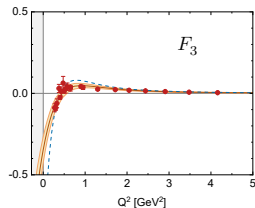
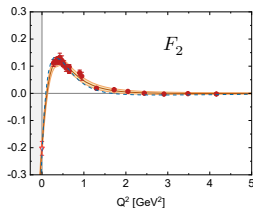
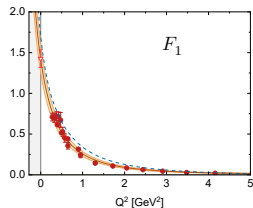
N(1520)

CLAS

PDG

MAID

Fit



Kinematics



$$= \sum_{i=1}^{18} c_i(\eta_+, \eta_-, \omega, \lambda) \bar{u}(p_f) X_i^{\mu\nu}(p, Q, Q') u(p_i)$$

18 CFFs

4 kinematic variables:

$$\eta_+ = \frac{Q^2 + Q'^2}{2m^2}$$

$$\eta_- = \frac{Q \cdot Q'}{m^2}$$

$$\omega = \frac{Q^2 - Q'^2}{2m^2}$$

$$\lambda = -\frac{p \cdot Q}{m^2}$$

18 Compton tensors, form minimal basis

- systematic derivation
- similar to Tarrach basis

[Tarrach, Nuovo Cim. A28 \(1975\)](#)

$$X'_i = U_{ij} X_j, \quad \det U = \text{const.}$$

- CFFs free of kinematics

$$X_1^{\mu\nu} = \frac{1}{m^4} t_{Q'p}^{\mu\alpha} t_{pQ}^{\alpha\nu},$$

$$X_2^{\mu\nu} = \frac{1}{m^2} t_{Q'Q}^{\mu\nu},$$

$$X_3^{\mu\nu} = \frac{1}{m^4} t_{Q'Q}^{\mu\alpha} t_{Q'Q}^{\alpha\nu},$$

$$X_4^{\mu\nu} = \frac{1}{m^6} t_{Q'Q}^{\mu\alpha} p^\alpha p^\beta t_{Q'Q}^{\beta\nu},$$

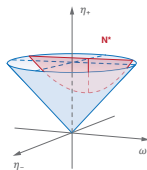
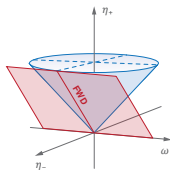
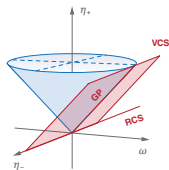
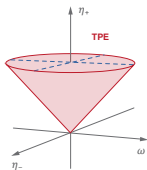
$$X_5^{\mu\nu} = \frac{\lambda}{m^4} (t_{Q'Q}^{\mu\alpha} t_{pQ}^{\alpha\nu} + t_{Q'p}^{\mu\alpha} t_{Q'Q}^{\alpha\nu}),$$

$$X_6^{\mu\nu} = \frac{1}{m^2} \varepsilon_{Q'Q}^{\mu\nu},$$

$$X_7^{\mu\nu} = \frac{1}{im^3} (t_{Q'Q}^{\mu\alpha} \varepsilon_{\gamma Q}^{\alpha\nu} - \varepsilon_{Q'\gamma}^{\mu\alpha} t_{Q'Q}^{\alpha\nu}),$$

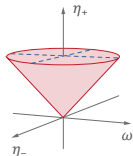
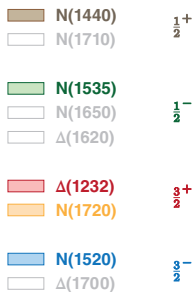
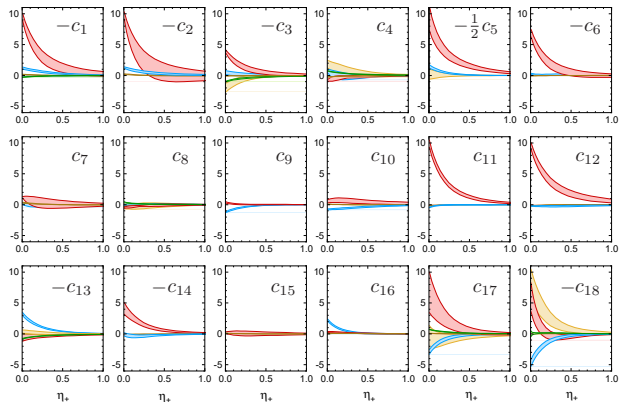
$$X_8^{\mu\nu} = \frac{\omega}{im^3} (t_{Q'Q}^{\mu\alpha} \varepsilon_{\gamma Q}^{\alpha\nu} + \varepsilon_{Q'\gamma}^{\mu\alpha} t_{Q'Q}^{\alpha\nu}),$$

⋮



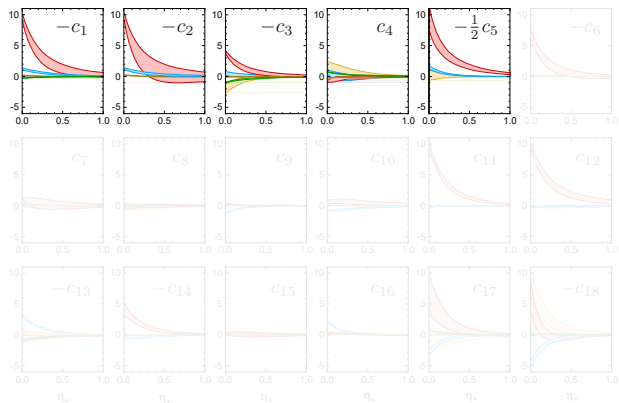
GE, Ramalho,
1806.04579

Compton form factors



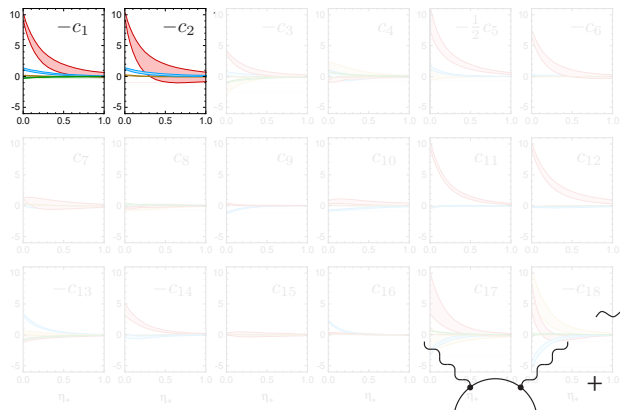
To be multiplied with
$$\frac{(m_R^2 - m^2)^2}{(s - m_R^2)(u - m_R^2)} = \frac{\delta^2}{(\eta_- + \delta)^2 - 4\lambda^2}$$

Compton form factors



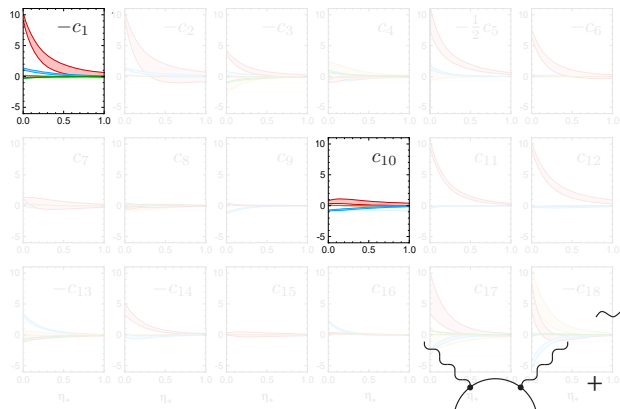
- CS on scalar particle

Compton form factors



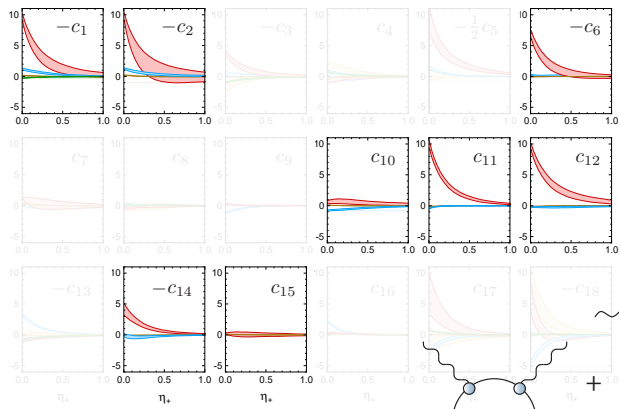
- CS on scalar particle
- CS on pointlike scalar

Compton form factors



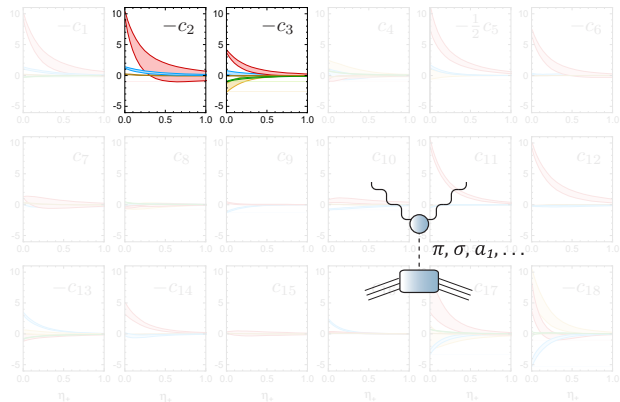
- CS on scalar particle
- CS on pointlike scalar
- CS on pointlike fermion

Compton form factors



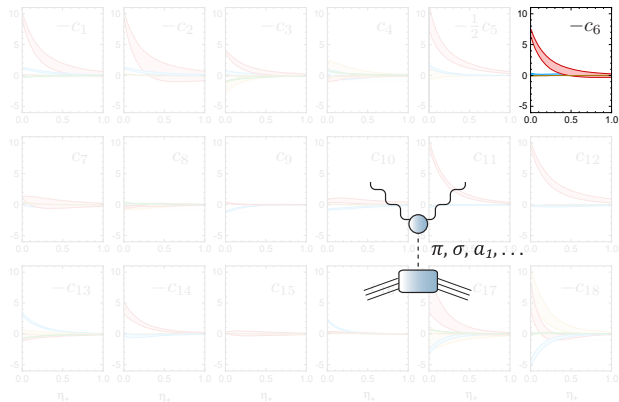
- CS on scalar particle
- CS on pointlike scalar
- CS on pointlike fermion
- **Nucleon Born poles** in s & u channel

Compton form factors



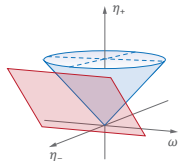
- CS on scalar particle
- CS on pointlike scalar
- CS on pointlike fermion
- Nucleon Born poles in s & u channel
- **Scalar pole** in t channel

Compton form factors

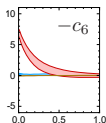
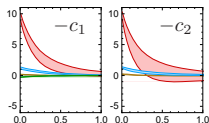


GE, Fischer, Weil, Williams,
PLB 774 (2017)

- CS on scalar particle
- CS on pointlike scalar
- CS on pointlike fermion
- Nucleon Born poles in s & u channel
- Scalar pole in t channel
- Pion pole in t channel ($\pi^0 \rightarrow \gamma^* \gamma^*$)

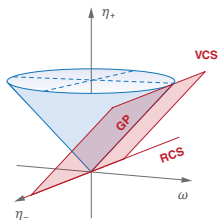
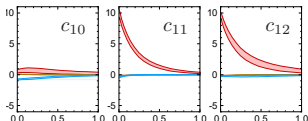


Polarizabilities



Scalar polarizabilities:

$$\begin{bmatrix} \alpha + \beta \\ \beta \end{bmatrix} = -\frac{\alpha_{\text{em}}}{m^3} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$

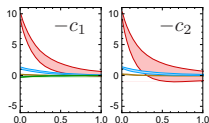


Spin polarizabilities:

$$\begin{bmatrix} \gamma_{E1E1} \\ \gamma_{M1M1} \\ \gamma_{E1M2} \\ \gamma_{M1E2} \end{bmatrix} = \frac{\alpha_{\text{em}}}{2m^4} \begin{bmatrix} c_6 + 4c_{11} - 4c_{12} \\ -c_6 - 2c_{10} + 4c_{12} \\ c_6 + 2c_{10} \\ -c_6 \end{bmatrix}$$

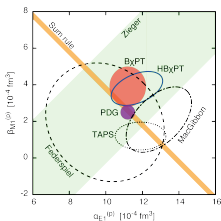
$$\begin{bmatrix} \gamma_0 \\ \gamma_\pi \end{bmatrix} = -\frac{2\alpha_{\text{em}}}{m^4} \begin{bmatrix} c_{11} \\ c_6 + c_{10} + c_{11} - 2c_{12} \end{bmatrix}$$

Polarizabilities



Scalar polarizabilities:

$$\begin{bmatrix} \alpha + \beta \\ \beta \end{bmatrix} = -\frac{\alpha_{\text{em}}}{m^3} \begin{bmatrix} c_1 \\ c_2 \end{bmatrix}$$



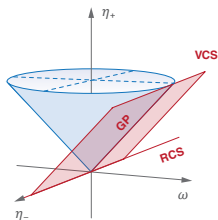
Hagelstein, Miskimen, Pascalutsa,
Prog. Part. Nucl. Phys. 88 (2016)

PDG:

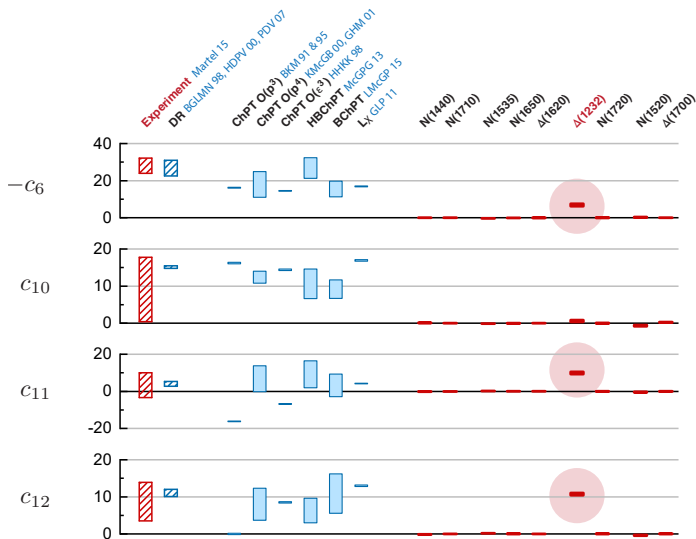
$$-c_1 = 20.3(4)$$

$$-c_2 = 3.7(6)$$

Large $\Delta(1232)$ contribution,
but also $N(1520)$ non-negligible

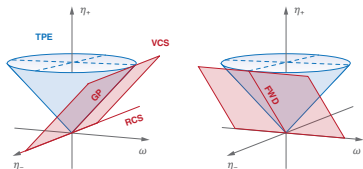


Spin polarizabilities

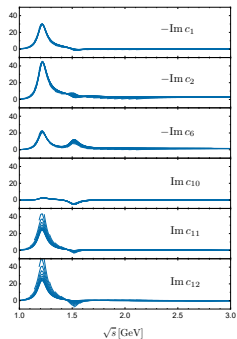


Only $\Delta(1232)$
important

General kinematics

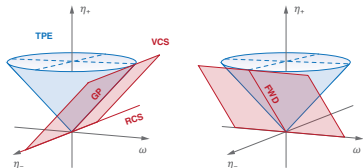


- Lorentz-invariant PW analyses?

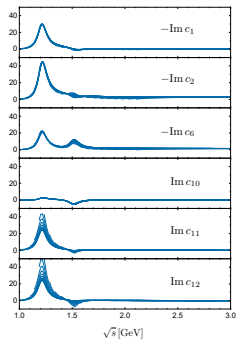


Resonance
contributions
in RCS

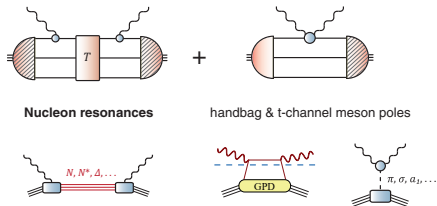
General kinematics



- Lorentz-invariant PW analyses?
- With minimal basis, only physical singularities; if no physical singularities, no momentum dependence!



Resonance contributions in RCS

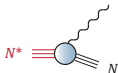


Compton scattering

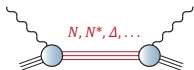


- kinematic variables
- tensor basis
- constraint-free **Compton FFs**

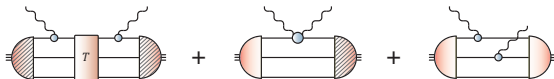
GE, Ramalho,
1806.04579



- general offshell transition vertices
- constraint-free **transition FFs**
- fits for transition FFs



- impact of higher resonances on Compton FFs
- only $\Delta(1232)$ and $N(1520)$ relevant for polarizabilities

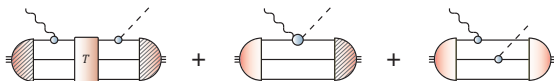
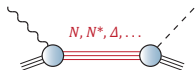


Meson electroproduction?



- kinematic variables
- tensor basis
- constraint-free **electroproduction amplitudes**

[GE, Sanchis-Alepuz, Williams, Alkofer, Fischer, PPNP 91 \(2016\)](#)



How important is the “**QCD background**”?