

NUCLEON TENSOR CHARGES

AND THEIR IMPLICATIONS



HUEY-WEN LIN

Outline

§ Consumer's Guide to Lattice Nucleon Calculations

§ Isovector Tensor Charges

↻ Impacts on transversity distribution

↻ BSM interactions in beta decay

§ Flavor-Dependent Tensor Charges

↻ nEDM applications



Lattice 101

§ Lattice QCD is an ideal theoretical tool for investigating the strong-coupling regime of quantum field theories

§ Physical observables are calculated from the path integral

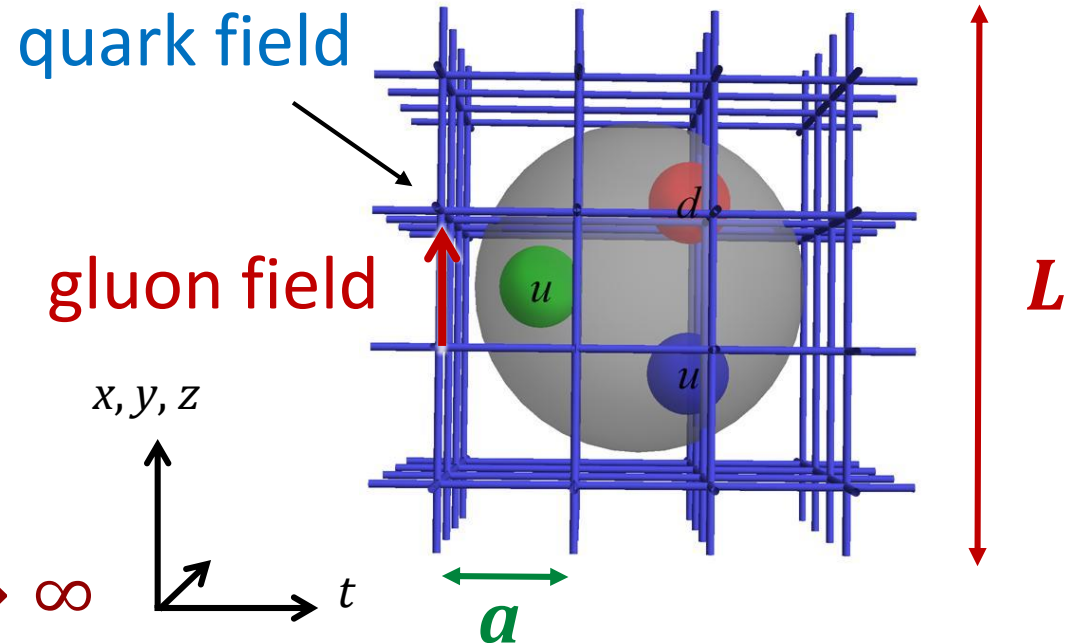
$$\langle 0|O(\bar{\psi}, \psi, A)|0\rangle = \frac{1}{Z} \int \mathcal{D}A \mathcal{D}\bar{\psi} \mathcal{D}\psi e^{iS(\bar{\psi}, \psi, A)} O(\bar{\psi}, \psi, A)$$

in **Euclidean** space

- ∞ Quark mass parameter (described by m_π)
- ∞ Impose a UV cutoff
discretize spacetime
- ∞ Impose an infrared cutoff
finite volume

§ Recover physical limit

$$m_\pi \rightarrow m_\pi^{\text{phys}}, \quad a \rightarrow 0, \quad L \rightarrow \infty$$



Are We There Yet?

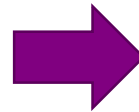
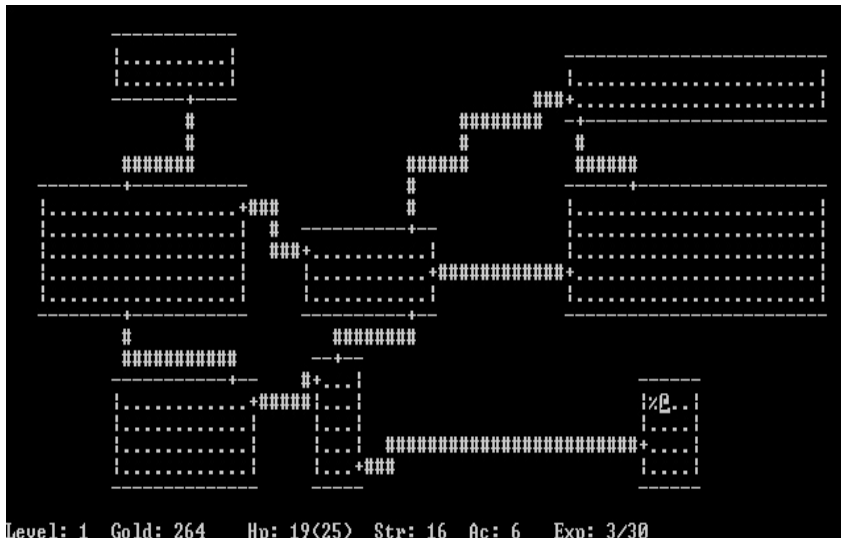
§ Lattice gauge theory was proposed in the 1970s by Wilson

∞ Why haven't we solved QCD yet?

§ Progress is limited by computational resources

1980s

Today



§ Greatly assisted by advances in algorithms

∞ Physical pion-mass ensembles are not uncommon!

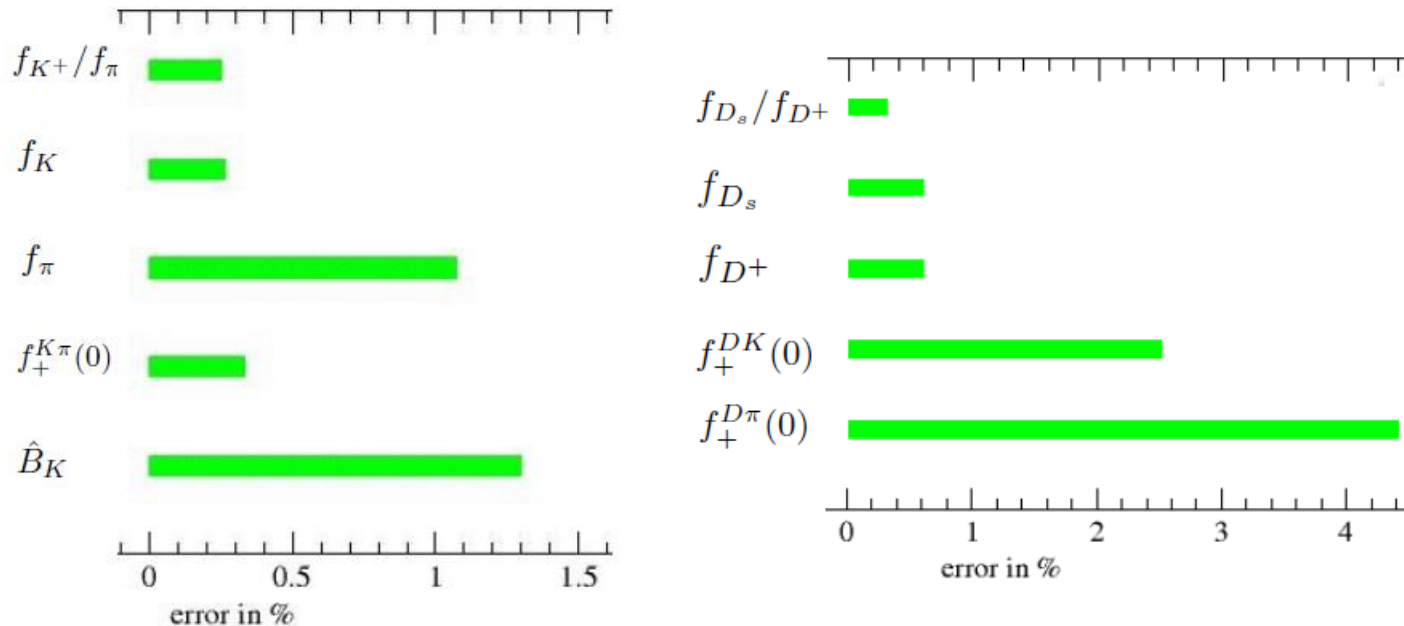
Successful Examples

§ Lattice flavor physics provides precise inputs from the SM

A. El-Khadra, Sep. 2015, INT workshop “QCD for New Physics at the Precision Frontier”

⇒ Very precise results in many meson systems

errors (in %) **(preliminary) FLAG-3 averages**



§ We are beginning to do precision calculations in nucleons

The Trouble with Nucleons

Nucleons are more complicated than mesons because...

§ Noise issue

- ↻ Signal diminishes at large t_E relative to noise
- ↻ Get worse when quark mass decreases

§ Excited-state contamination

- ↻ Nearby excited state: Roper(1440)

§ Hard to extrapolate in pion mass

- ↻ Δ resonance nearby; multiple expansions, poor convergence...
- ↻ Less an issue in the physical pion-mass era

§ Requires larger volume and higher statistics

- ↻ Ensembles are not always generated with nucleons in mind
- ↻ **High-statistics:** large measurement and long trajectory

The Trouble with Nucleons

Nucleons are more complicated than mesons because...

§ Noise issue

↪ Signal d

↪ Get wor

§ Excited-s

↪ Nearby c

§ Hard to e

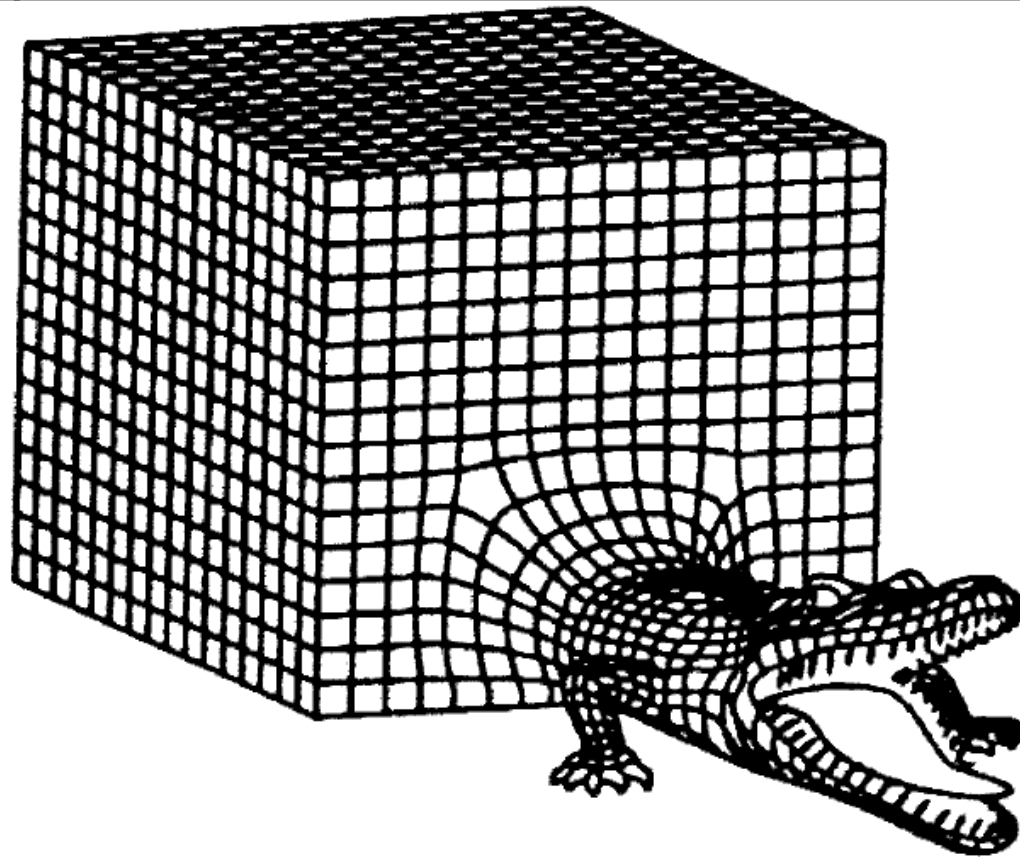
↪ Δ resona

↪ Less an

§ Requires

↪ Ensemb

↪ High-st

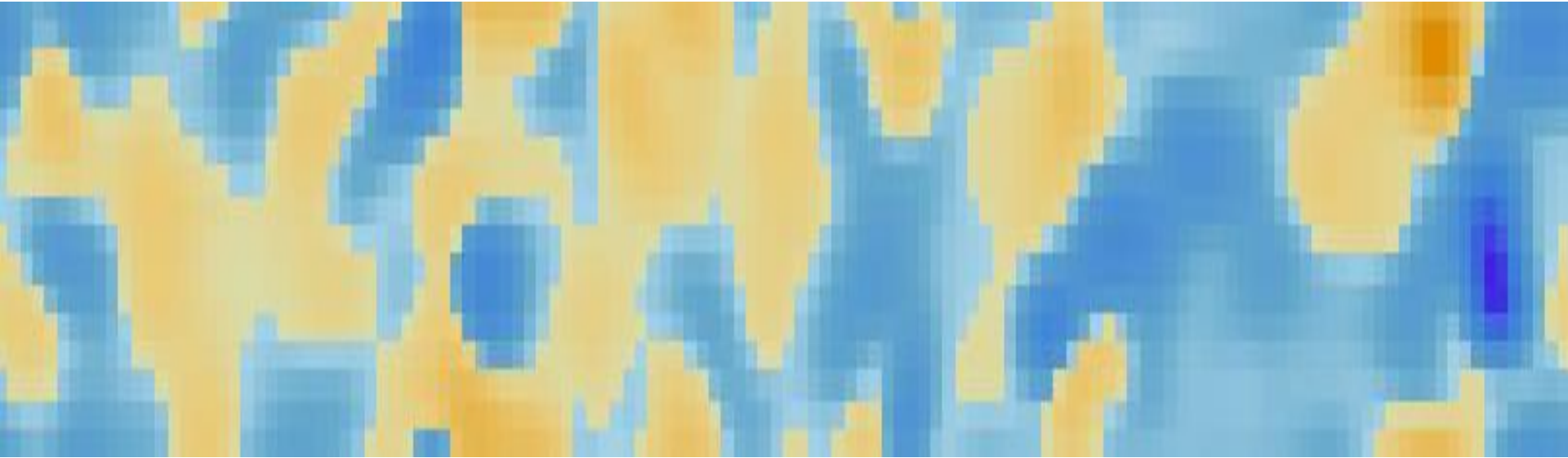


convergence...

ns in mind

PROCEED WITH CAUTION

Nucleon Matrix Elements

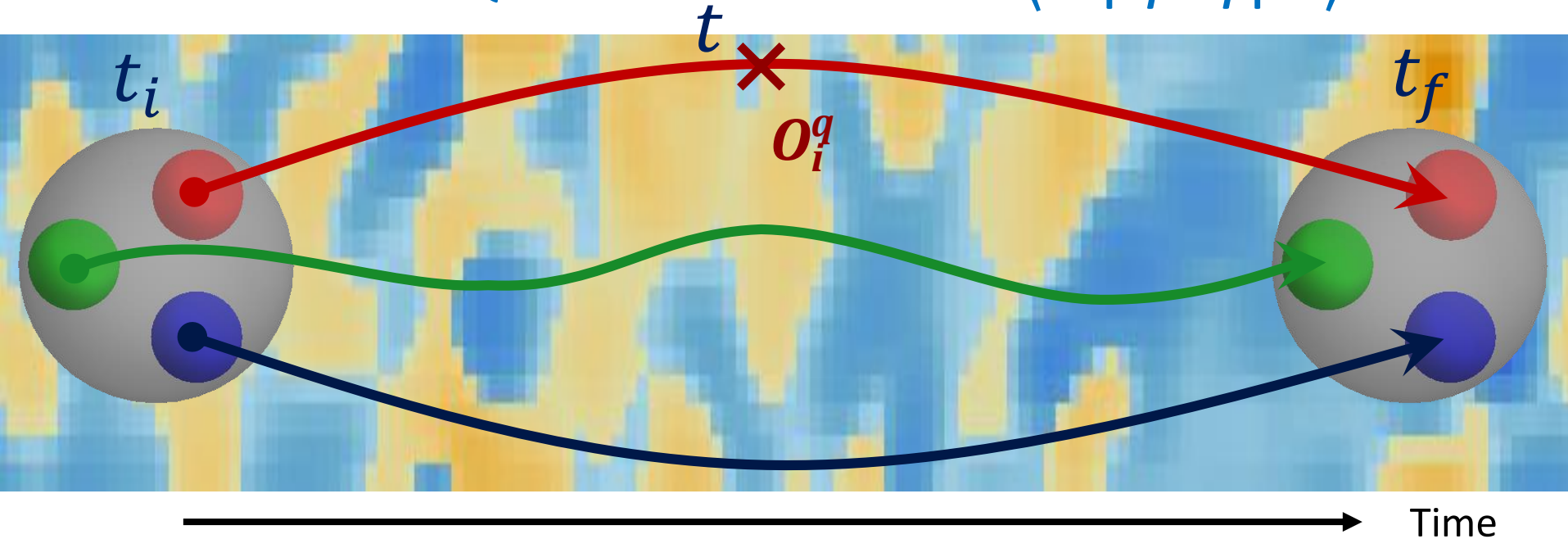


§ Pick a QCD vacuum

↻ Gauge/fermion actions, flavour $(2, 2+1, 2+1+1)$, m_π , a , L , ...

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N | \bar{q} \Gamma q | N \rangle$



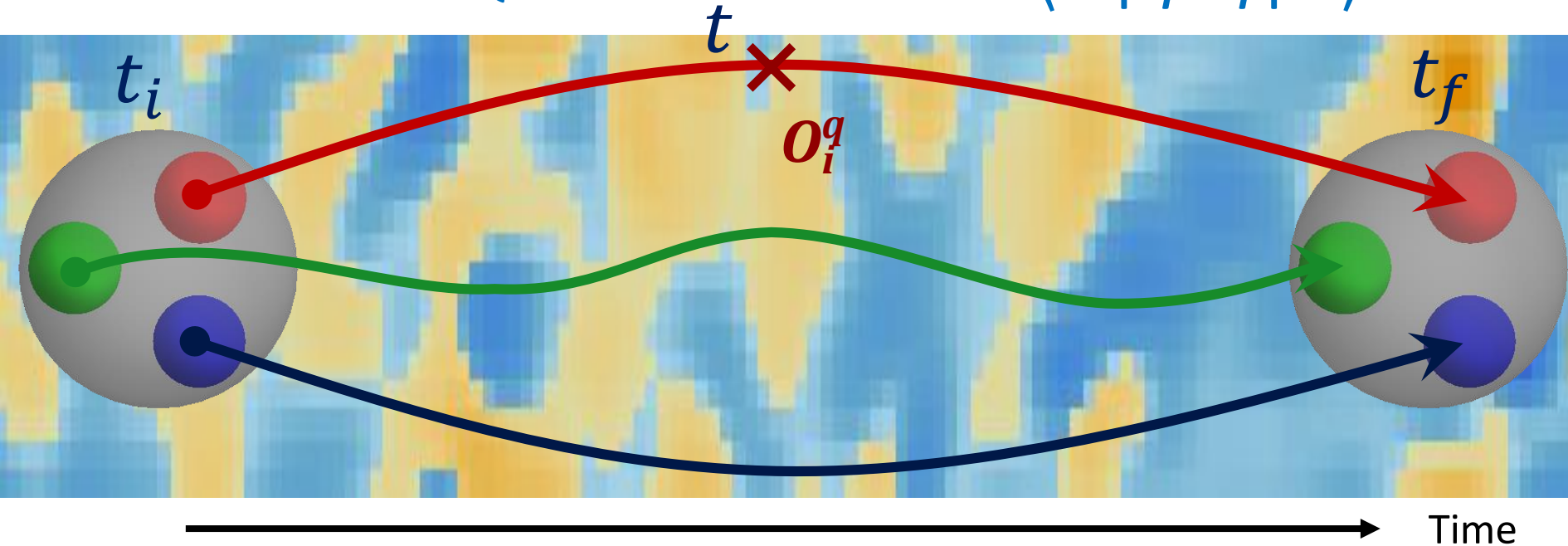
§ Construct correlators (hadronic observables)

⌘ Requires “quark propagator”

Invert Dirac-operator matrix (rank $O(10^{12})$)

Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N | \bar{q} \Gamma q | N \rangle$



§ Analysis (extract couplings)

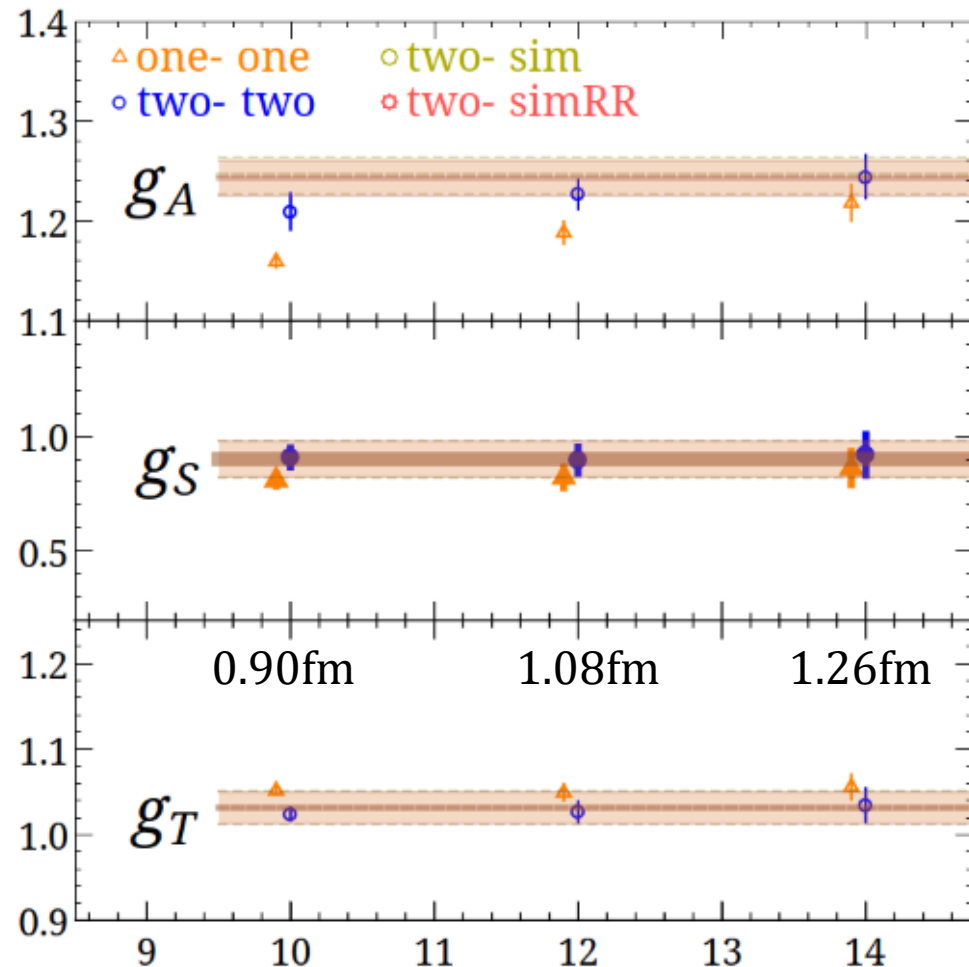
$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t_f - t_i)} \\ + \mathcal{A}_0 \mathcal{A}_1^* \langle 0 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_0(t - t_i)} e^{-M_1(t_f - t)} + \mathcal{A}_0^* \mathcal{A}_1 \langle 1 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_1(t - t_i)} e^{-M_0(t_f - t)} \\ + |\mathcal{A}_1|^2 \langle 1 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_1(t_f - t_i)}$$
$$C^{2\text{pt}}(t_f, t_i) = |\mathcal{A}_0|^2 e^{-M_0(t_f - t_i)} + |\mathcal{A}_1|^2 e^{-M_1(t_f - t_i)} + \dots$$

Analysis

§ An example from PNDME

∞ Move the **excited-state systematic** into the statistical error

$a = 0.09$ fm, 310-MeV pion



$$C^{3\text{pt}}(t_f, t, t_i) = |\mathcal{A}_0|^2 \langle 0 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t_f - t_i)}$$

$$+ \mathcal{A}_0 \mathcal{A}_1^* \langle 0 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_0(t-t_i)} e^{-M_1(t_f-t)}$$

$$+ \mathcal{A}_0^* \mathcal{A}_1 \langle 1 | \mathcal{O}_\Gamma | 0 \rangle e^{-M_0(t-t_i)} e^{-M_0(t_f-t)}$$

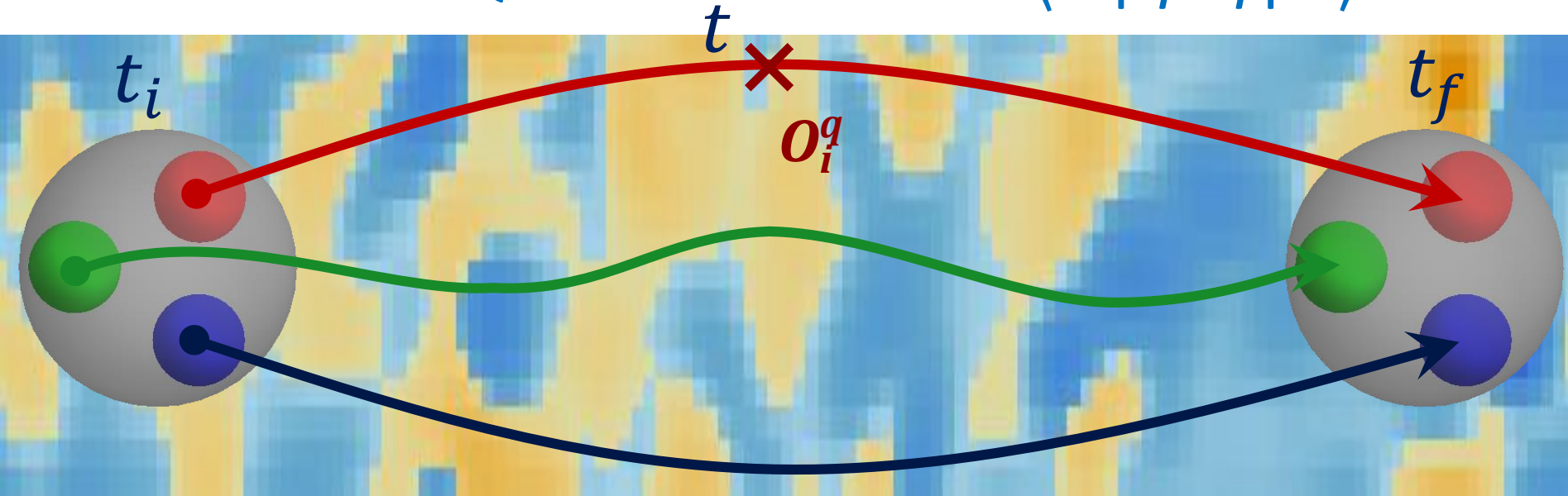
$$+ |\mathcal{A}_1|^2 \langle 1 | \mathcal{O}_\Gamma | 1 \rangle e^{-M_1(t-t_i)} e^{-M_1(t_f-t)}$$

∞ Much stronger effect at finer lattice spacing!

∞ Needs to be studied case by case

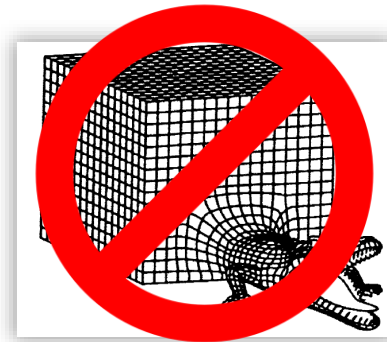
Nucleon Matrix Elements

Lattice-QCD calculation of $\langle N | \bar{q} \Gamma q | N \rangle$



§ Systematic Uncertainty (nonzero a , finite L , etc.)

- ∞ Contamination from excited states
- ∞ Nonperturbative renormalization
e.g. RI/SMOM scheme in $\overline{\text{MS}}$ at 2 GeV
- ∞ Extrapolation to the continuum limit
($m_\pi \rightarrow m_\pi^{\text{phys}}$, $L \rightarrow \infty$, $a \rightarrow 0$)



Isovector Tensor Charge



PNDME

Precision Neutron-Decay Matrix Elements (2010-)

<https://sites.google.com/site/pndmelqcd/>

Tanmoy Bhattacharya



Rajan Gupta



HWL



Vincenzo Cirigliano



+



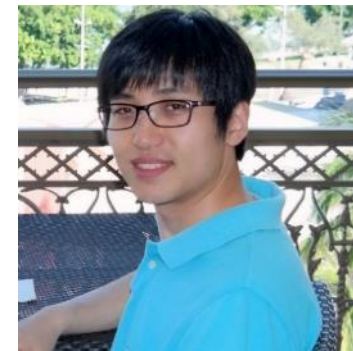
Saul Cohen



Anosh Joseph



Yong-Chull Jang



Boram Yoon

Precision Nucleon Couplings

- § Much effort has been devoted to controlling systematics
- § A state-of-the art calculation (PNDME): **2016**

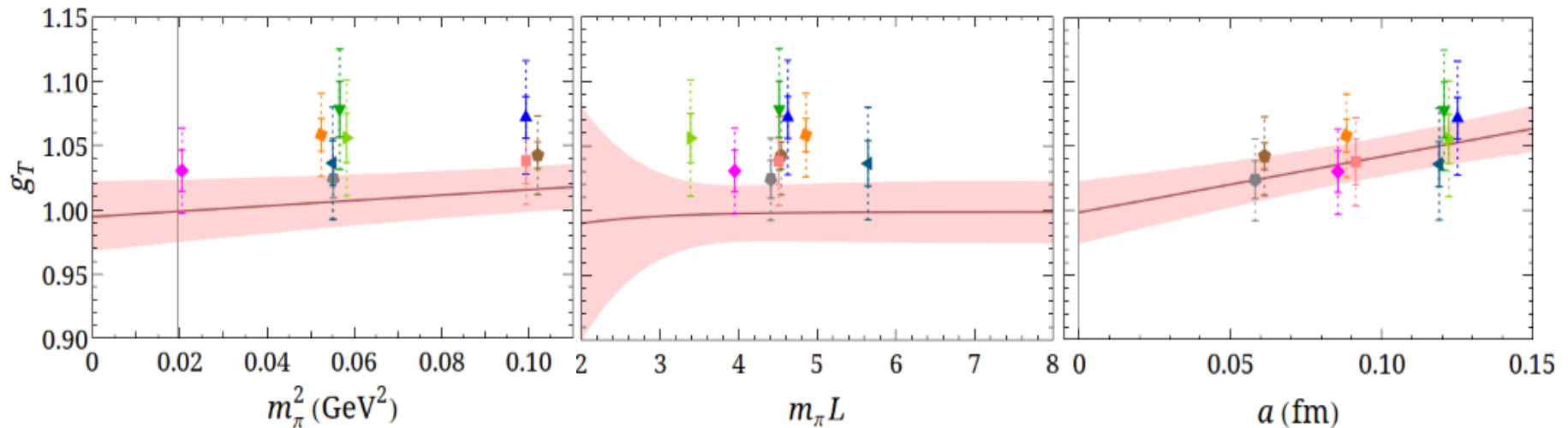
a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	24k
0.12	$32^3 \times 64$	4.38	220	8,10,12	7.6k
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	64.6k
0.09	$32^3 \times 96$	4.51	310	10,12,14	7.0k
0.09	$48^3 \times 96$	4.79	220	10,12,14	7.1k
0.09	$64^3 \times 96$	3.90	130	10,12,14	56.5k
0.06	$48^3 \times 144$	4.52	310	16,20,22,24	64.0k
0.06	$64^3 \times 144$	4.41	220	16,20,22,24	41.6k
We thank MILC collaboration for sharing their 2+1+1 HISQ lattices					

Precision Nucleon Couplings

§ A state-of-the-art calculation (PNDME)

↻ Extrapolate to the **continuum** limit ($m_\pi \rightarrow m_\pi^{\text{phys}}, L \rightarrow \infty, a \rightarrow 0$)
PNDME, 1606.07049

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



2016: First extrapolation to the physical limit
of a nucleon matrix element!

Precision Nucleon Couplings

§ 2018: 4 lattice spacings, 2 physical pion mass, $M_\pi \leq 320$ MeV

a (fm)	V	$M_\pi L$	M_π (MeV)	t_{sep}	# Meas.
0.15	$16^3 \times 48$	3.93	310	5,6,7,8,9	122.7K
0.12	$24^3 \times 64$	4.55	310	8,10,12	64.8k
0.12	$24^3 \times 64$	3.29	220	8,10,12	60.5K
0.12	$32^3 \times 64$	4.38	220	8,10,12	47.6K
0.12	$40^3 \times 64$	5.49	220	8,10,12,14	128.6K
0.09	$32^3 \times 96$	4.51	310	10,12,14	114.9K
0.09	$48^3 \times 96$	4.79	220	10,12,14	123.4K
0.09	$64^3 \times 96$	3.90	130	8,10,12,14,16	165.1K
0.06	$48^3 \times 144$	4.52	310	18,20,22,24	64.0K
0.06	$64^3 \times 144$	4.41	220	18,20,22,24	41.6K
0.06	$96^3 \times 192$	3.80	130	16,18,20,22	43.2K

Precision Nucleon Couplings

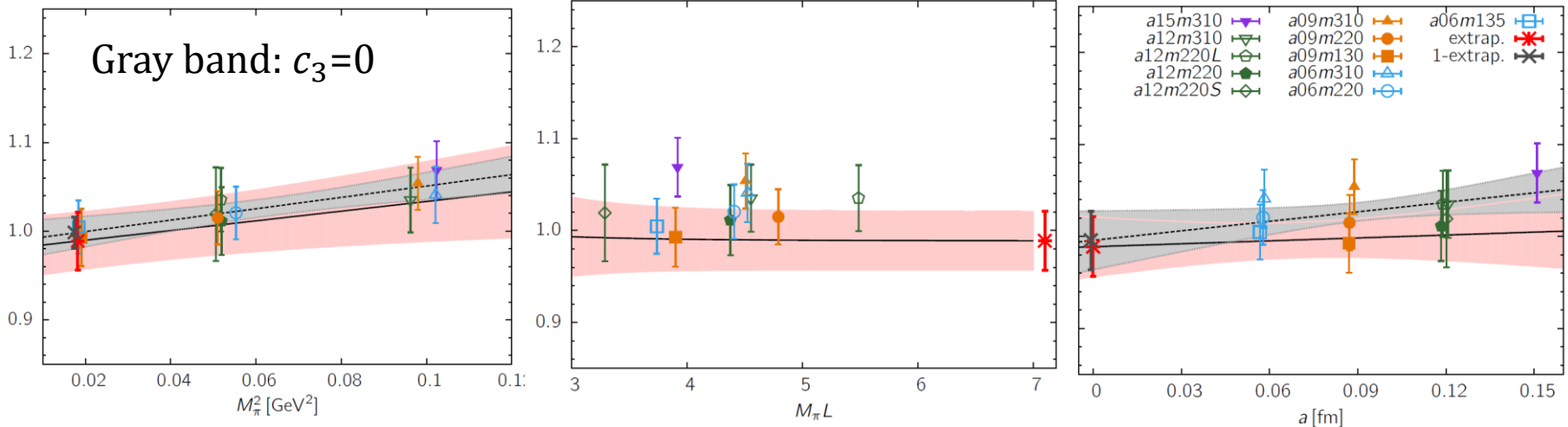
§ Much effort has been devoted to controlling systematics

§ A state-of-the art calculation (PNDME)

PNDME, 1806.09006

↻ Extrapolate to the physical limit (varying ansatz)

$$g_T(a, m_\pi, L) = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$



2018: Still the only collaboration that has a full continuum-extrapolated g_T



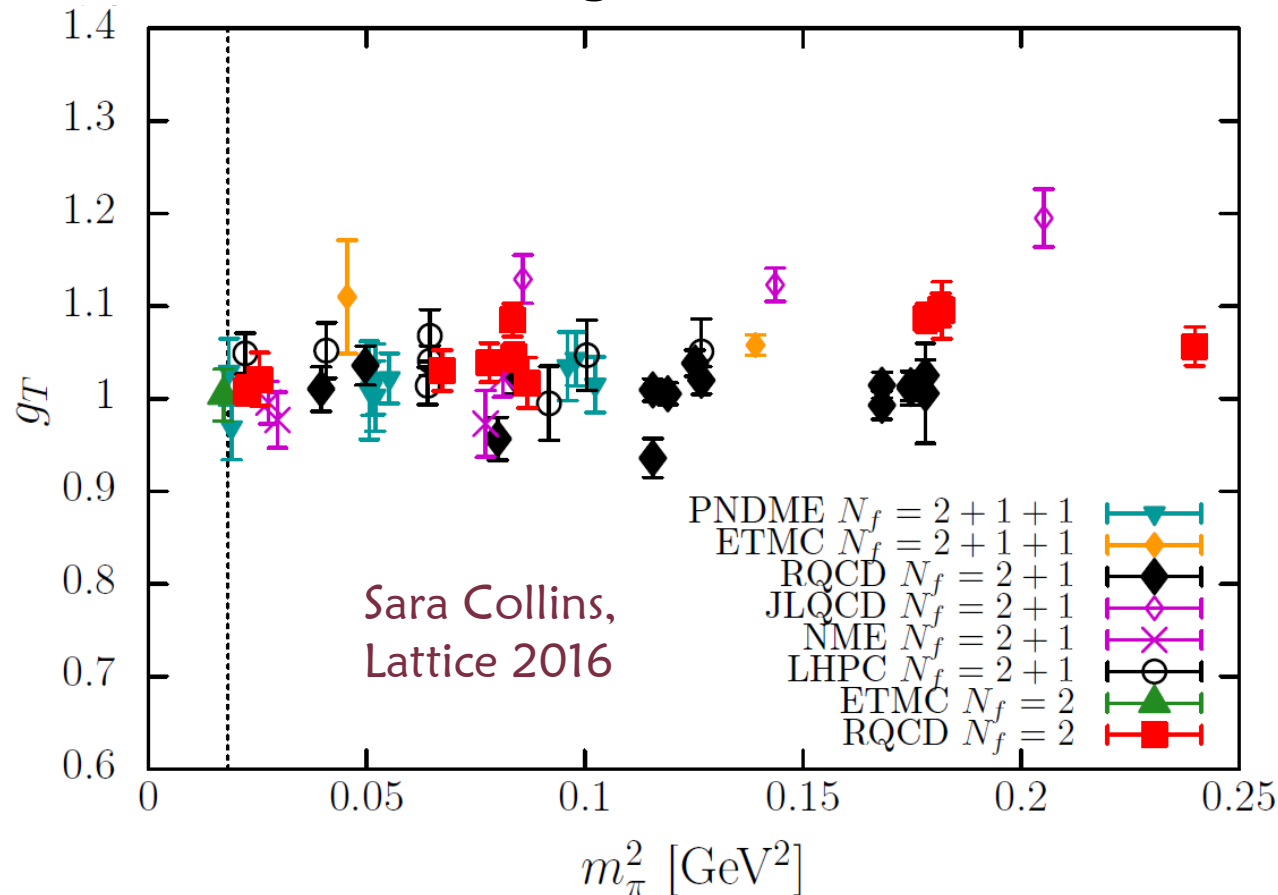
Yong-Chull Jang

Precision Nucleon Couplings

§ Usually more than one LQCD calculation

∞ For example, tensor charge

∞ Lattice results should agree in the continuum limit



Jeremy Green,
Lattice 2018

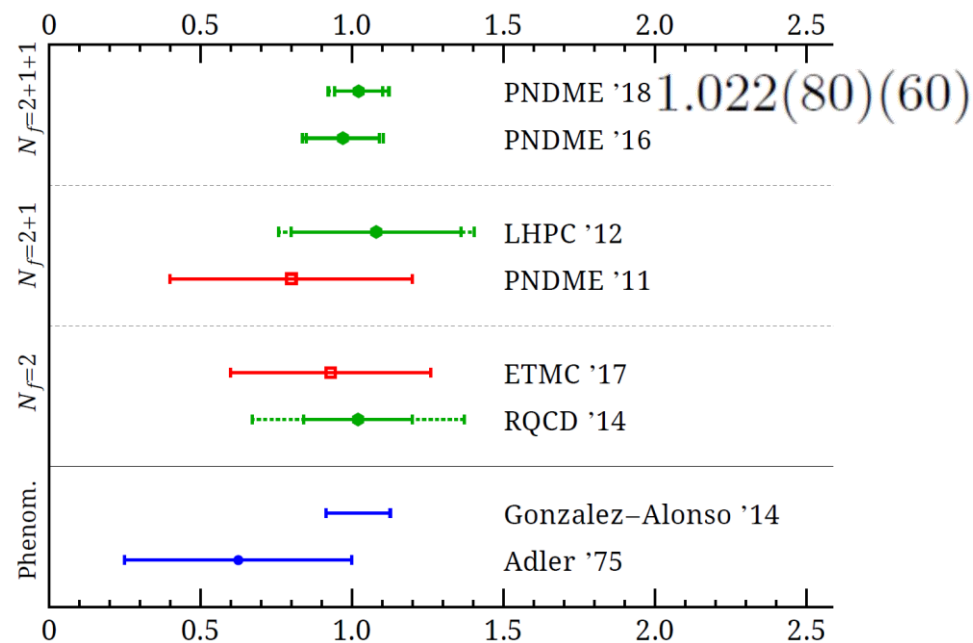
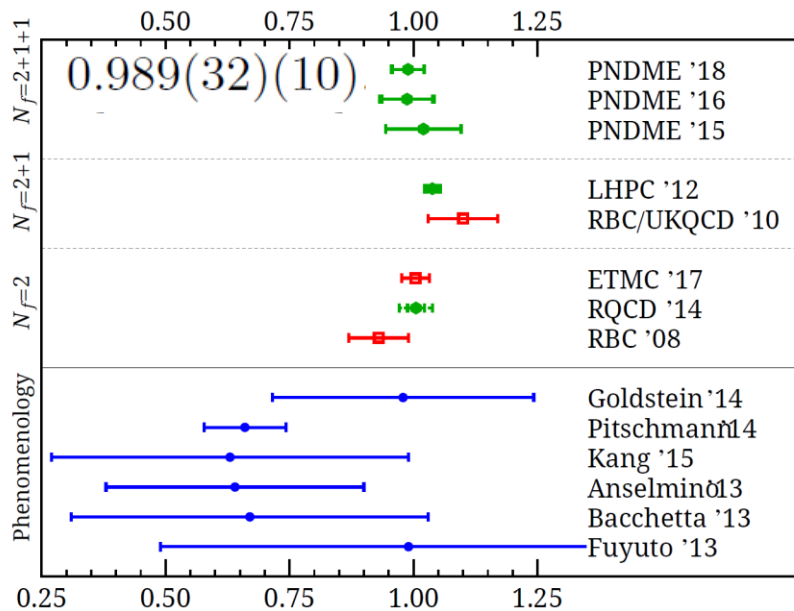
Precision Nucleon Couplings

FLAG rating system PNDME, 1506.06411; 1606.07049

New: excited-state rating

Collaboration	Ref.	publication status	N_f	chiral extrapolation	continuum extrapolation	finite volume	excited state	renormalization	g_T
PNDME'15	This work	P	2+1+1	★	★	★	★	★	1.020(76) ^a
ETMC'13	[30]	C	2+1+1	■	○	○	■	★	1.11(3) ^b
LHPC'12	[28]	A	2+1	★	○	★	○	★	1.037(20) ^c
RBC/UKQCD'10	[29]	A	2+1	○	■	★	★	★	1.10(7) ^d
RQCD'14	[31]	P	2	★	★	★	○	★	1.005(17)(29)) ^e
ETMC'13	[30]	C	2	★	■	○	■	○	1.114(46) ^f
RBC'08	[32]	P	2	■	■	★	■	★	0.93(6) ^g

PNDME, 1806.09006



Precision Nucleon Couplings

FLAG rating system: PNDME 1506.06411; 1606.07049

New: FLAG nucleon matrix elements

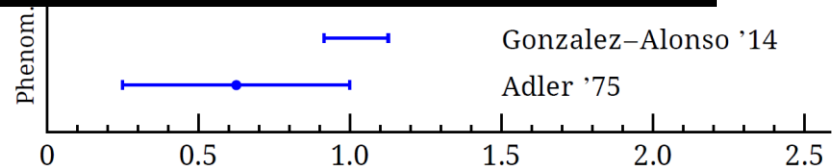
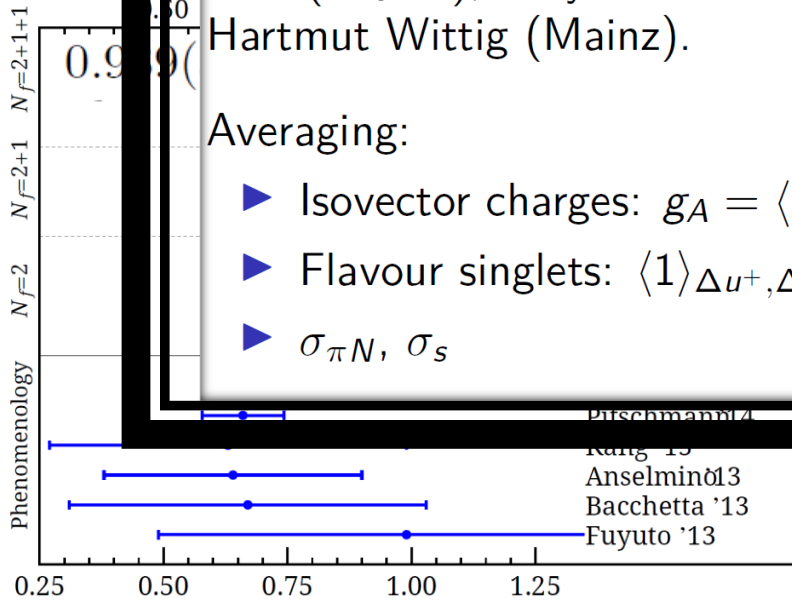
Sara Collins, Trento workshop of Mapping PDF and DA

Nucleon matrix element working group:

S. C (RQCD), Amy Nicholson (CaLat), Rajan Gupta (PNDME), Hartmut Wittig (Mainz).

Averaging:

- ▶ Isovector charges: $g_A = \langle 1 | \Delta_{u^+} - \Delta_{d^+} \rangle$, g_S , $g_T = \langle 1 | \delta_{u^-} - \delta_{d^-} \rangle$
- ▶ Flavour singlets: $\langle 1 | \Delta_{u^+}, \Delta_{d^+}, \Delta_{s^+} \rangle$, $\langle 1 | \delta_{u^-}, \delta_{d^-}, \delta_{s^-} \rangle$
- ▶ $\sigma_{\pi N}$, σ_S

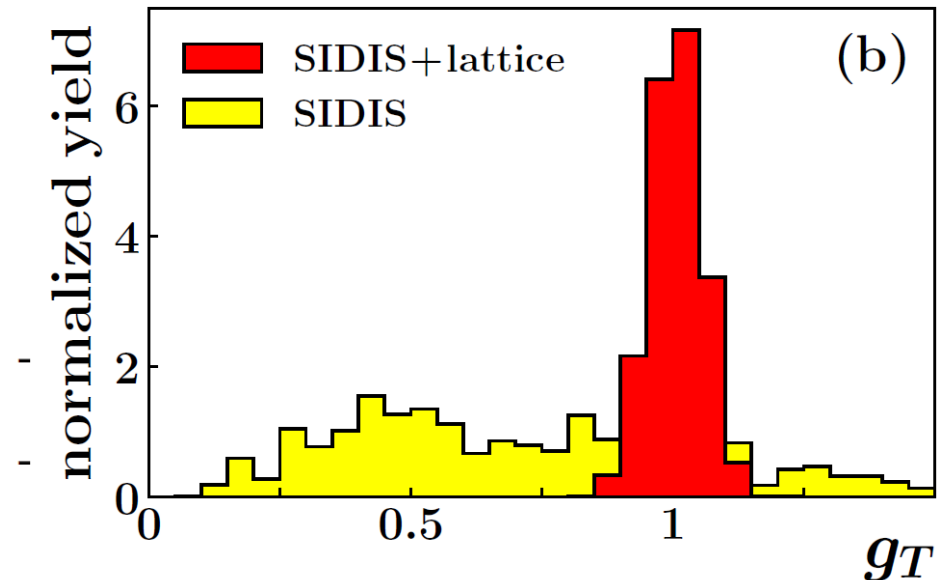
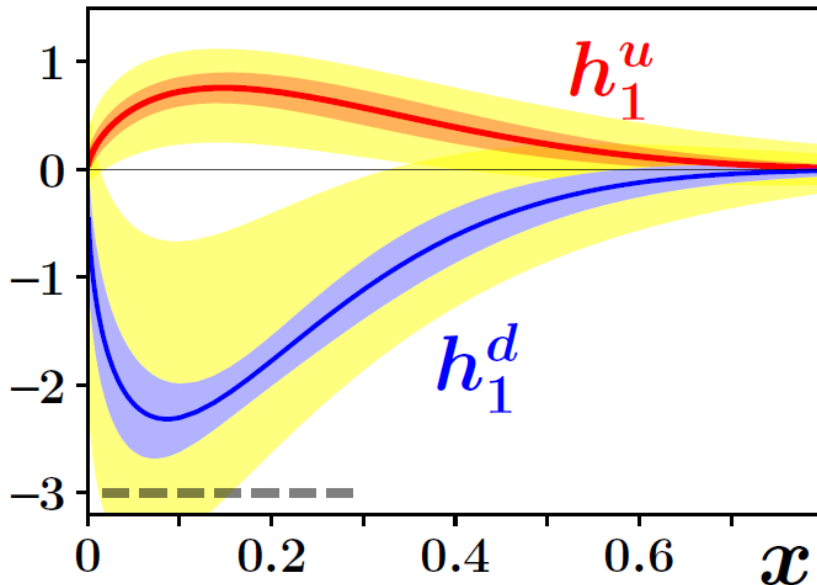


0.20(76) ^a
1(3) ^b
0.37(20) ^c
0(7) ^d
0.05(17)(29) ^e
0.14(46) ^f
0.03(6) ^g

From Charges to PDFs

§ Improved transversity distribution with LQCD g_T

- ∞ Global analysis with 12 extrapolation forms: $g_T = 1.006(58)$
- ∞ Use to constrain the global analysis fits to SIDIS π^\pm production data from proton and deuteron targets



Lin, Melnitchouk, Prokudin, Sato, 1710.09858, Phys. Rev. Lett. 120, 152502 (2018)

New Interactions

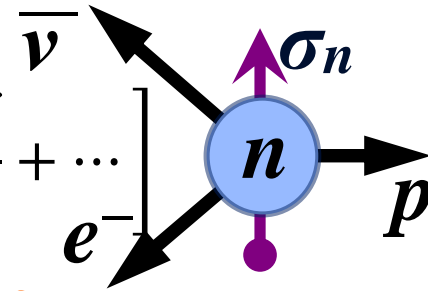
§ Neutron beta decay could be related to new interactions:

$$H_{\text{eff}} = G_F \left(J_{V-A}^{\text{lept}} \times J_{V-A}^{\text{quark}} + \sum_i \varepsilon_i^{\text{BSM}} \hat{O}_i^{\text{lept}} \times \hat{O}_i^{\text{quark}} \right)$$

∞ ε_S and ε_T are related to the masses of the new TeV-scale particles

∞ Parameters sensitive to new physics

$$d\Gamma \propto F(E_e) \left[1 + A \frac{\vec{\sigma}_n \cdot \vec{p}_e}{E_e} + b \frac{m_e}{E_e} + \left(B_0 + B_1 \frac{m_e}{E_e} \right) \frac{\vec{\sigma}_n \cdot \vec{p}_\nu}{E_\nu} + \dots \right]$$



Fierz interference term:

Deviations from the leading-order e^- spectrum

Energy-dependent part of the **neutrino asymmetry parameter** with neutron spin

$$\{b, B\}_{\text{BSM}} = f_0(\varepsilon_{S,T} g_{S,T})$$

Precision LQCD input
($m_\pi \approx 140$ MeV, $a \rightarrow 0$)

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

New Interactions

§ Neutron beta decay could be related to new interactions:

$$H_{\text{eff}} = G_F \left(J_{V-A}^{\text{lept}} \times J_{V-A}^{\text{quark}} + \sum_i \varepsilon_i^{\text{BSM}} \hat{O}_i^{\text{lept}} \times \hat{O}_i^{\text{quark}} \right)$$

∞ ε_S and ε_T are related to the masses of the new TeV-scale particles

∞ Parameters

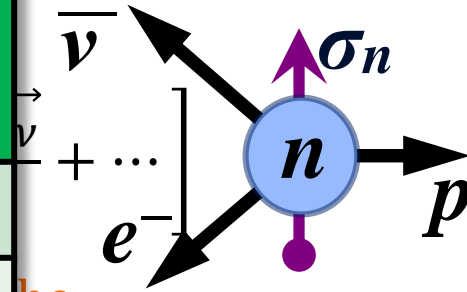
$$d\Gamma \propto F(E_e)$$

Fierz inter

Deviations f

leading-ord

Ongoing and Future Experiments	Expected Precision
UCNb & UCNB at LANL	10^{-3} to 10^{-4}
Nab at ORNL	10^{-3}
FRMII in Munich, ...	
CENPA ${}^6\text{He}(b_{\text{GT}})$	10^{-3} to 10^{-4}



the parameter

$$\{D, D\}_{\text{BSM}} = \mathcal{O}(\varepsilon_{S,T}, g_{S,T})$$

precision LQCD input
($m_\pi \approx 140$ MeV, $a \rightarrow 0$)

$$\varepsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Beta Decays & BSM

§ Given precision $g_{S,T}$ and O_{BSM} , predict new-physics scales

Low-Energy

Expt \rightarrow

$$O_{\text{BSM}} = f_O(\epsilon_{S,T} g_{S,T})$$

Precision LQCD input
($m_\pi \rightarrow 140$ MeV, $a \rightarrow 0$) \leftarrow

$$\epsilon_{S,T} \propto \Lambda_{S,T}^{-2}$$

Upcoming precision

low-energy experiments

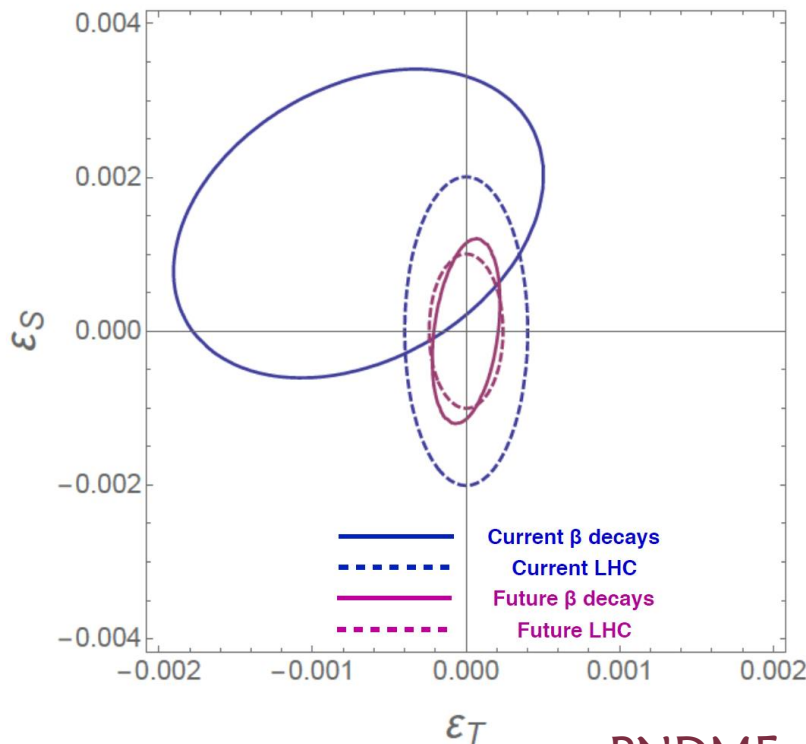
LANL/ ORNL UCN neutron
decay exp't

$$|B_1 - b|_{\text{BSM}} < 10^{-3}$$

$$|b|_{\text{BSM}} < 10^{-3}$$

CENPA: ${}^6\text{He}(b_{\text{GT}})$ at 10^{-3}

Also see talk by A. Garcia, E. Mereghetti



Plots by Vincenzo Cirigliano

PNDME, PRD85 054512 (2012);
1306.5435; 1606.07049; 1806.09006

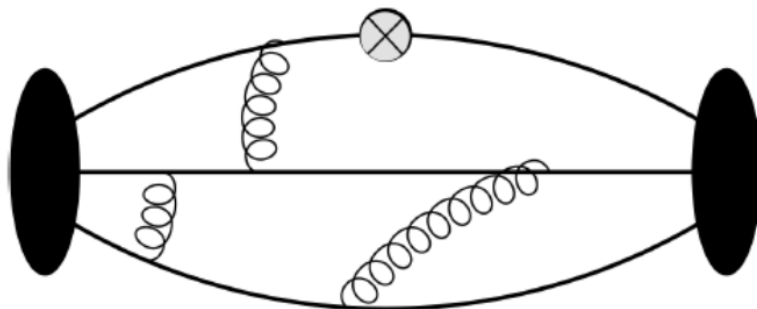
Flavor-Dependent Tensor Charges



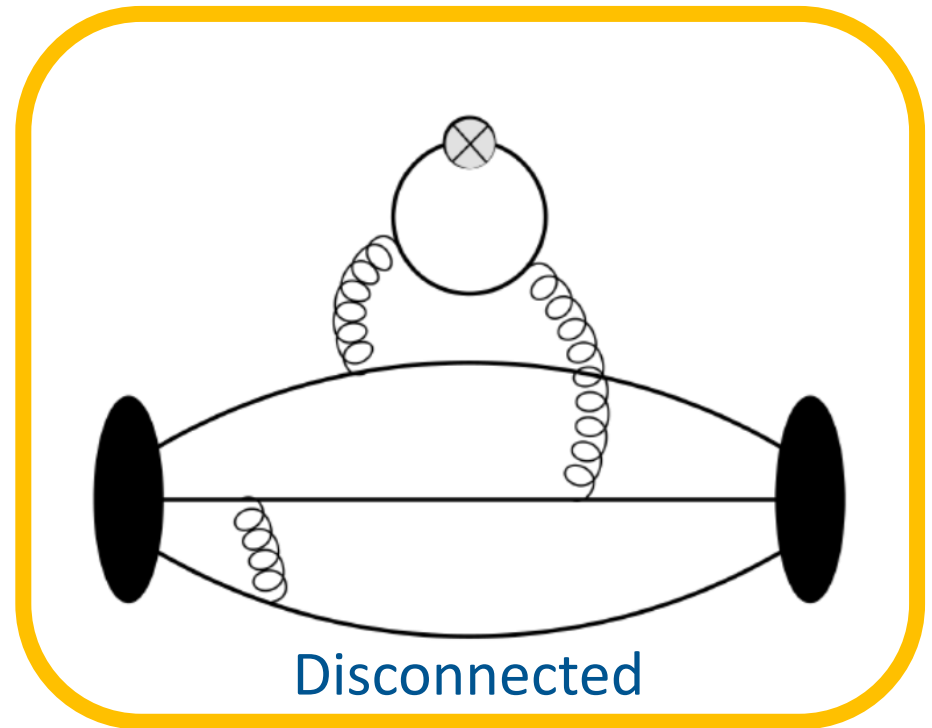
Flavor-Dependent Quark Spin

§ New type of diagram is needed: “disconnected”

- ⌘ Historically, notoriously noisy to calculate on the lattice
- ⌘ Recent developments offer new methods and increasing computational resources
- ⌘ Truncated solver, hopping-parameter expansion, hierarchical probing, ...



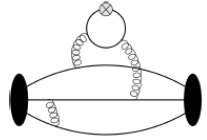
Connected



Disconnected

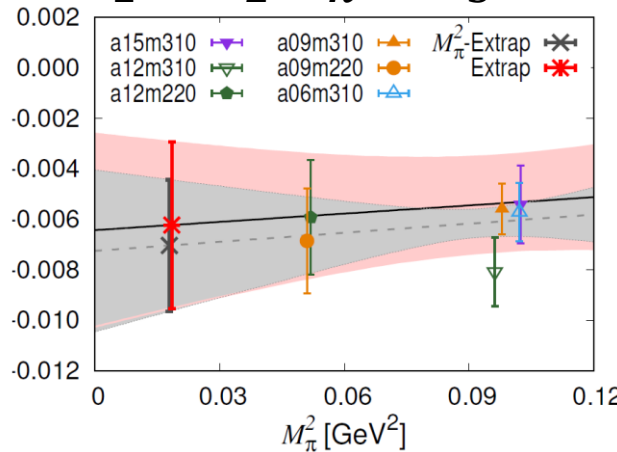
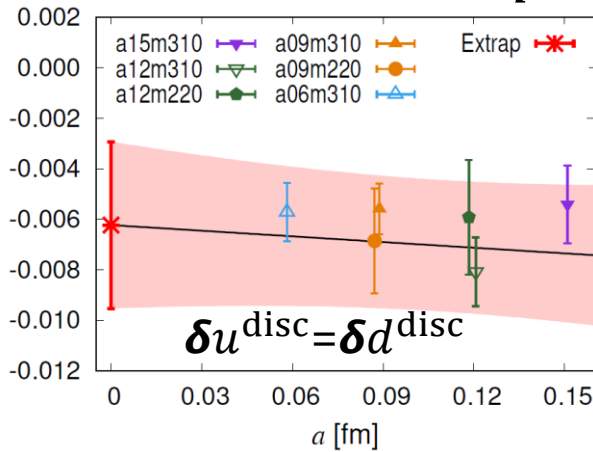
Continuum Extrapolation

§ Up and down quark “disconnected” contribution



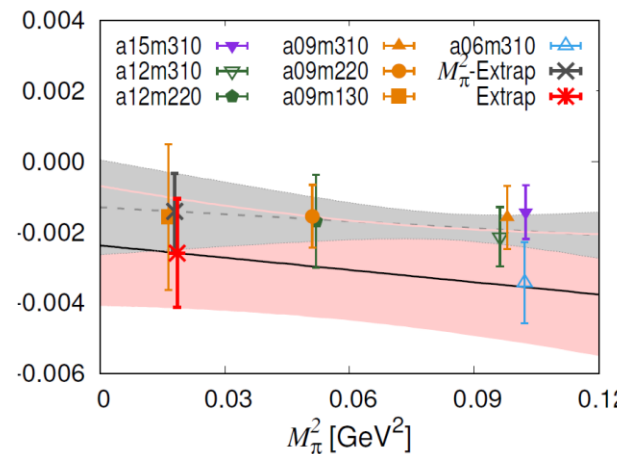
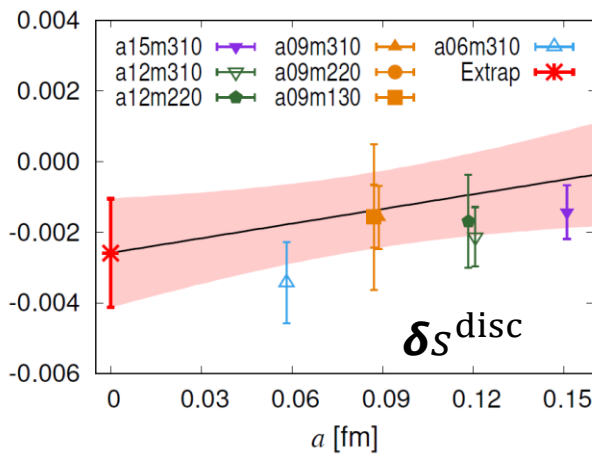
$$\delta q^{\text{disc}} = c_1 + c_2 m_\pi^2 + c_3 a + c_4 e^{-m_\pi L}$$

PNDME, 1806.09006, 1808.07597



First time in LQCD

Mild dependence on a and pion mass!

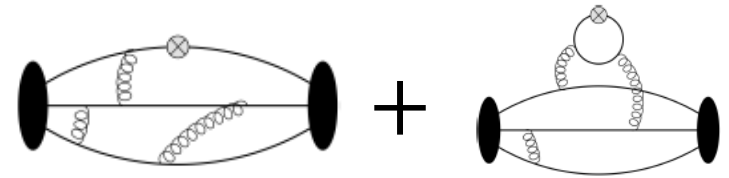


Yong-Chull Jang

Quark Contribution

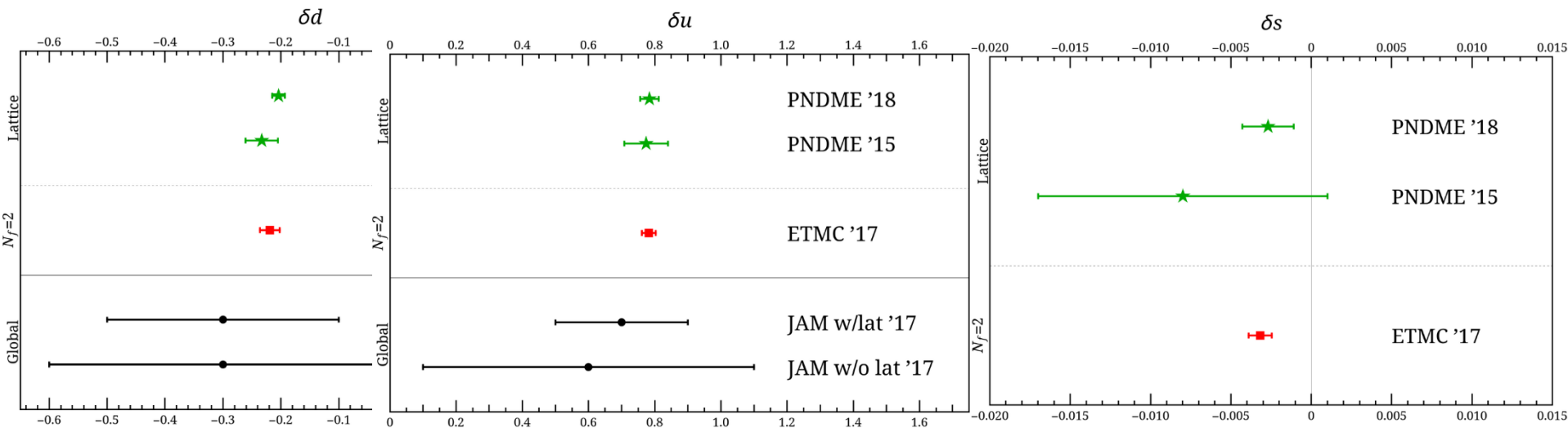
& Sum up both contributions

	g_T^u	g_T^d	g_T^s
Connected	0.790(27)	-0.198(10)	
Disconnected	-0.0064(33)	-0.0064(33)	-0.0027(16)
PDNME'18 (Sum)	0.784(28)	-0.204(11)	-0.0027(16)
ETMC'17 [14]	0.782(21)	-0.219(17)	-0.00319(72)
PNDME'15 [5]	0.774(66)	-0.233(28)	0.008(9)



Calculation from one lattice ensemble only
No cont. extrapolation errors

PNDME, 1806.09006, 1808.07597



Proton tensor charges from a Poincaré-covariant Faddeev equation

1806.01287

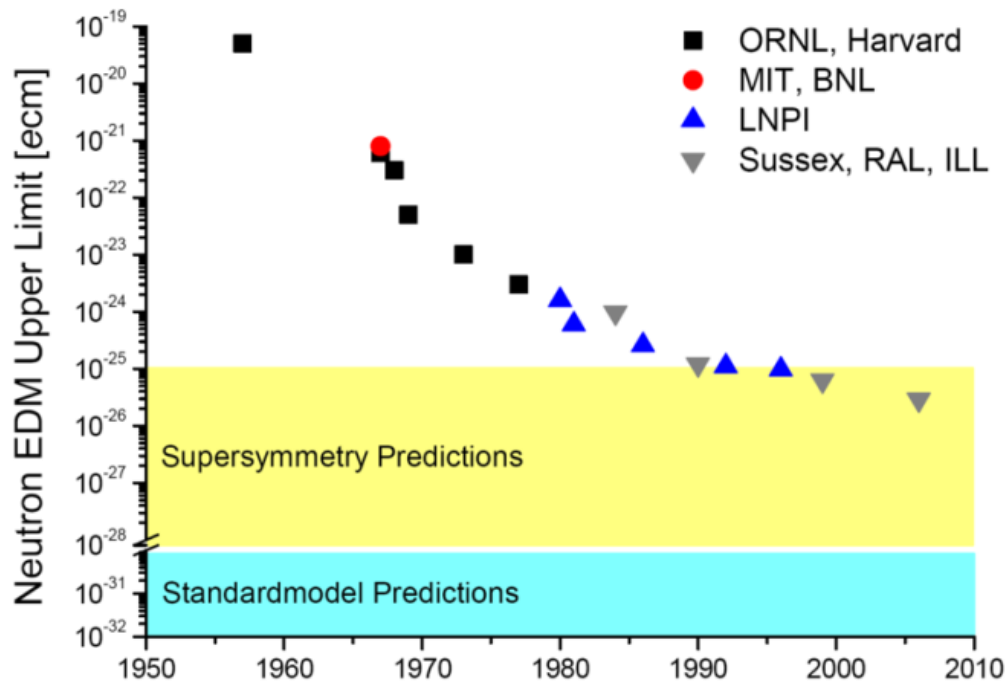
Qing-Wu Wang,¹ Si-Xue Qin,^{2,*} Craig D. Roberts,³ and Sebastian M. Schmidt⁴

$$\delta_T u = 0.912_{(47)}^{(42)}; \quad \delta_T d = -0.218_{(5)}^{(4)}$$

Electric Dipole Moment

§ Why do we care?

- ∞ CP-violating effect \Rightarrow Key ingredient for baryogenesis
 \Rightarrow Why matter exists
- ∞ Extremely small in SM: $\approx 10^{-31}$ e-cm (expect to probe 10^{-28} soon)
- ∞ Good candidate to constrain BSM models



$nEDM$

§ Lattice community are working on various contributions

§ Lagrangian $L = L_{\text{QCD}}^{\text{CP Even}} + L_{\Theta} + L_{\text{quark}}^{\text{dim-5}} + L_{\text{chromo-quark}}^{\text{dim-5}} + \dots$

$$i\Theta \frac{g^2}{16\pi^2} \int d^4x G^{\mu\nu} \tilde{G}_{\mu\nu}$$

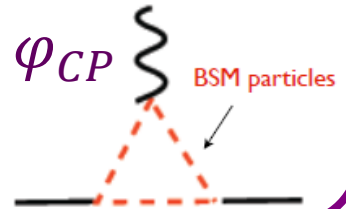
CP-even QCD vacuum
with θ -term expansion; noisy

RBC, J/E, CP-PACS(2005),
CP-PACS(2006, 2010), QCDSF(2011), ...

CP-odd QCD vacuum 1502.02295
with dynamical quarks by QCDSF

$$-0.0038(2)(9) \theta \text{ e}\cdot\text{fm}$$

Induced by a variety of
BSM scenarios

$$d_i \propto \frac{m_i}{\Lambda^2} \sin \varphi_{CP}$$


BSM particles

$$\bar{q} \sigma_{\mu\nu} \gamma_5 \lambda^A G^{\mu\nu A} q + \dots$$

A few works in progress

T. Bhattacharya et al, 1502.07325

$$-\frac{i}{2} \bar{q} \sigma_{\mu\nu} \gamma_5 q F^{\mu\nu}$$

This talk focuses on the quark EDM

Electric Dipole Moment

§ Quark EDM (d_q) in nucleon comes from

$$d_N = d_u g_T^{(n,u)} + d_d g_T^{(n,d)} + d_s g_T^{(n,s)}$$

↪ Hadronic contribution: $\langle N | \bar{q} \sigma_{\mu\nu} q | N \rangle$, $q \in \{u, d, s\}$

§ Extrapolate to the continuum limit PNDME, 1808.07597

$$g_T^u = 0.784(28), g_T^d = -0.204(11), g_T^s = 0.0027(16)$$

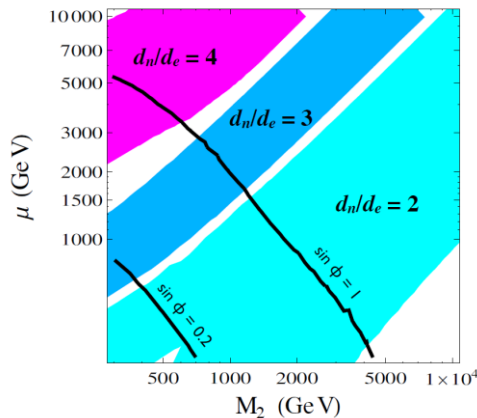
§ Implications for new physics?

↪ Take split SUSY for example

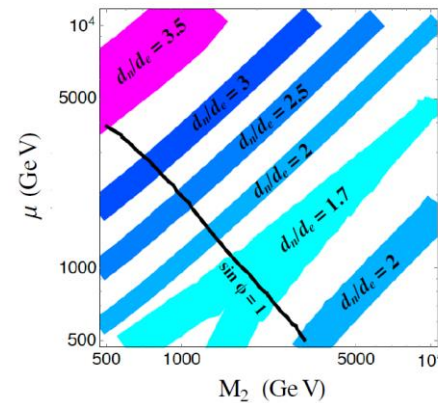
Wells, 2003;

Arkani-Hamed and Dimopoulos, 2004;

Giudice and Romanino, 2004



1506.04196



1808.07597

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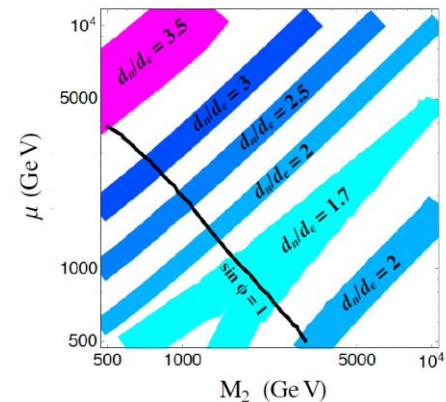
⇒ Take split SUSY for example

⇒ Using our lattice inputs, we can derive an upper limit for the neutron EDM in split SUSY PNDME, 1808.07597

$$|d_n| < 3 \times 10^{-28} e \cdot \text{cm}$$

using $|d_e| < 8.7 \times 10^{-29} e \cdot \text{cm}$ with 90% confidence

ACME Coll., Science Vol. 343 no. 6168 pp. 269-272 (2014)



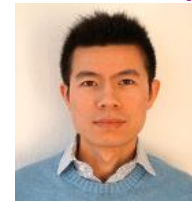
Summary

§ Exciting era using LQCD to study nucleon structure

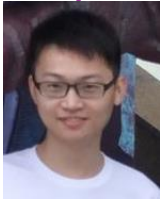
- ↪ Well-studied systematics → precision nucleon structures
- ↪ Address neglected disconnected contributions
obtaining flavor-dependent quantities
- ↪ BSM applications with fundamental symmetry community and PDFs

§ Overcoming longstanding obstacle to full x -distribution (LP^3)

- ↪ Progress made in **first lattice pion PDF** (1804.01483) & **meson distribution amplitudes** (1702.0008, 1712.10025)
- ↪ More systematics study planned in the near future



J. Zhang



R. Zhang

§ Stay tuned for many more exciting results from LQCD



Thanks to MILC collaboration for sharing their 2+1+1 HISQ lattices

The work of HL is sponsored by NSF CAREER Award under grant PHY 1653405

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