

# Emergent mass and the critical phenomena

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# Particle Data Group



Citation: C. Patrignani *et al.* (Particle Data Group), Chin. Phys. C, 40, 100001 (2016) and 2017 update

**g**  
or gluon

$$I(J^P) = 0(1^-)$$

SU(3) color octet

Mass  $m = 0$ .

Theoretical value. A mass as large as a few MeV may not be precluded, see YNDURAIN 95.

| VALUE   | DOCUMENT ID | TECN     | COMMENT       |
|---|-------------|----------|---------------|
| ● ● ● We do not use the following data for averages, fits, limits, etc. ● ● ● |             |          |               |
|   | ABREU       | 92E DLPH | Spin 1, not 0 |
|   | ALEXANDER   | 91H OPAL | Spin 1, not 0 |
|   | BEHREND     | 82D CELL | Spin 1, not 0 |
|   | BERGER      | 80D PLUT | Spin 1, not 0 |
|   | BRANDELIK   | 80C TASS | Spin 1, not 0 |

## gluon REFERENCES

|           |     |              |                            |                  |
|-----------|-----|--------------|----------------------------|------------------|
| YNDURAIN  | 95  | PL B345 524  | F.J. Yndurain              | (MADU)           |
| ABREU     | 92E | PL B274 498  | P. Abreu <i>et al.</i>     | (DELPHI Collab.) |
| ALEXANDER | 91H | ZPHY C52 543 | G. Alexander <i>et al.</i> | (OPAL Collab.)   |
| BEHREND   | 82D | PL B110 329  | H.J. Behrend <i>et al.</i> | (CELLO Collab.)  |
| BERGER    | 80D | PL B97 459   | C. Berger <i>et al.</i>    | (PLUTO Collab.)  |
| BRANDELIK | 80C | PL B97 453   | R. Brandelik <i>et al.</i> | (TASSO Collab.)  |



# In QCD: Gluons become massive!



南开大学  
Nankai University

PHYSICAL REVIEW

VOLUME 125, NUMBER 1

JANUARY 1, 1962

## Gauge Invariance and Mass

JULIAN SCHWINGER

*Harvard University, Cambridge, Massachusetts, and University of California, Los Angeles, California*

(Received July 20, 1961)

It is argued that the gauge invariance of a vector field does not necessarily imply zero mass for an associated particle if the current vector coupling is sufficiently strong. This situation may permit a deeper understanding of nucleonic charge conservation as a manifestation of a gauge invariance, without the obvious conflict with experience that a massless particle entails.

- Schwinger  
1962

PHYSICAL REVIEW D

VOLUME 26, NUMBER 6

15 SEPTEMBER 1982

## Dynamical mass generation in continuum quantum chromodynamics

John M. Cornwall

*Department of Physics, University of California, Los Angeles, California 90024*

(Received 30 April 1982)

- Cornwall  
1982

$$\Delta_{\mu\nu}^{-1}(q) = \text{[Feynman diagrams (a) through (e)]}$$

$$\Pi_{\mu\nu}(q)$$

$$\Pi_{\mu\nu}(q) = P_{\mu\nu}(q)\Pi(q)$$

$$P_{\mu\nu}(q) = g_{\mu\nu} - q_\mu q_\nu / q^2$$

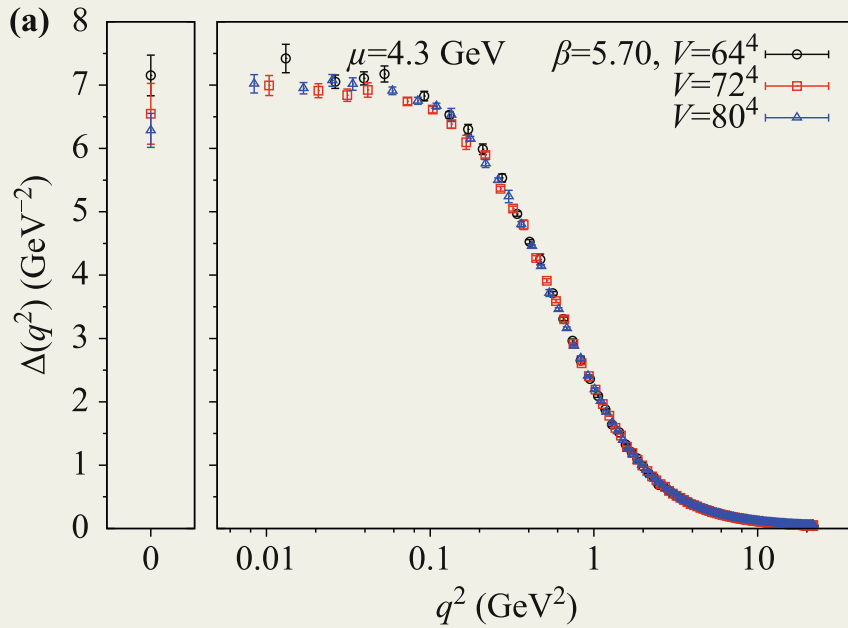
- Binosi &  
Papavassiliou

Phys. Rept. 479

(2009)1-152

Pinch Technique: Theory  
and Applications

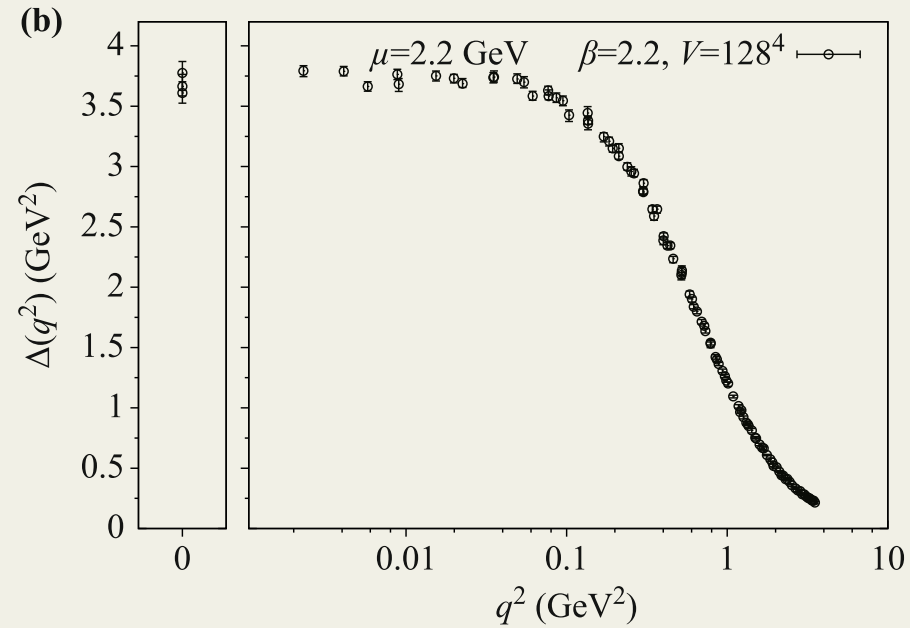
a) SU(3)



I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, Lattice gluodynamics computation of Landau gauge Green's functions in the deep infrared, *Phys. Lett. B* 676, 69 (2009), arXiv: 0901.0736 [hep-lat]

I. L. Bogolubsky, E. M. Ilgenfritz, M. Muller-Preussker, and A. Sternbeck, The Landau gauge gluon and ghost propagators in 4D SU(3) gluodynamics in large lattice volumes, arXiv: 0710.1968 [hep-lat]

b) SU(2)



A. Cucchieri and T. Mendes, Numerical test of the Gribov-Zwanziger scenario in Landau gauge, *PoS QCD-TNT 09*, 026 (2009), arXiv: 1001.2584 [hep-lat]



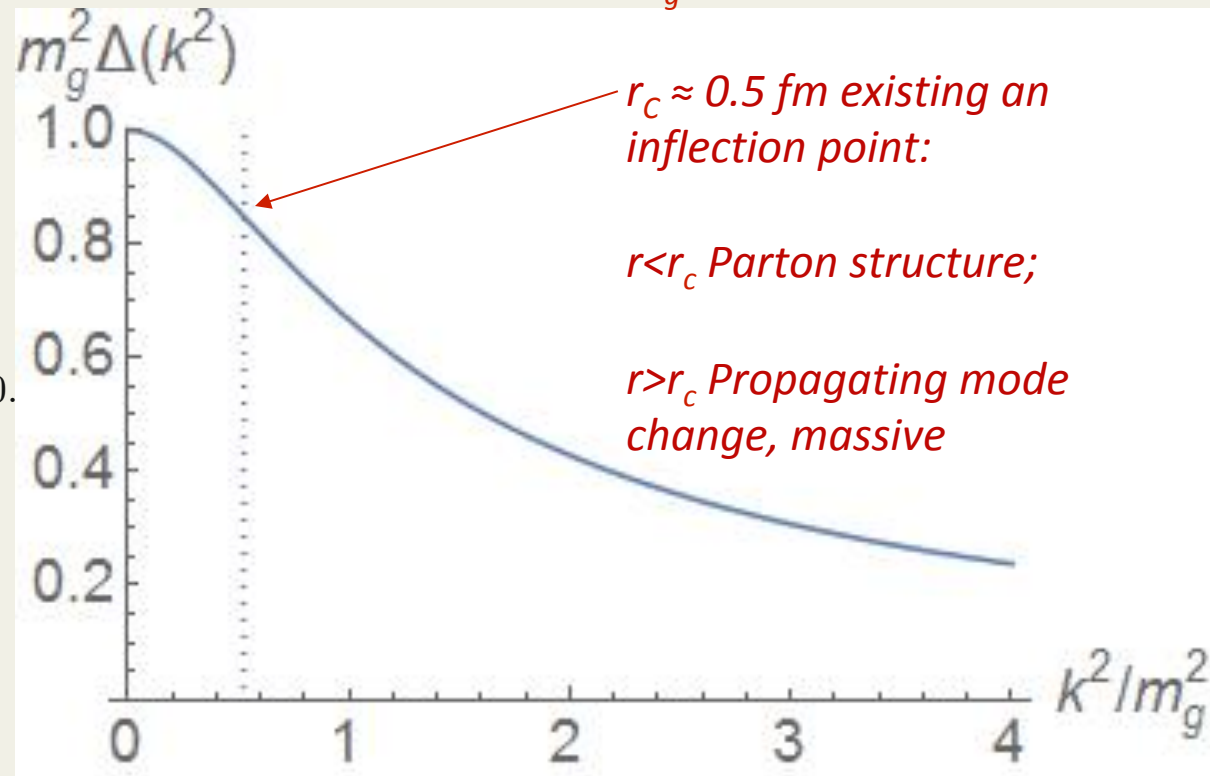
## Spectral representation

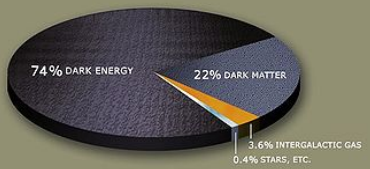
$$\Delta(q^2) = \int_0^\infty d\sigma \frac{\rho(\sigma)}{q^2 + \sigma},$$

if

$$\Delta''(q_\star^2) = 2 \int_0^\infty d\sigma \frac{\rho(\sigma)}{(q_\star^2 + \sigma)^3} = 0.$$

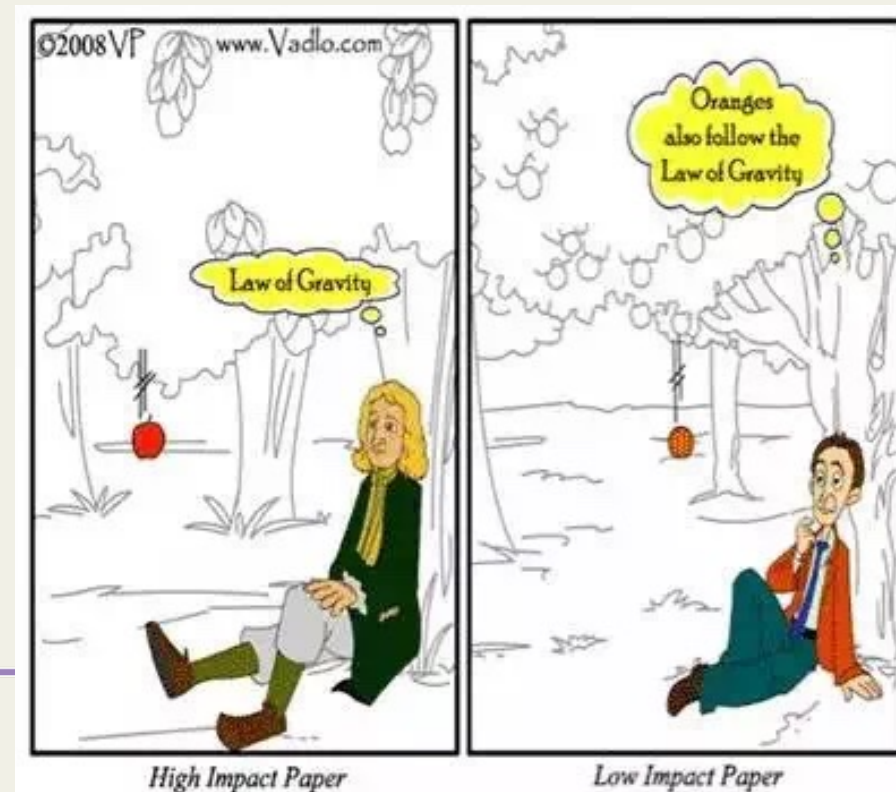
Non positive definite



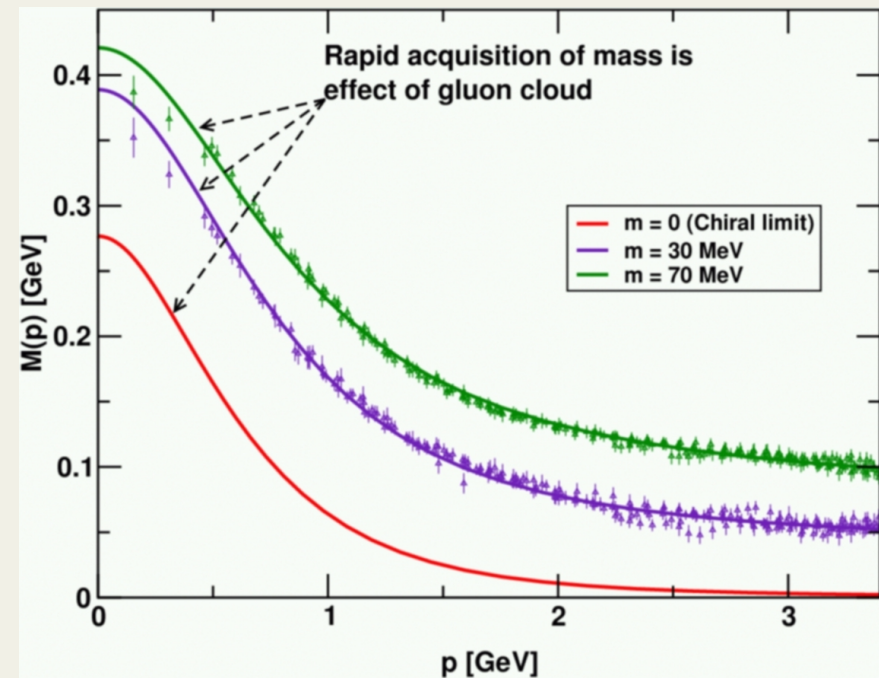
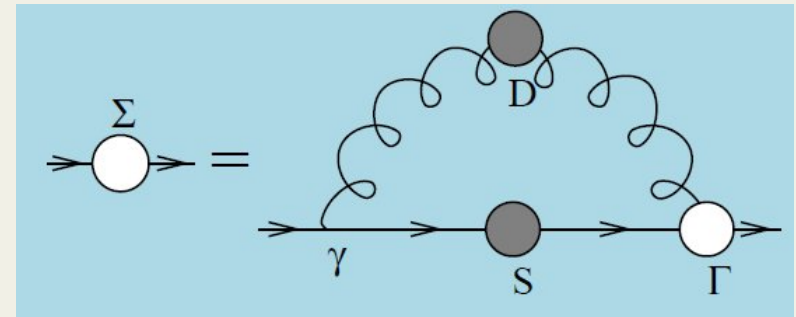


- .....
- Can we quantitatively understand quark and gluon confinement in quantum chromodynamics and the existence of a mass gap?

Quantum chromodynamics, or QCD, is the theory describing the strong nuclear force. Carried by gluons, it binds quarks into particles like protons and neutrons. Apparently, the tiny subparticles are permanently confined: one can't pull a quark or a gluon from a proton because the strong force gets stronger with distance and snaps them right back inside.



- Is a crucial emergent phenomenon in QCD
- Expressed in hadron wave functions not in vacuum condensates
- Contemporary theory indicates that it is responsible for more than 98% of the visible mass in the Universe; namely, given that classical massless-QCD is a conformally invariant theory, then DCSB is the origin of *mass from nothing*.
- **Dynamical**, not spontaneous
  - Add nothing to QCD ,  
*No Higgs field, nothing!*  
Effect achieved purely through quark+gluon dynamics.



## Pion's dichotomy

### Goldstone boson and Bound State

Maris, Roberts and Tandy, Phys. Lett. **B420**(1998) 267-273

#### ➤ Pion's Bethe-Salpeter amplitude

#### Solution of the Bethe-Salpeter equation

$$\Gamma_{\pi^j}(k; P) = \tau^{\pi^j} \gamma_5 \left[ iE_{\pi}(k; P) + \gamma \cdot P F_{\pi}(k; P) + \gamma \cdot k k \cdot P G_{\pi}(k; P) + \sigma_{\mu\nu} k_{\mu} P_{\nu} H_{\pi}(k; P) \right]$$

#### ➤ Dressed-quark propagator

$$S(p) = \frac{1}{i\gamma \cdot p A(p^2) + B(p^2)}$$

#### ➤ Axial-vector Ward-Takahashi identity entails(chiral limit)

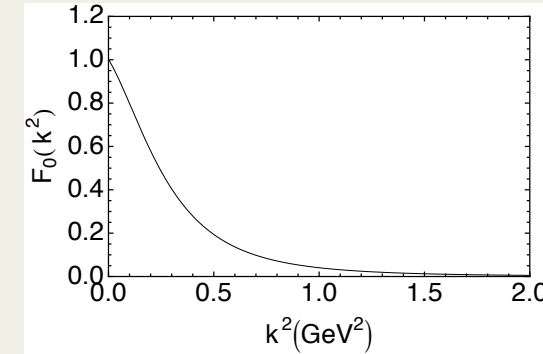
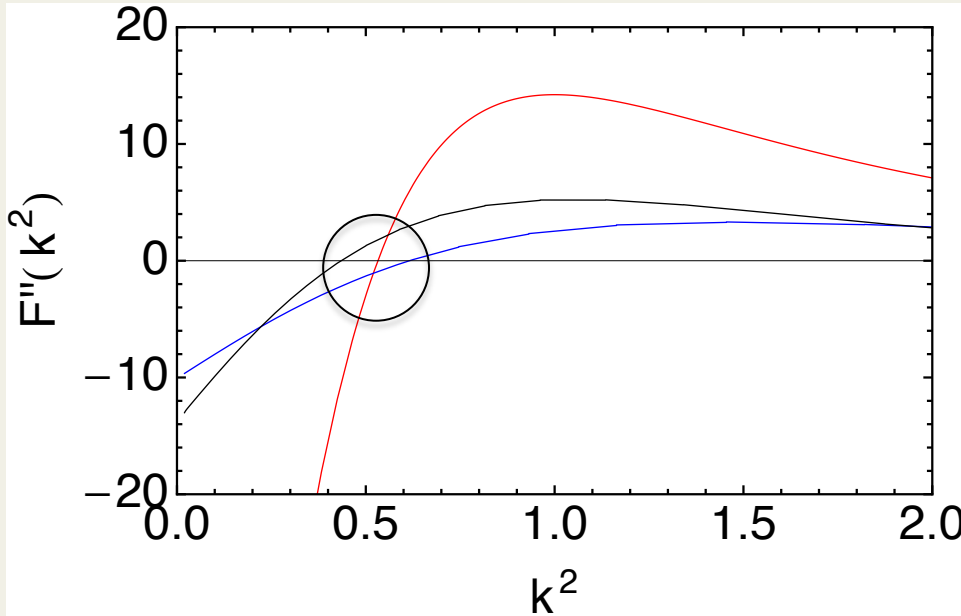
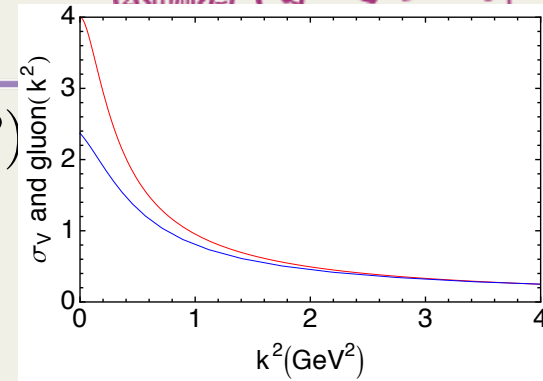
$$f_{\pi} E(k; P | P^2 = 0) = B(k^2) + (k \cdot P)^2 \frac{d^2 B(k^2)}{d^2 k^2} + \dots$$

- Given the dichotomy of pion the fine-tuning should not play any role in an explanation of pion properties;
- Descriptions of pion within frameworks that cannot faithfully express symmetries and their breaking patterns(such as constituent-quark models) are unreliable;
- Hence, pion properties are an almost direct measure of the dressed-quark mass function.

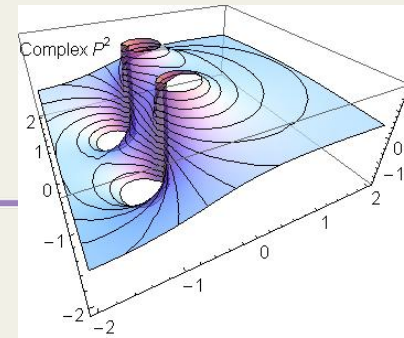
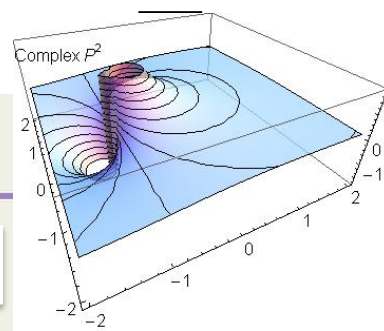
# Properties of Pion BS Wave Function

$$\chi_\pi(k; P) = \gamma_5 (iE(k; P) + \gamma \cdot PF(k; P) + \gamma \cdot kG(k; P))$$

- Infrared Behaviors



- Inflection points
- Black line: F function
- Red line: running gluon propagator
- Blue line: vector part of propagator





# Gluon mass scale--->Quark--->LFWFs

$m_g \approx 0.5 \text{ GeV}$  : A dynamical mass scale generation



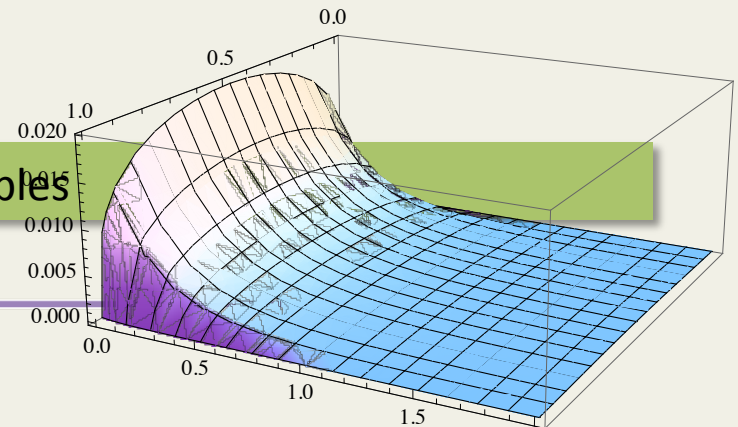
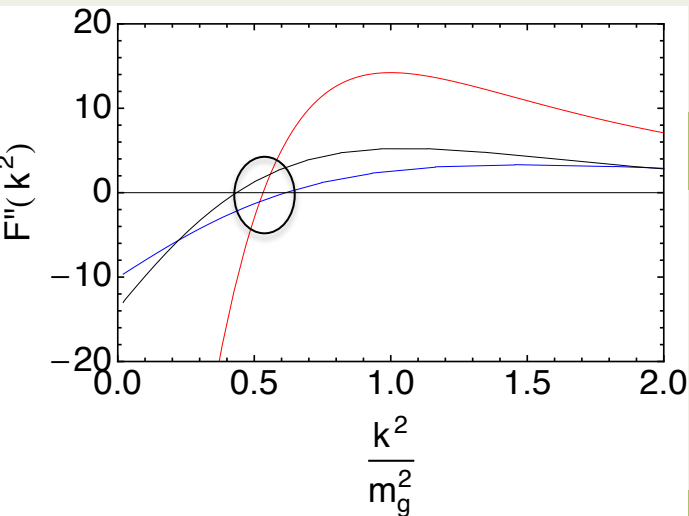
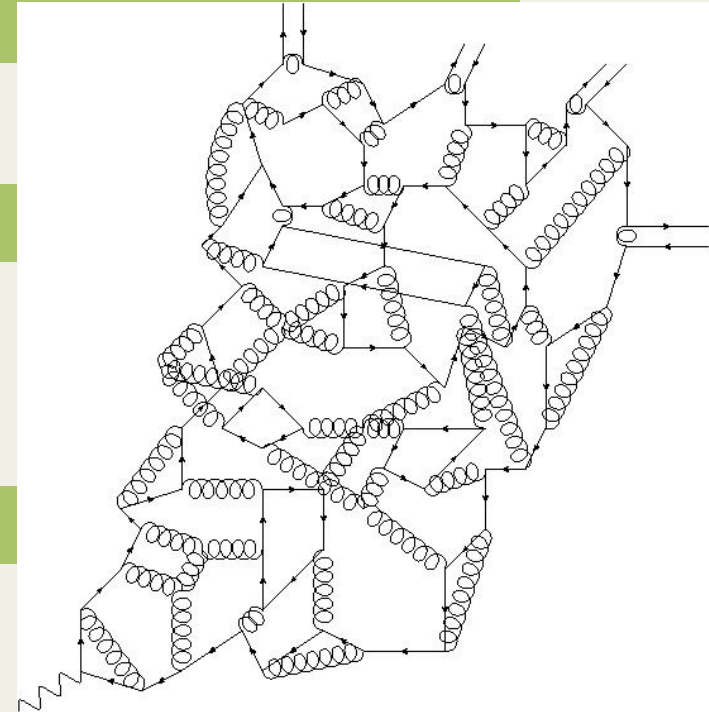
$l_c \approx 0.5$  : maximum wavelength




Quark and Hadron



LFWFs and observables



# Transition from Dynamical Chiral Symmetry Breaking to Explicit Chiral Symmetry Breaking

$$\mathcal{L}_{\text{QCD}} = \bar{\psi}_i (i(\gamma^\mu D_\mu)_{ij} - m \delta_{ij}) \psi_j - \frac{1}{4} G_{\mu\nu}^a G_a^{\mu\nu}$$


- Current quark mass, Higgs boson coupling;
- $m=0$ , chiral limit, physics controlled by the emergent phenomena: confinement and DCSB;
- The chiral symmetry is broken explicitly if the coupling of Higgs boson increasing;
- Question: could we describe the transition area and does it has the any relation with gluon mass  $m_g$ ?



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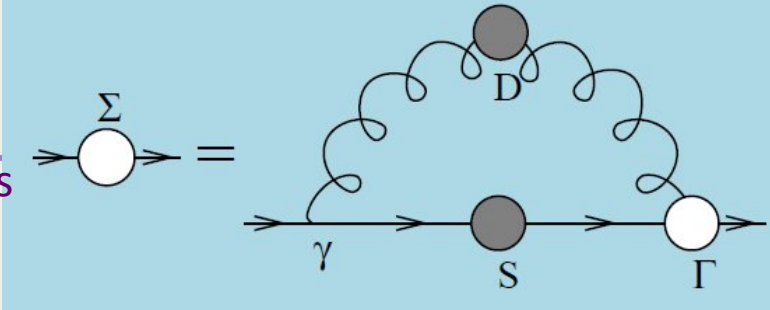
## Three examples:

- Multisolution of gap equation and the critical mass;
- The mass dependence of parton distribution amplitudes of pseudoscalar mesons
- Observables

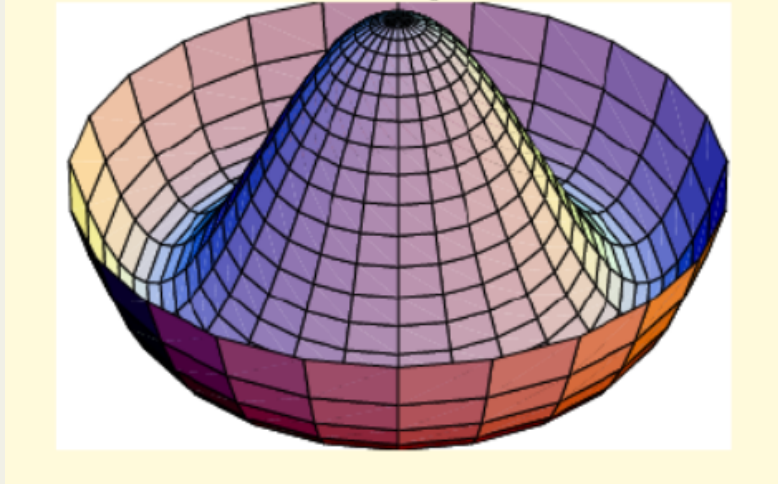


# Example-I

Multi-solutions of quark Gap equation and a critical mass



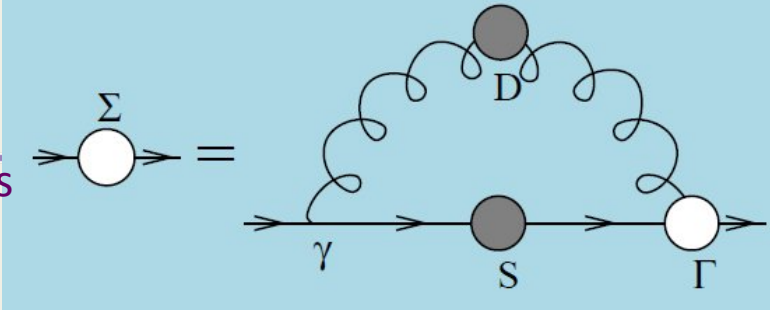
CJT effective potential



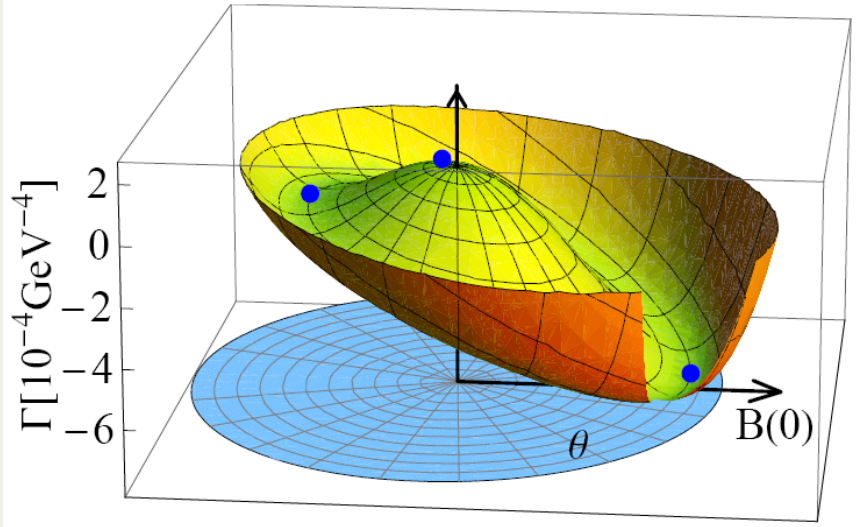
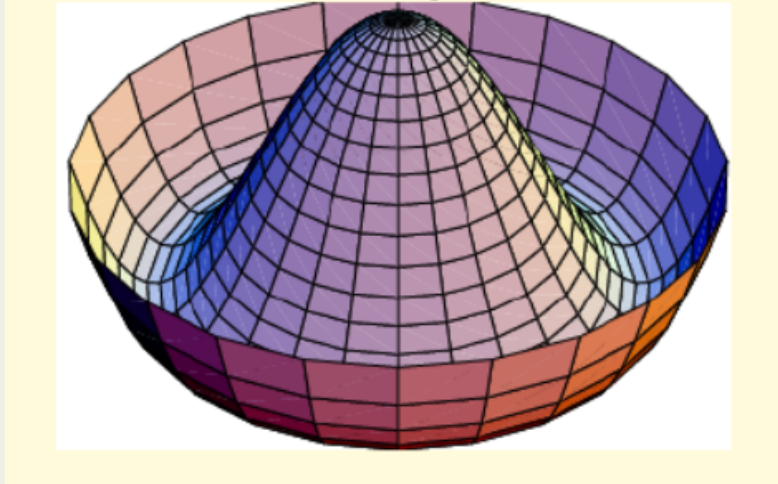
## In chiral limit

# Example-I

Multi-solutions of quark Gap equation and a critical mass



CJT effective potential



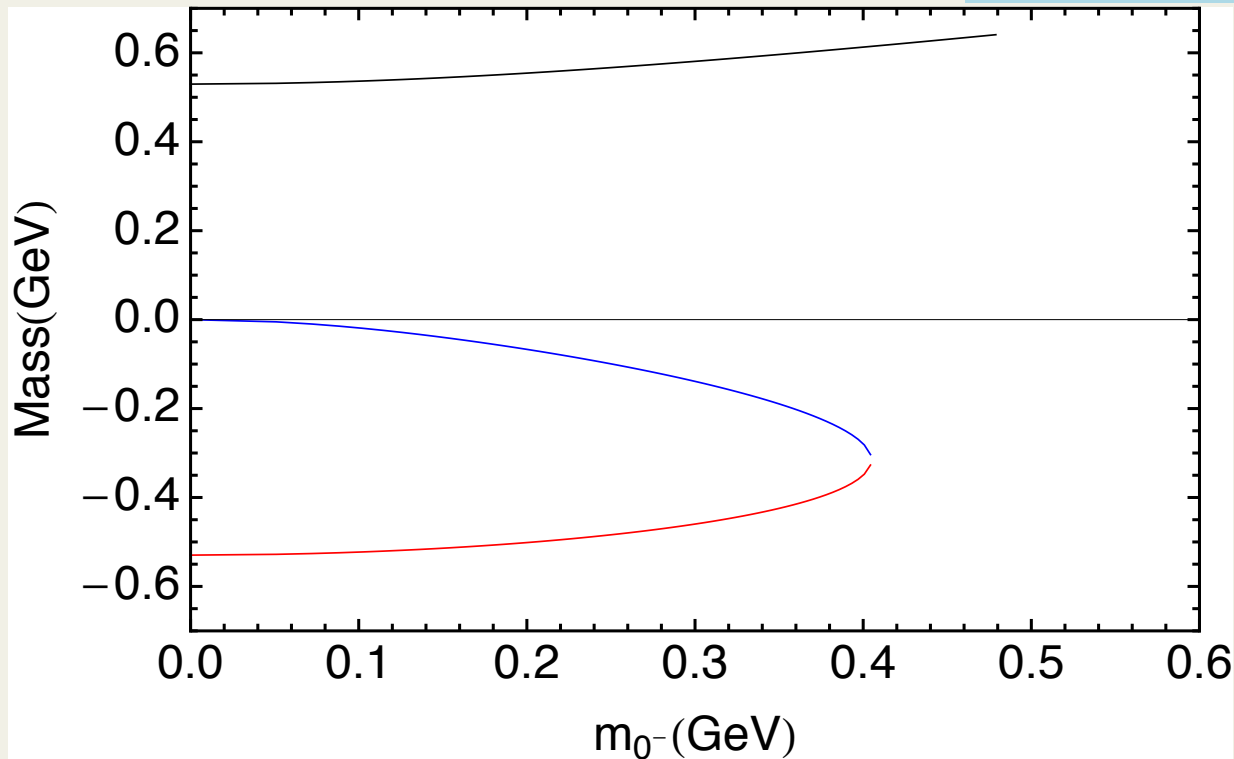
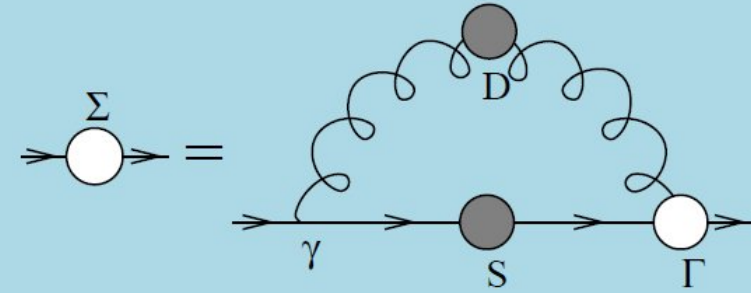
**In chiral limit**

**Beyond chiral limit**

# Critical mass

Dynamical chiral symmetry breaking and a critical mass

Lei Chang, *et al*, PRC 75 (2007) 015201



- Critical current quark mass, the related pseudoscalar meson mass is around  $\sim 0.4$  GeV;
- Below the critical mass there are three different quark propagators
- Analytical convergence radius, without considering log or fraction power

# Example-II

Mass dependence of pseudoscalar meson's parton distribution amplitudes

Consider (without gauge link...)

$$\begin{aligned} & \langle 0 | \psi(-x) \gamma_5 \gamma_\mu \psi(x) | \pi(q) \rangle \\ &= f_\pi \int_0^1 du e^{-i(2u-1)x \cdot q} \left( \tilde{q}_\mu \left[ \phi_\pi^{(2)}(u) + \frac{1}{4} x^2 \phi_\pi^{(4)}(u) + \mathcal{O}(x^4) \right] + \frac{1}{2} \frac{x_\mu}{x \cdot q} \psi_\pi^{(4)}(u) \right), \end{aligned}$$

Equally

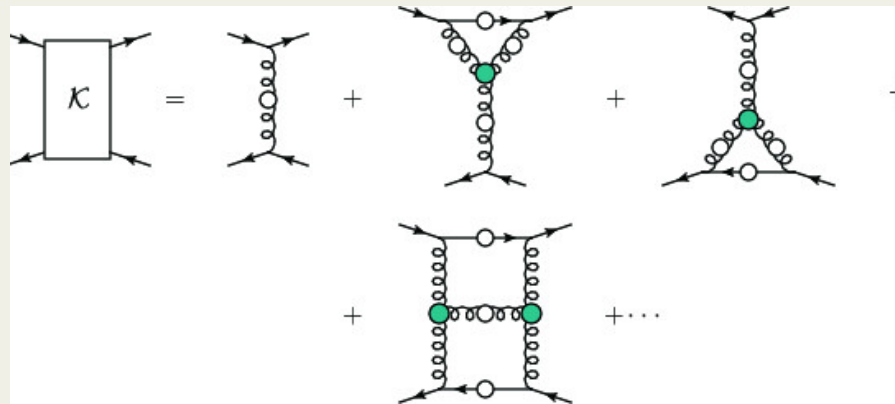
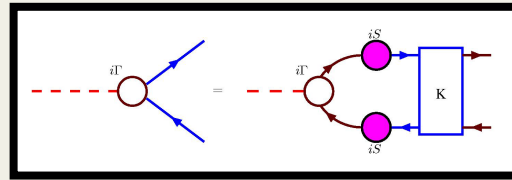
$$\langle 0 | \psi(-x) \gamma_5 \gamma_\mu \psi(x) | \pi(q) \rangle = Z_2 \text{tr}_{\text{CD}} \int_{dk}^\Lambda e^{-ix \cdot k - ix \cdot (k-q)} \gamma_5 \gamma_\mu S(k) \Gamma_\pi(k; q) S(k-q),$$

Definition for the twist-2

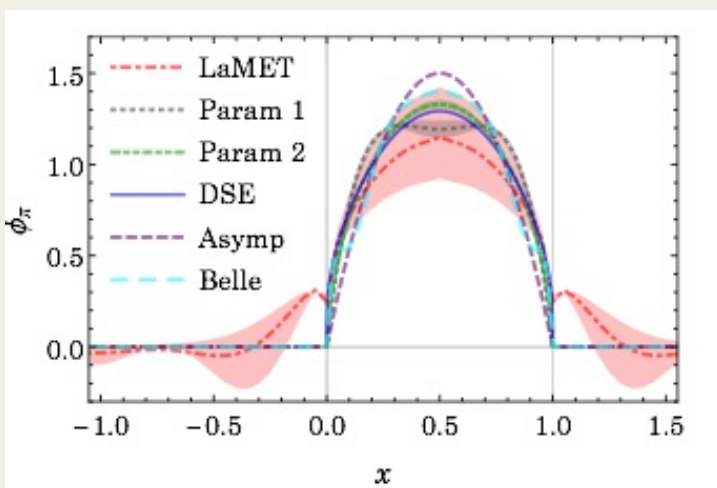
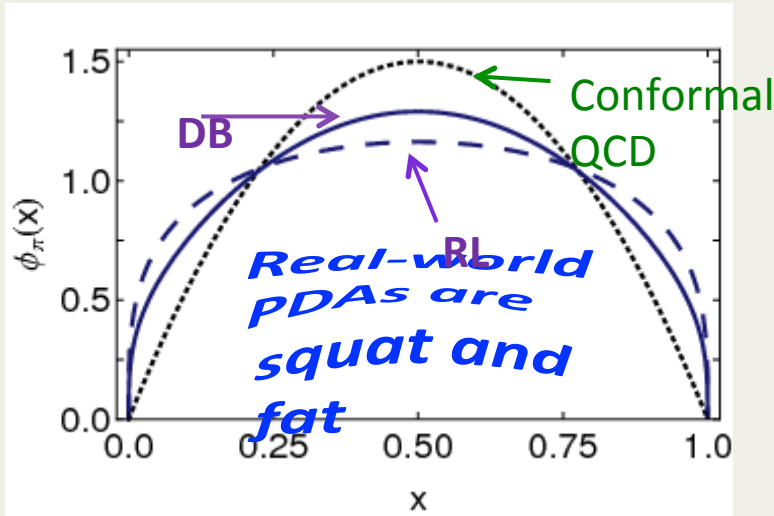
$$f_\pi \phi_\pi^{(2)}(u) = Z_2 \text{tr}_{\text{CD}} \int_{dk}^\Lambda \delta(u n \cdot q - n \cdot k) \gamma_5 \gamma \cdot n S(k) \Gamma_\pi(k; q) S(k-q),$$

- PDAs depends on the BS wave function...model calculation.
- Fix renormalization scale  $\mu=2\text{GeV}$
- Moments calculation
- Current quark mass dependence of PDA can be read from the corresponding BS wave function

## Bethe-Salpeter Equations



- Landau gauge
- Truncate kernel
- Fix renormalization point
- Modeling Interaction



Pion Distribution Amplitude from Lattice QCD

- Continuum-QCD prediction: marked broadening of  $\phi_\pi(x)$ , which owes to DCSB
- Scale evolution quite slow

Imaging dynamical chiral symmetry breaking: pion wave function on the light front, Lei Chang, *et al.*, Phys. Rev. Lett. 110 (2013) 132001

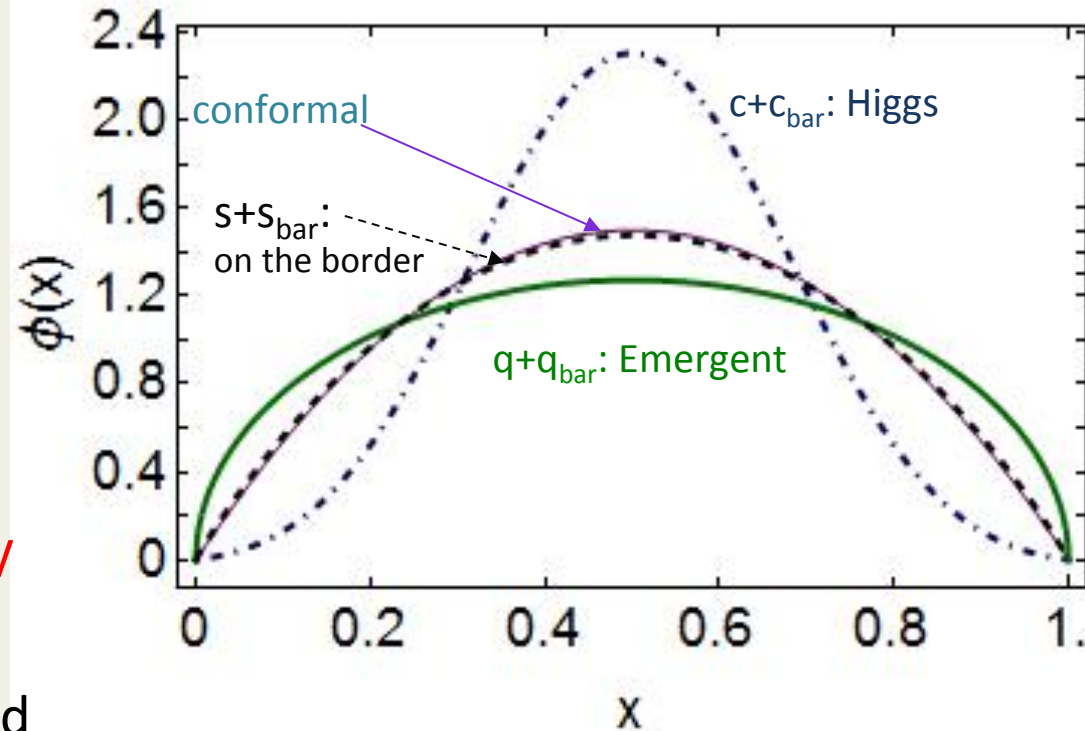
- Lattice simulation: arxiv: 1702.00008(林慧雯)
- X-d Ji, large momentum effective theory

DSE & IQCD predictions are practically indistinguishable; Favor no-humped behavior

# Emergent Mass vs. Higgs Mechanism

Parton distribution amplitudes of S-wave heavy-quarkonia  
Minghui Ding, et al, Phys. Lett. B **753** (2016) pp. 330-335

$\mu=2\text{GeV}$



- When does Higgs mechanism begin to influence mass generation?
- limit  $m_{\text{quark}} \rightarrow \infty$   
 $\varphi(x) \rightarrow \delta(x-1/2)$
- limit  $m_{\text{quark}} \rightarrow 0$   
 $\varphi(x) \sim (8/\pi) [x(1-x)]^{1/2}$
- Transition boundary lies just above  $m_{\text{strange}}$ , **the related ps meson mass is around 700MeV**
- Comparison between distributions of light-quarks and those involving strange-quarks is obvious place to find signals for strong-mass generation

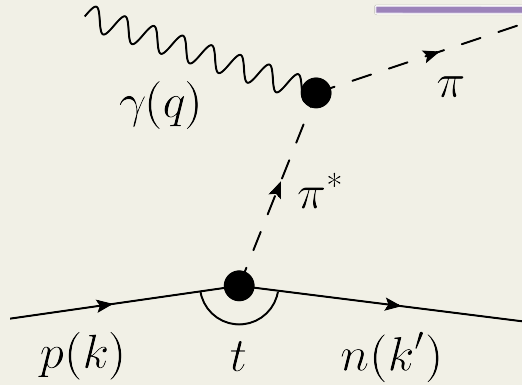
# Example-III

Mass dependence of electromagnetic form factors

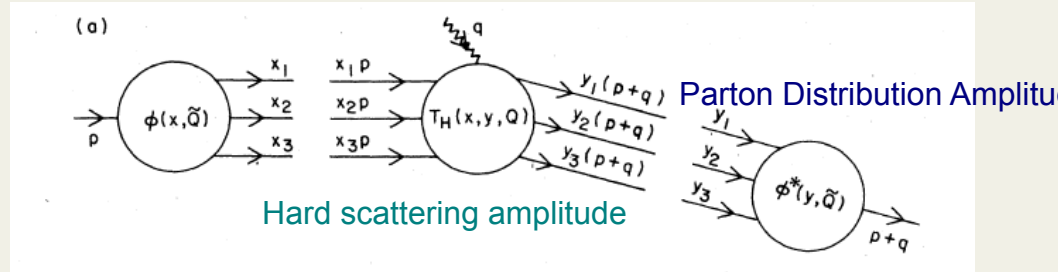
Muyang Chen's talk, Friday



# Measure pion elastic form factor in space-like region

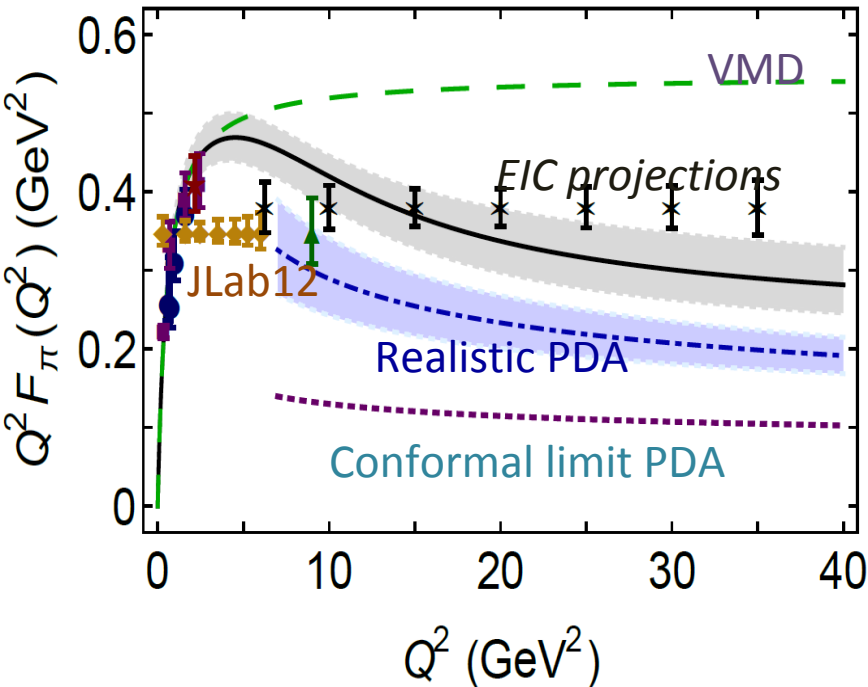


Jefferson Lab PAC 30 Proposal



(G.R. Farrar and D.R. Jackson, PRL43 (1979) 246;  
P. Lepage and S. Brodsky, PLB 87 (1979) 359)

Measurement of the Charged Pion Form Factor to High  $Q^2$



$$Q^2 F_\pi(Q^2) \stackrel{Q^2 \gg \Lambda_{\text{QCD}}^2}{\sim} 16 \pi f_\pi^2 \alpha_s(Q^2) w_\pi^2; \quad w_\pi = \frac{1}{3} \int_0^1 dx \frac{1}{x} \varphi_\pi(x)$$

Performing asymptotic valence-quark distribution amplitude  $6x(1-x)$

$$Q^2 F = 0.15 \quad \text{at } Q^2 = 4 \text{ GeV}^2$$

A factor 2.7 smaller than the empirical value  $0.41 \text{ GeV}^2$  quoted at  $Q^2 = 2.45 \text{ GeV}^2$

A factor 3 smaller than the case of BSE

# Messure $\pi_0$ - $\gamma$ transition form factor in space like region

★ Ultraviolet scale---pion gamma transition form factor at lowest-order pQCD and leading twist 2

$$Q^2 F(Q^2) = 2f_\pi \frac{1}{3} \int_0^1 \frac{\varphi_\pi^{(2)}(x)}{1-x}$$

See Khepani's talk(DSE approach)

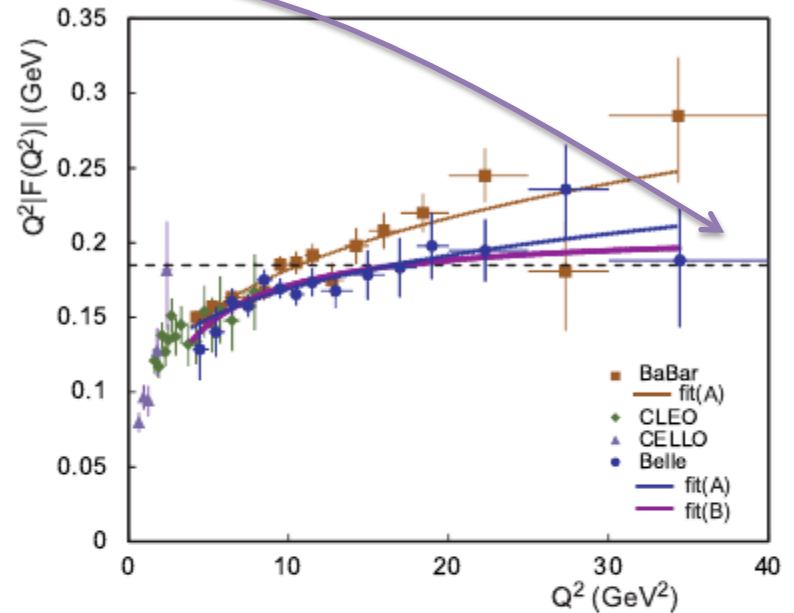


FIG. 24: Comparison of the results for the product  $Q^2|F(Q^2)|$  for the  $\pi^0$  from different experiments. The error bars are a quadratic sum of statistical and systematic uncertainties. For the Belle and BaBar results, only a  $Q^2$ -dependent systematic-error component is included. The two curves denoted fit(A) use the BaBar parameterization while the curve denoted fit(B) uses Eq.(23) (see the text). The dashed line shows the asymptotic prediction from pQCD ( $\sim 0.185$  GeV).

$$f_{0-} \quad w_{0-}$$

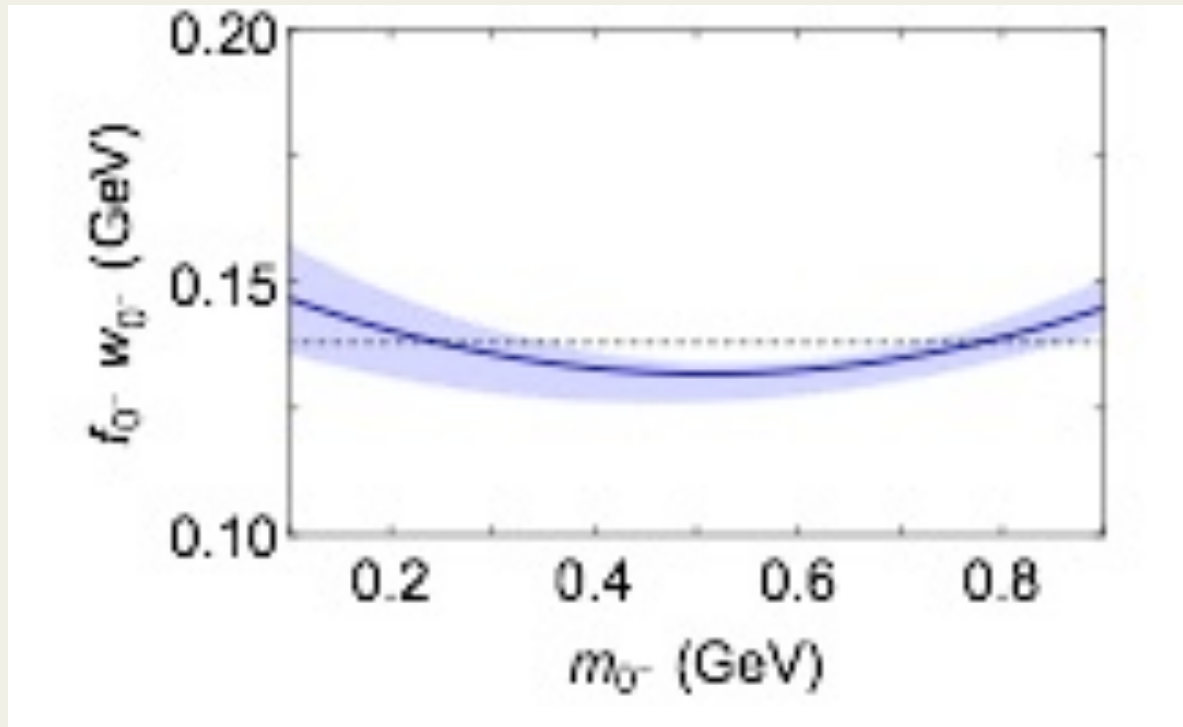
$$f_{0-} \varphi(x) = Z_2 \text{tr}_{CD} \int_{dk} \delta(n \cdot k - x n \cdot P) \gamma_5 \gamma \cdot n S(k) \Gamma_{0-}(k, k - P) S(k - P)$$

$$w_{0-} = \int_0^1 dx \frac{\varphi(x)}{3(1-x)}$$

$\mu=2\text{GeV}$   
without evolution

## Mass-dependence of pseudoscalar meson elastic form factors

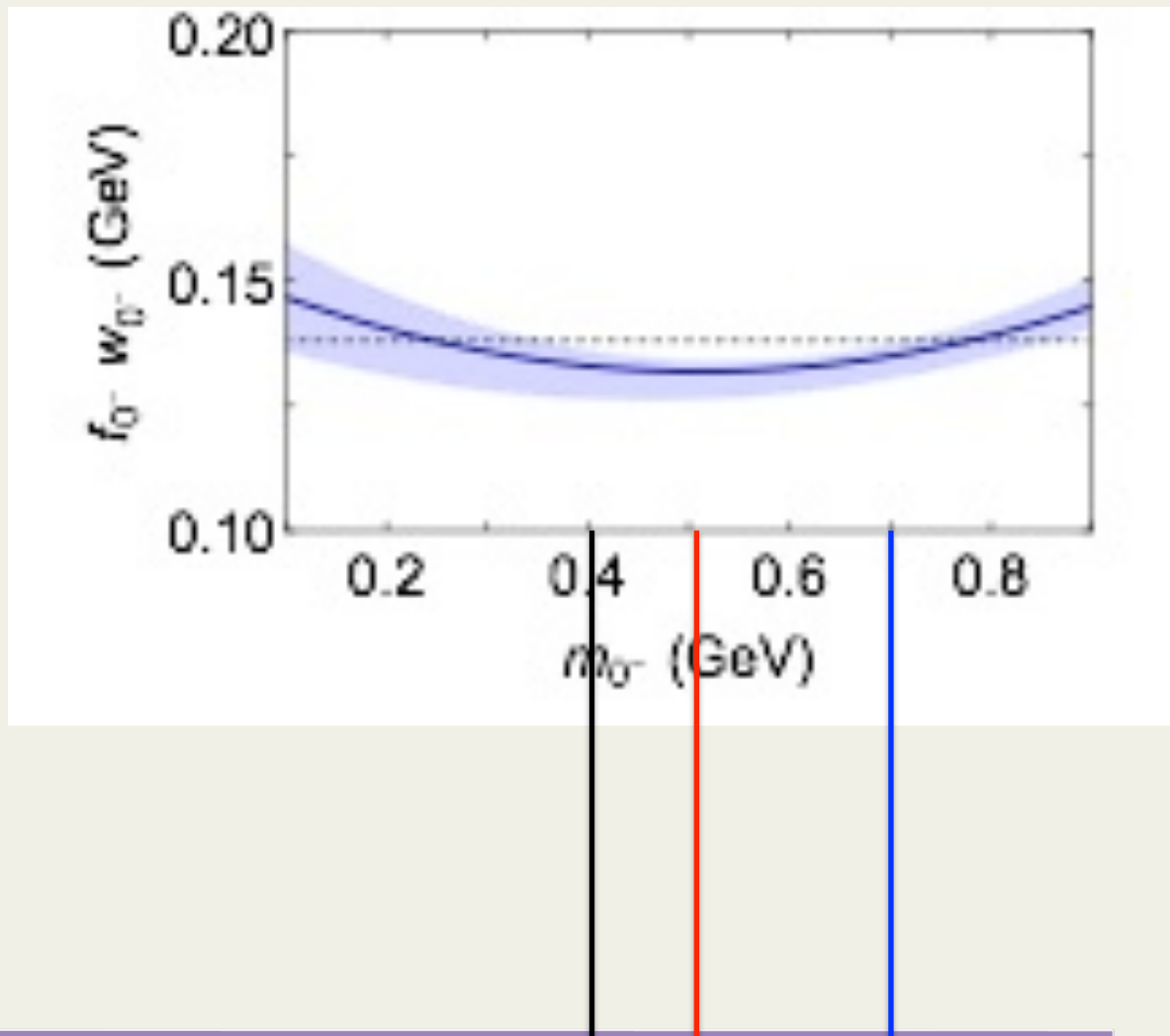
Muyang Chen, et.al, arXiv:1808.09461



- A consistent rainbow-ladder truncation
- Minimum exist around 0.5GeV, the gluon mass

# Mass-dependence of pseudoscalar meson elastic form factors

Muyang Chen, et.al, arXiv:1808.09461



- Gluon mass generation...
- QCD is well-defined at UV momenta owing to asymptotic freedom; QCD is IR finite, owing to dynamical generation of gluon mass-scale;
- Maximum wavelength for gluon/quark;
- Critical behavior for the wave function...
- Transition from dynamical breaking to explicit breaking.

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THANKS FOR YOUR  
ATTENTION