Three-gluon vertex: the new frontier

Joannis Papavassiliou

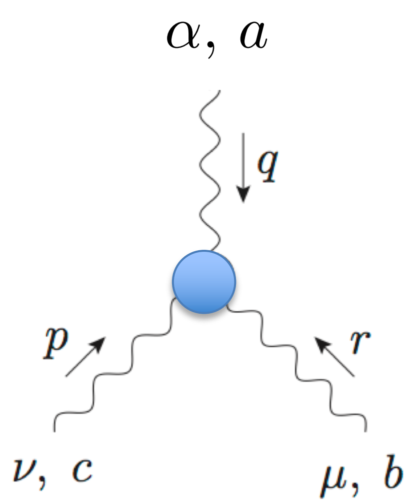
Department of Theoretical Physics and IFIC
University of Valencia-CSIC

TNT V

Emergent mass and its consequences in the Standard Model
17-21 September, 2018
*ECT**

(based on work with A. C. Aguilar and M. Ferreira)

Three-gluon vertex



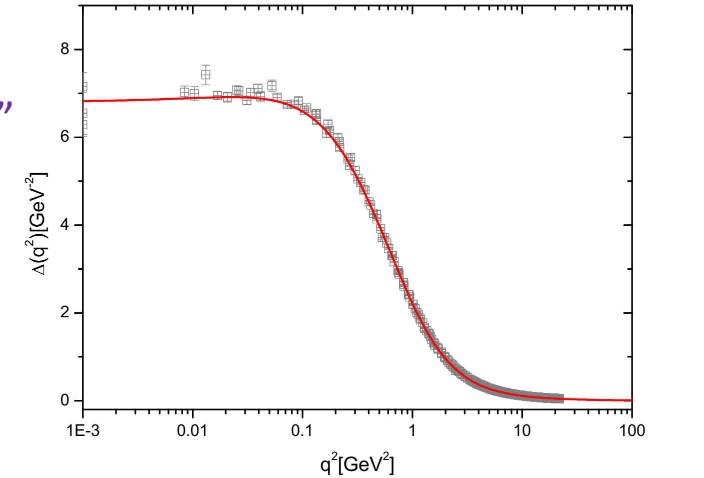
$$= \quad g f^{abc} \mathrm{I}\Gamma_{\alpha\mu\nu}(q,r,p)$$

- Purely non-Abelian (no QED analogue)
- Crucial for asymptotic freedom
- Host of a plethora of tightly interwoven nonperturbative effects

Emergent mass in the gauge sector

"Saturation" ⟷ "dynamical gluon mass generation"

J. M. Cornwall, Phys. Rev. D, 1453 (1982)

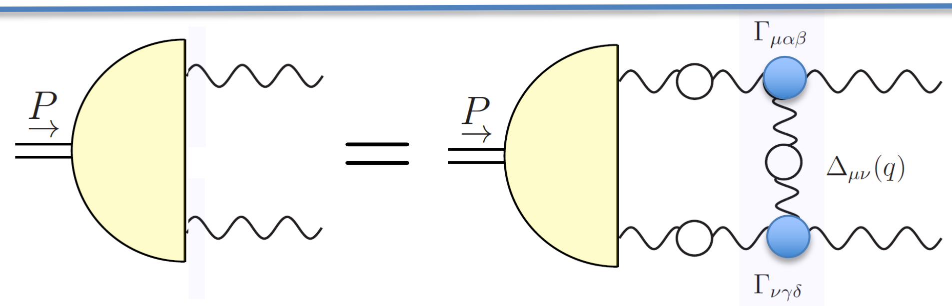


Existence of "massive" solutions depends crucially on the presence of massless poles in $\mathbb{\Gamma}$

A. C. Aguilar, D. Binosi, J. P. , Phys. Rev. D 78, 025010 (2008)

Glueballs

The infrared suppression of $\mathbb{\Gamma}$ seems crucial for getting "correct" glueball masses from the BSE



J. Meyers, E.S. Swanson, Phys. Rev. D 87, 036009 (2013)

H.Sanchis-Alepuz, C. S. Fischer, *et al*, Phys.Rev. D92, 034001 (2015).

Hybrids and Exotics

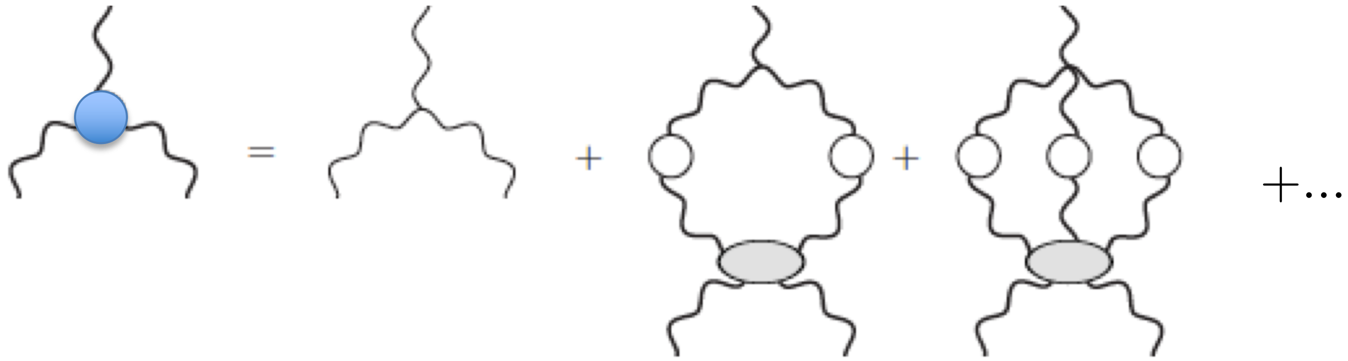
Systems with a "valence" gluon G : $Q\bar{Q}G, \; QQQG$

G plays a dual role : both force field and matter

The infrared behavior of $\mathbb{\Gamma}$ affects their appearance in the spectrum

S.S.Xu, Z.F.Cui, L.Chang, J.P., C.D.Roberts, H.S.Zong, arXiv:1805.06430

"Conventional" approach: solve the SDE of the three-gluon vertex



R. Alkofer, M.Q. Huber, K. Schwenzer, Eur. Phys. J. C62 , 761 (2009); Phys. Rev. D81, 105010 (2010)
M. Pelaez, M. Tissier, N. Wschebor, Phys. Rev. D88 (2013)
G. Eichmann, R. Williams, R. Alkofer, M. Vujinovic,Phys. Rev. D89,105014 (2014)
A. Blum, M.Q. Huber, M. Mitter, L. von Smekal, Phys. Rev. D 89, 061703(R) (2014)
A. K. Cyrol, L. Fister, M. Mitter, J. M. Pawlowski, N. Strodthoff, Phys. Rev. D94, 054005 (2016)
R. Williams, C. S. Fischer, and W.Heupel, Phys. Rev. D 93, no. 3, 034026 (2016)

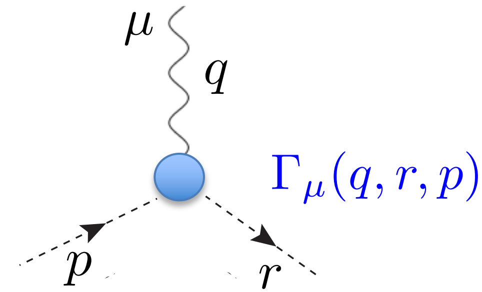
Instead …

Reconstruct $\mathbb{\Gamma}^{abc}_{\alpha\mu\nu}$ *from its Slavnov-Taylor identity (STI)*

Non-perturbative aspects make the construction rather subtle

The general idea: a sophisticated version of the Gauge Technique

Prototype: three-particle vertex of scalar QED



It satisfies the Ward-Takahashi identity

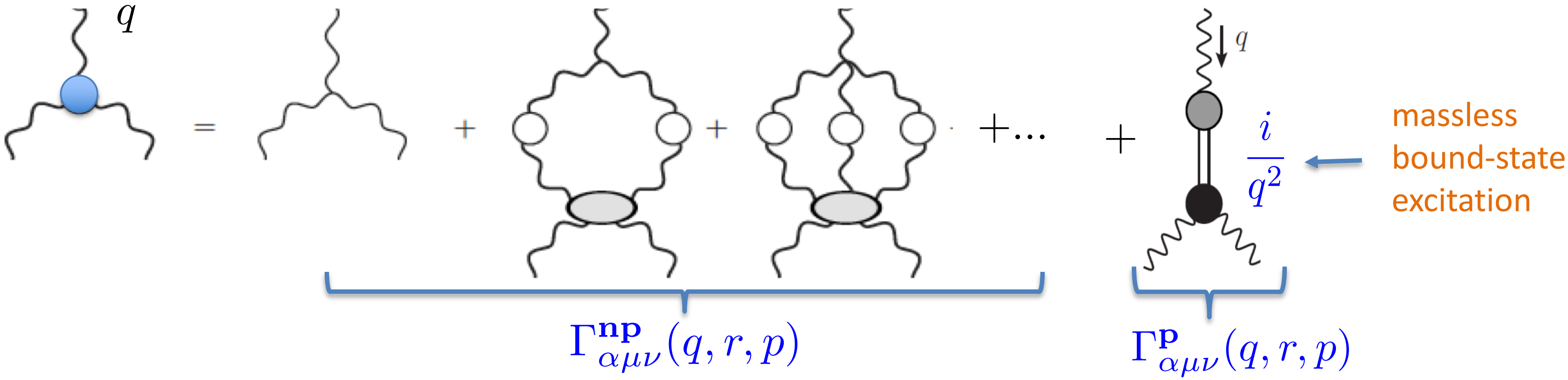
$$q^\mu \Gamma_\mu(q,r,p) = \mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)$$

Write as $\Gamma_\mu(q,r,p) = \underbrace{\Gamma^L_\mu(q,r,p)}_{\text{“longitudinal”}} + \underbrace{\Gamma^T_\mu(q,r,p)}_{\text{“transverse”}}$

$$\Gamma^L_\mu(q,r,p) = \frac{(r-p)_\mu}{p^2-r^2}[\mathcal{D}^{-1}(p^2) - \mathcal{D}^{-1}(r^2)]$$ satisfies WTI

$$\Gamma^T_\mu(q,r,p) = \underbrace{A(q,r,p)}_{\text{“undetermined”}}[(q.p)r_\mu - (q.p)p_\mu]$$ automatically conserved

$\mathrm{I}\Gamma$ *and the gluon mass*



- $\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)$ contains massless poles and triggers the Schwinger mechanism
- Responsible for mass generation and subsequent saturation of $\Delta(0)$
- The poles are formed dynamically → Bound states satisfying a special BSE
- $\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)$ is longitudinally coupled :

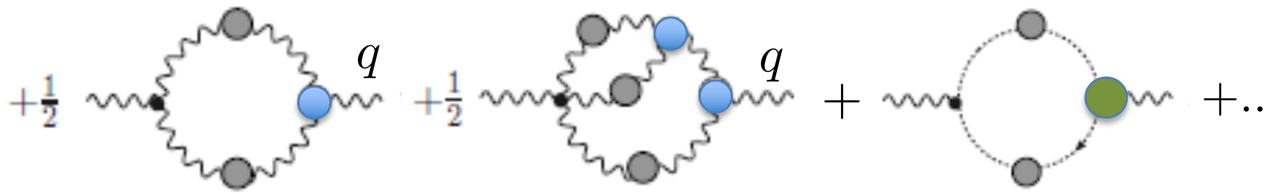
$$\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = \left(\frac{q_\alpha}{q^2}\right) A_{\mu\nu}(q,r,p) + \left(\frac{r_\mu}{r^2}\right) B_{\alpha\nu}(q,r,p) + \left(\frac{p_\nu}{p^2}\right) C_{\alpha\mu}(q,r,p)$$

$$P^{\mu\nu}(q) = g^{\mu\nu} - \frac{q^\mu q^\nu}{q^2}\,.$$

$$P_{\alpha\alpha'}(q)P_{\mu\mu'}(r)P_{\nu\nu'}(p)\,\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = 0$$

Substitute $\mathrm{I\!I}\Gamma$ into the SDE of the gluon propagator

$$\Delta^{-1}(q^2) = \Delta_0^{-1}(q^2)$$



$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

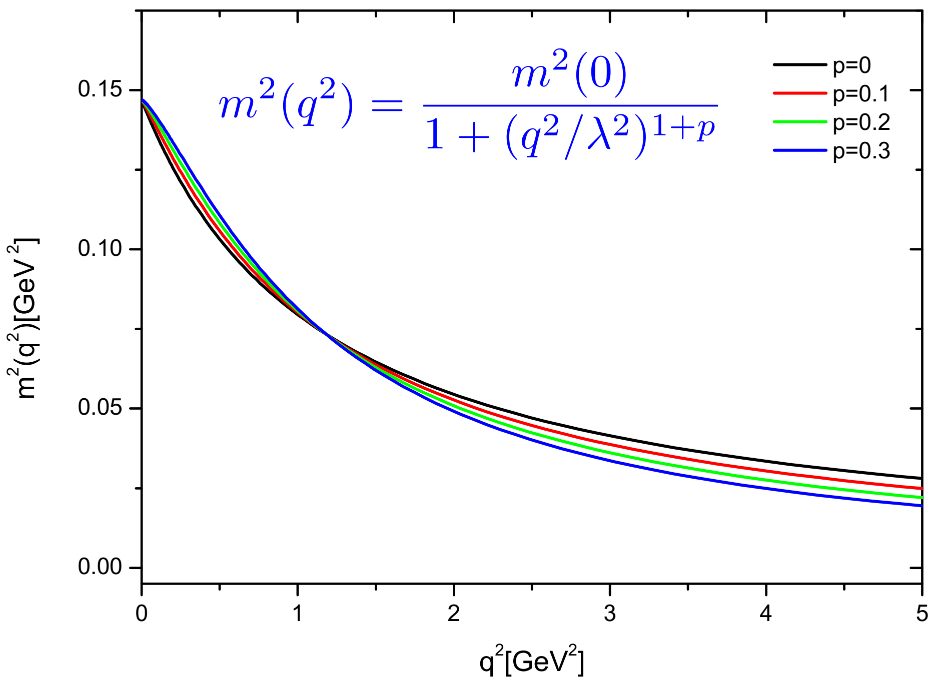
kinetic term — $q^2 J(q^2)$

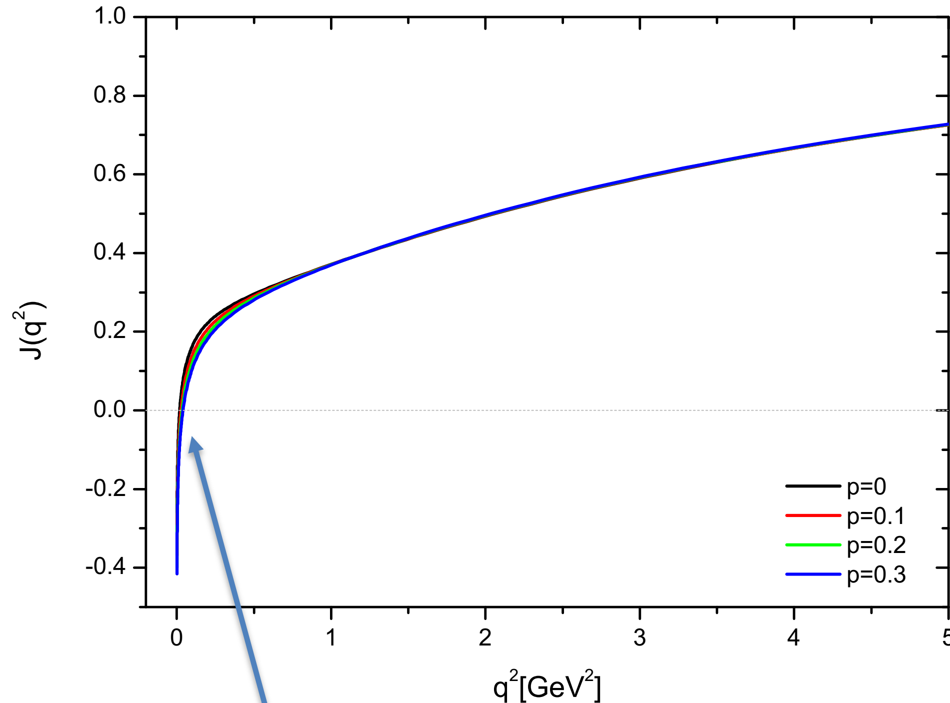
running mass — $m^2(q^2)$

$$J(q^2) = 1 + \int_k \mathcal{K}_1(q, k, m^2, J)$$

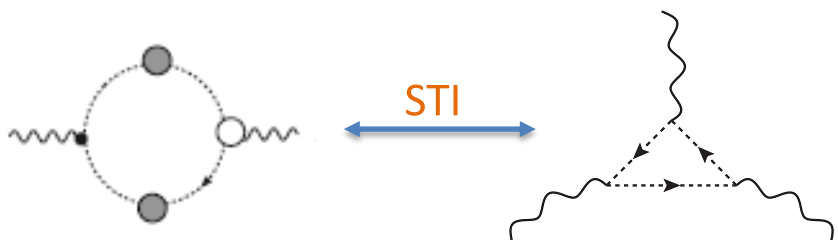
$$m^2(q^2) = \int_k \mathcal{K}_2(q, k, m^2, J)$$

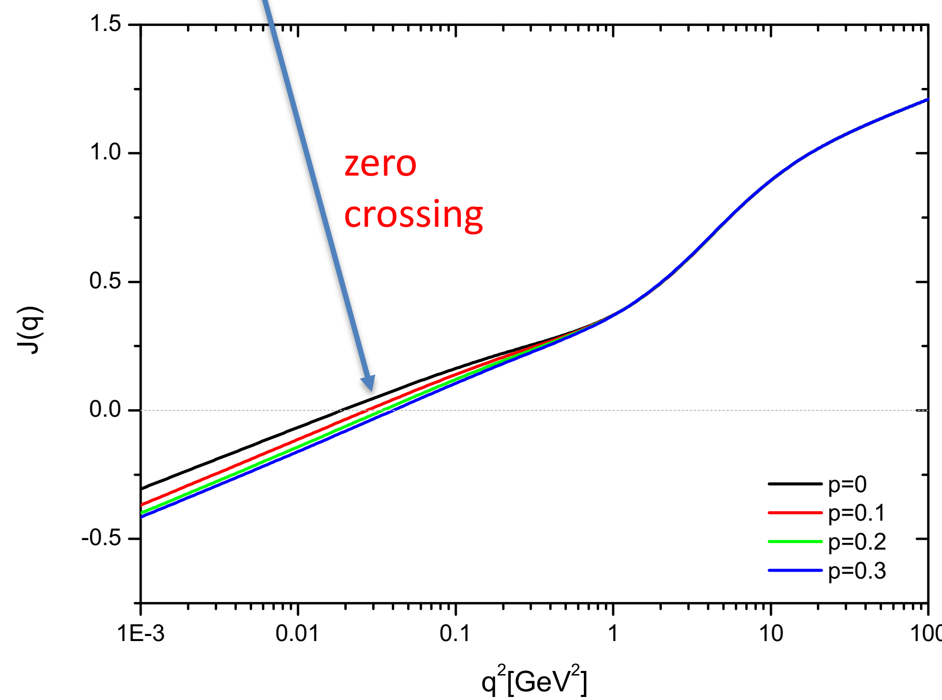
System of coupled equations





p	Crossing [MeV]
0	139
0.1	166
0.2	187
0.3	202





Zero crossing “passes” from $J(q^2)$ to $\Gamma^{\mathbf{np}}$

- The STIs must be realized in part by means of a longitudinally coupled pole term

$$\mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = \Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) + \Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)$$

$$\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$$

$$q^{\alpha}\Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) = p^2 J(p^2) P_{\mu\nu}(p) - r^2 J(r^2) P_{\mu\nu}(r)$$

$$q^{\alpha}\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = m^2(p^2) P_{\mu\nu}(p) - m^2(r^2) P_{\mu\nu}(r)$$

sum by parts

Full (abelianized) STI: $q^{\alpha}\mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p) = \Delta^{-1}(p^2) P_{\mu\nu}(p) - \Delta^{-1}(r^2) P_{\mu\nu}(r)$

Note that the kinematic structure of $\Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p)$ and $\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)$ is very different

$P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = 0$ vs $P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,\Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) \neq 0$

The Slavnov-Taylor identity of $\Gamma^{\mathbf{np}}_{\alpha\mu\nu}$

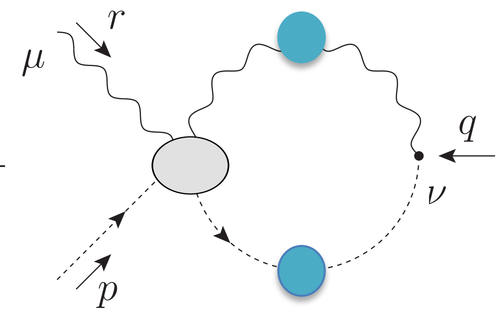
$$q^{\alpha}\Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) = F(q^2)\left[p^2 J(p^2) P^{\alpha}_{\nu}(p) H_{\alpha\mu}(p,q,r) - r^2 J(r^2) P^{\alpha}_{\mu}(r) H_{\alpha\nu}(r,q,p)\right]$$

$F(q^2)$: ghost dressing function

$H_{\alpha\mu}(p,q,r)$: Ghost-gluon kernel

$$D(q) \equiv \frac{iF(q)}{q^2}$$

$D(q)$: ghost propagator

$$H_{\nu\mu}(q,p,r) = g_{\mu\nu} +$$ 

$$H_{\nu\mu}(q,p,r) = g_{\mu\nu}A_1 + q_{\mu}q_{\nu}A_2 + r_{\mu}r_{\nu}A_3 + q_{\mu}r_{\nu}A_4 + r_{\mu}q_{\nu}A_5$$

Bose-symmetric Ball-Chiu basis

$$\Gamma_{\mathbf{np}}^{\alpha\mu\nu}(q,r,p) = \underbrace{\Gamma_L^{\alpha\mu\nu}(q,r,p)}_{\text{Saturates the STI}} + \underbrace{\Gamma_T^{\alpha\mu\nu}(q,r,p)}_{\text{Automatically conserved}}$$

$$\Gamma_L^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{10} X_i(q,r,p)\ell_i^{\alpha\mu\nu}$$

$$\ell_1^{\alpha\mu\nu} = (q-r)^\nu g^{\alpha\mu} \qquad \ell_2^{\alpha\mu\nu} = -p^\nu g^{\alpha\mu} \qquad \ell_3^{\alpha\mu\nu} = (q-r)^\nu[q_\mu r^\alpha - (q\cdot r)g_{\alpha\mu}]$$

$$\ell_4^{\alpha\mu\nu} = (r-p)^\alpha g^{\mu\nu} \qquad \ell_5^{\alpha\mu\nu} = -q^\alpha g^{\mu\nu} \qquad \ell_6^{\alpha\mu\nu} = (r-p)^\alpha[r^\nu p^\mu - (r\cdot p)g^{\mu\nu}],$$

$$\ell_7^{\alpha\mu\nu} = (p-q)^\mu g^{\alpha\nu} \qquad \ell_8^{\alpha\mu\nu} = -r^\mu g^{\alpha\nu} \qquad \ell_9^{\alpha\mu\nu} = (p-q)^\mu[p^\alpha q^\nu - (p\cdot q)g^{\alpha\nu}]$$

$$\ell_{10}^{\alpha\mu\nu} = q^\nu r^\alpha p^\mu + q^\mu r^\nu p^\alpha$$

$$\Gamma_T^{\alpha\mu\nu}(q,r,p) = \sum_{i=1}^{4} Y_i(q,r,p)t_i^{\alpha\mu\nu}$$

$$t_1^{\alpha\mu\nu} = [(q\cdot r)g^{\alpha\mu} - q^\mu r^\alpha][(r\cdot p)q^\nu - (q\cdot p)r^\nu]$$

$$t_2^{\alpha\mu\nu} = [(r\cdot p)g^{\mu\nu} - r^\nu p^\mu][(p\cdot q)r^\alpha - (r\cdot q)p^\alpha]$$

$$t_3^{\alpha\mu\nu} = [(p\cdot q)g^{\nu\alpha} - p^\alpha q^\nu][(q\cdot r)p^\mu - (p\cdot r)q^\mu]$$

$$t_4^{\alpha\mu\nu} = g^{\mu\nu}[(p\cdot q)r^\alpha - (r\cdot q)p^\alpha] + g^{\nu\alpha}[(q\cdot r)p^\mu - (p\cdot r)q^\mu] + g^{\alpha\mu}[(r\cdot p)q^\nu - (q\cdot p)r^\nu] + p^\alpha q^\mu r^\nu - r^\alpha p^\mu q^\nu$$

The Ball-Chiu solution

Define:

$$a_{qpr} := F(r)J(p)A_1(p,r,q)$$

$$b_{qpr} := F(r)J(p)A_3(p,r,q)$$

$$d_{qpr} := F(r)J(p)[A_4(p,r,q) - A_3(p,r,q)]$$

Matching tensorial sturctures

\+ *an important constraint*

$$X_1(q,r,p) = \frac{1}{4}[2(a_{pqr} + a_{prq}) + p^2(b_{qrp} + b_{rqp}) + 2(q \cdot p\, d_{prq} + r \cdot p\, d_{pqr}) + (q^2 - r^2)(b_{rpq} + b_{pqr} - b_{qpr} - b_{prq})]$$

$$X_2(q,r,p) = \frac{1}{4}[2(a_{prq} - a_{pqr}) - (q^2 - r^2)(b_{qrp} + b_{rqp}) + 2(q \cdot p\, d_{prq} - r \cdot p\, d_{pqr}) + p^2(b_{prq} - b_{pqr} + b_{qpr} - b_{rpq})]$$

$$X_3(q,r,p) = \frac{1}{q^2 - r^2}[a_{rpq} - a_{qpr} + r \cdot p\, d_{qpr} - q \cdot p\, d_{rpq}],$$

$$X_{10}(q,r,p) = -\frac{1}{2}[b_{qrp} + b_{rpq} + b_{pqr} - b_{qpr} - b_{rqp} - b_{prq}]$$

The "abelianized" solution

Turn off the ghost sector

$F(q^2) \to 1$

$H_{\mu\nu} \to g_{\mu\nu}$

$$X_1^{\mathrm{Ab}}(q,r,p) = \frac{1}{2}[J(r) + J(q)]$$

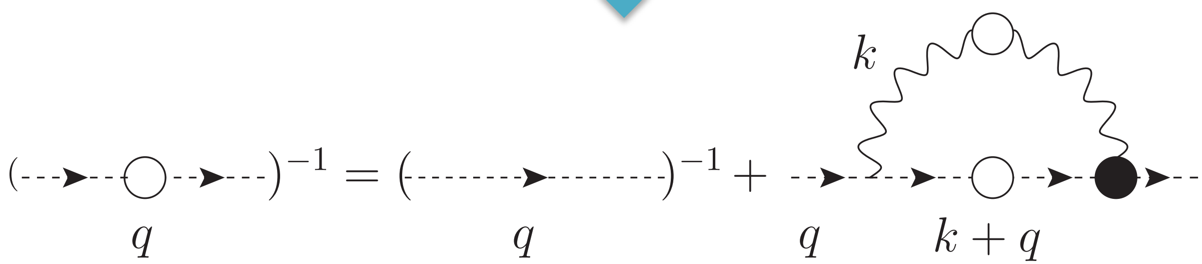
$$X_2^{\mathrm{Ab}}(q,r,p) = \frac{1}{2}[J(q) - J(r)]$$

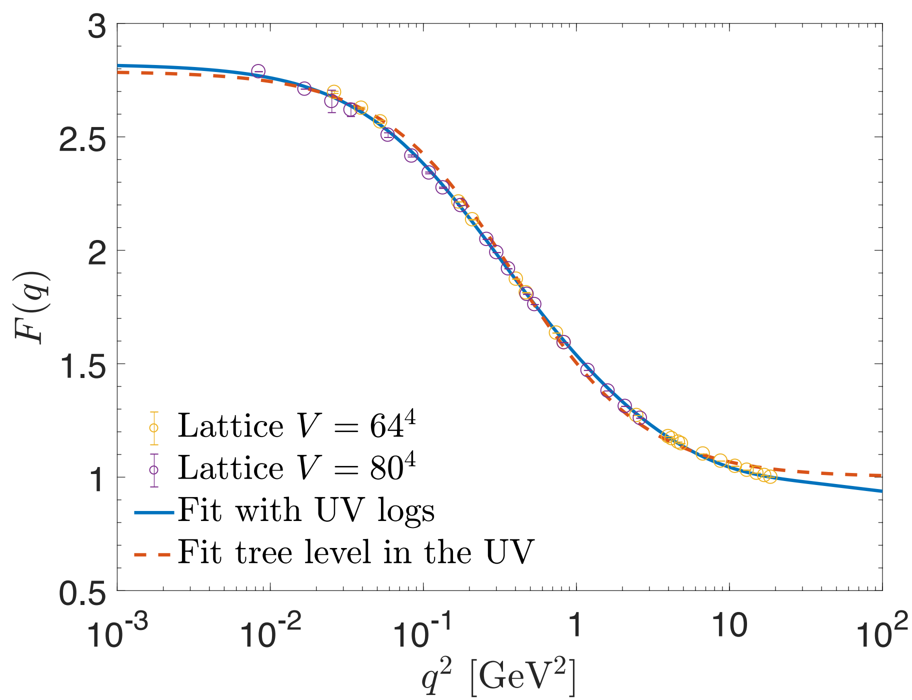
$$X_3^{\mathrm{Ab}}(q,r,p) = \frac{[J(q) - J(r)]}{q^2 - r^2},$$

$$X_{10}^{\mathrm{Ab}}(q,r,p) = 0$$

Collecting the ingredients of the ghost sector

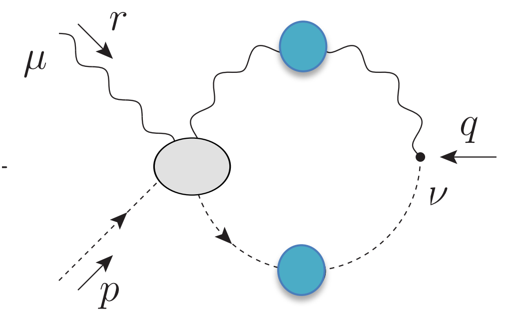
1 $\left\{F(q^2)\right\}$ From lattice simulations and the SDE of the ghost propagator



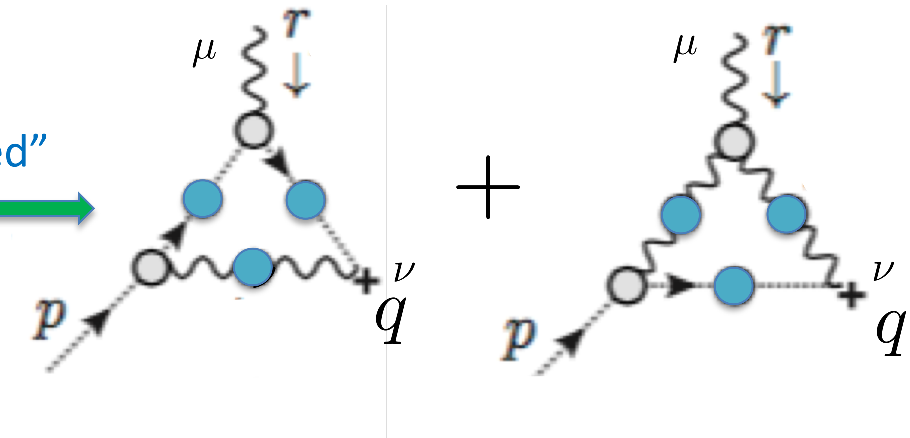


The ghost-gluon kernel

- $\{A_1, A_3\,, A_4\}$ computed from the SDE of the ghost-gluon kernel



"One-loop dressed" approximation



(from previous steps) ⟹ *Exact constraint*

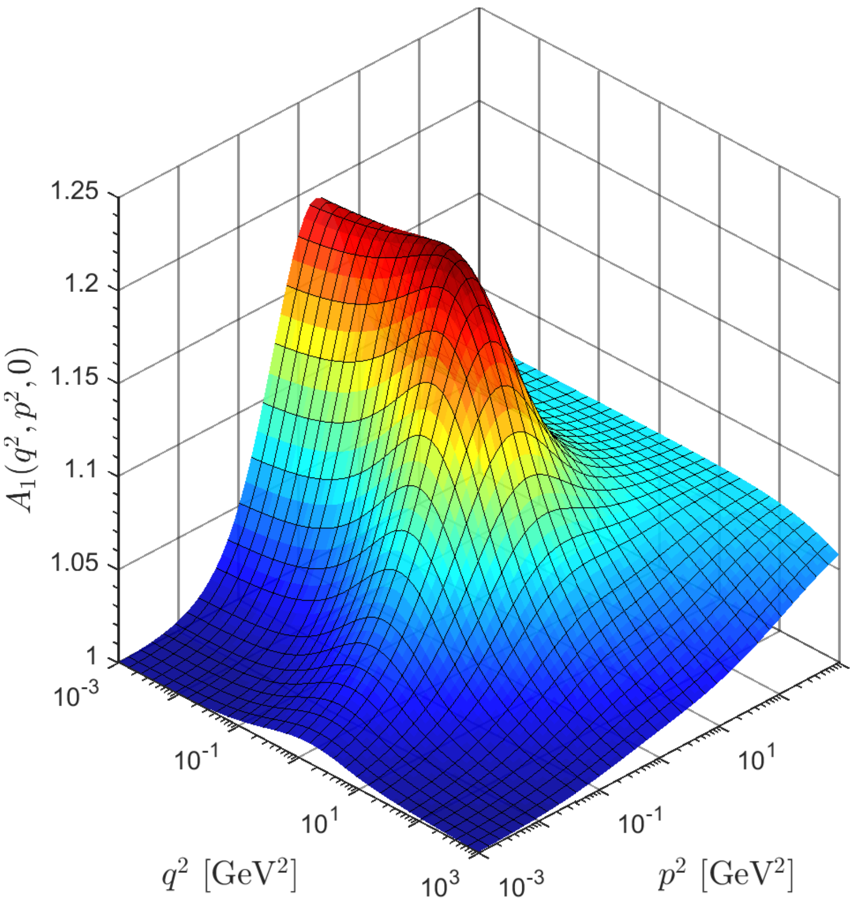
$$\mathcal{R}(q^2,p^2,r^2) = \frac{F(r)[A_1(q,r,p) + p^2 A_3(q,r,p) + (q\cdot p)A_4(q,r,p)]}{F(p)[A_1(q,p,r) + r^2 A_3(q,p,r) + (q\cdot r)A_4(q,p,r)]}$$

$$\mathcal{R}(q^2,p^2,r^2) = 1$$

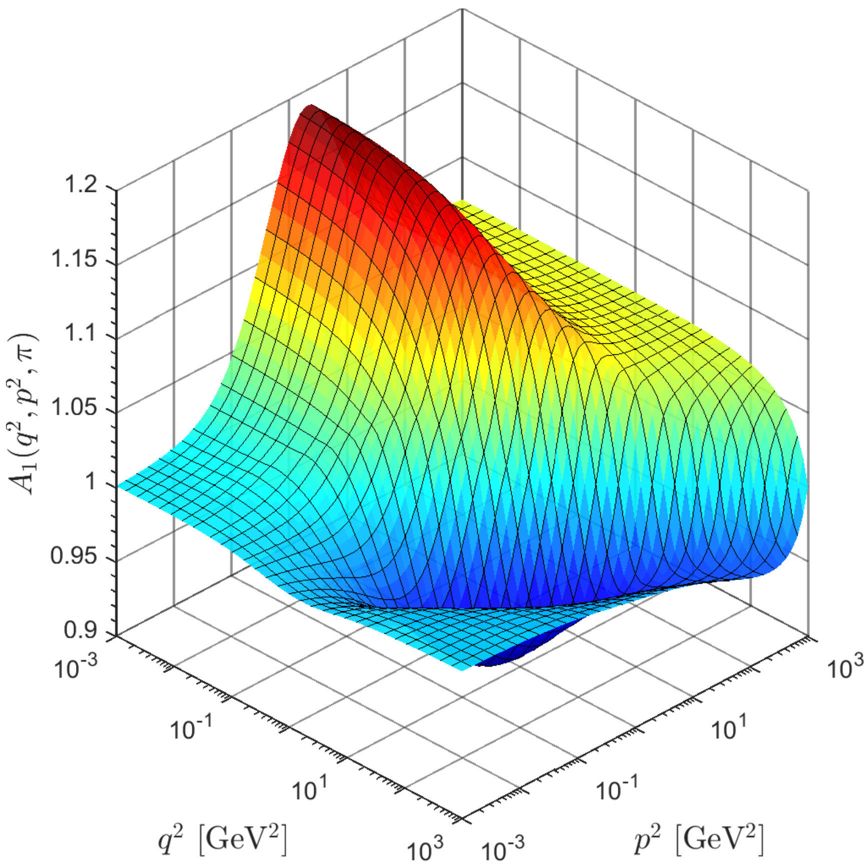
Non-trivial check of the trunction

$$H_{\nu\mu}(q,p,r) = g_{\mu\nu} A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + r_\mu q_\nu A_5$$

$\theta = 0$

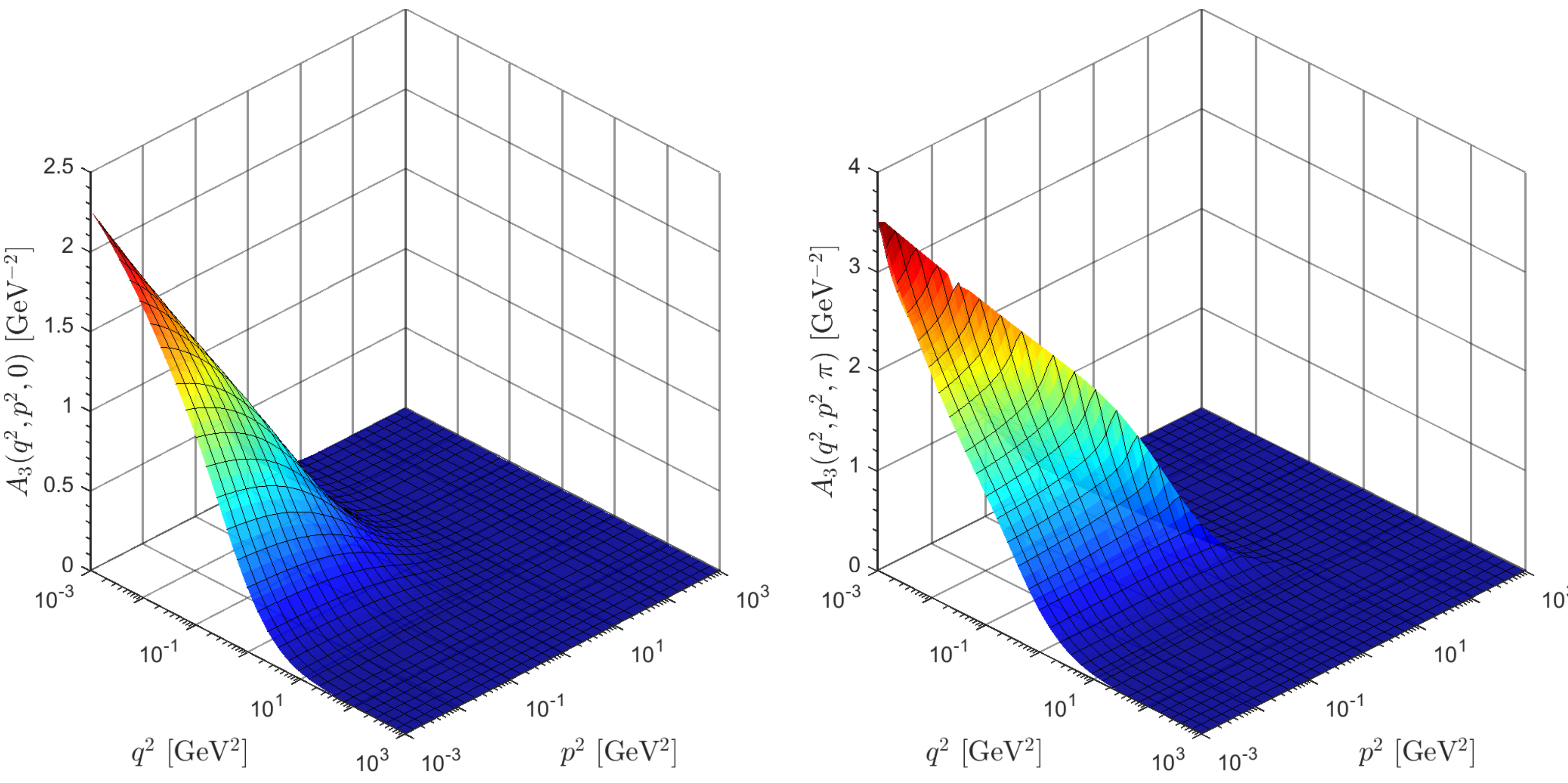


$\theta = \pi$

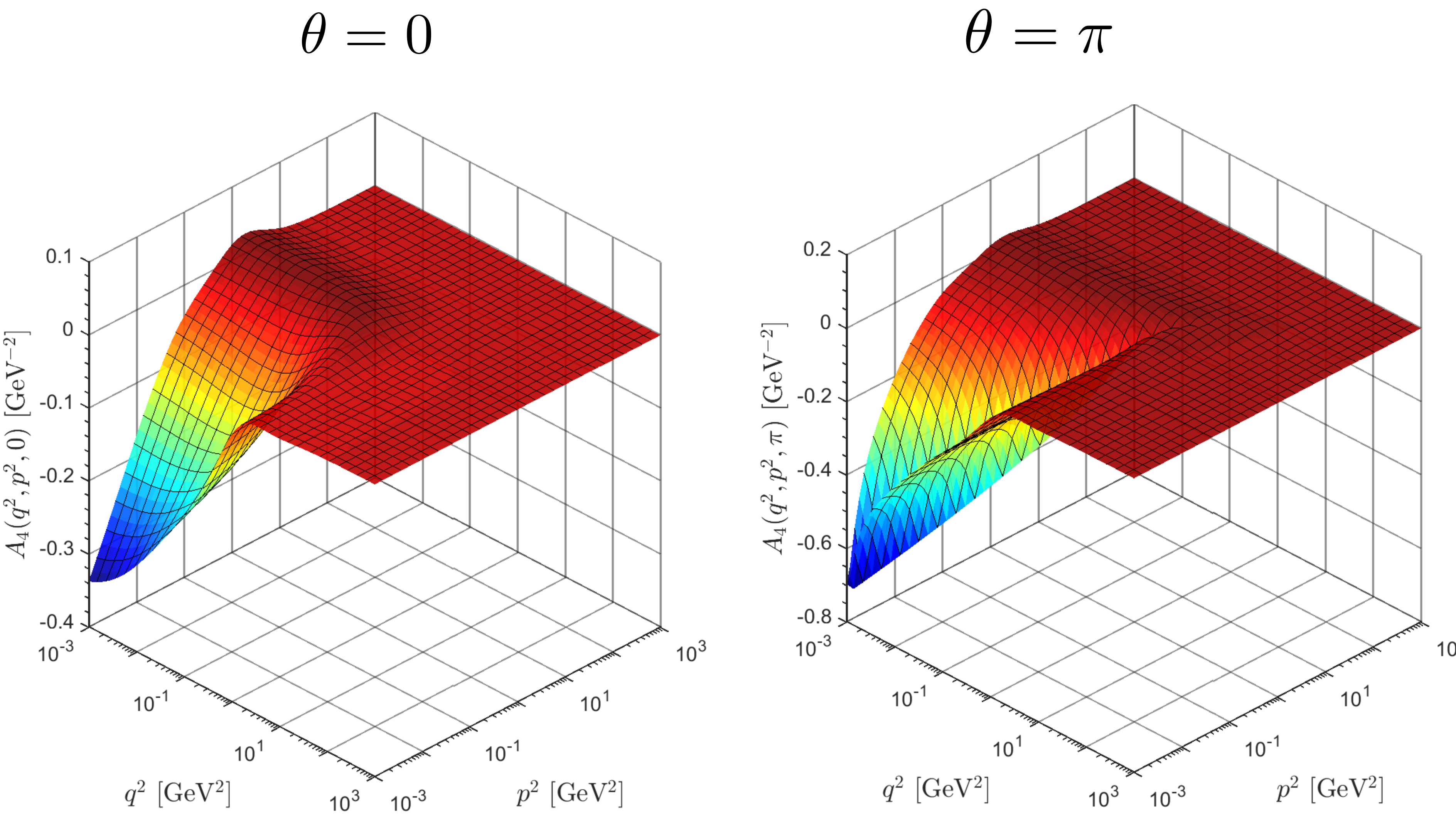


$$H_{\nu\mu}(q,p,r) = g_{\mu\nu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + r_\mu q_\nu A_5$$

$\theta = 0$ $\qquad\qquad$ $\theta = \pi$

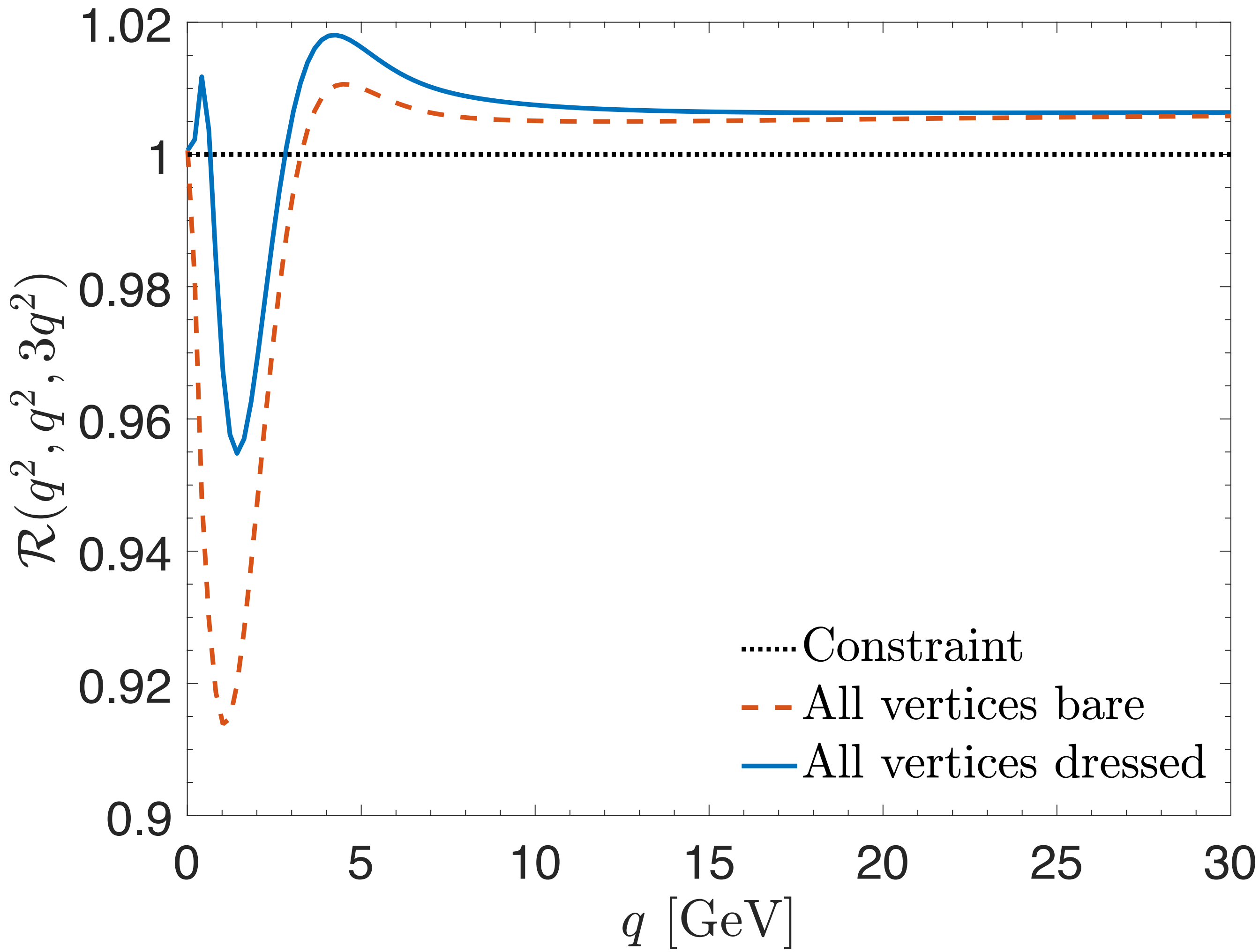


$$H_{\nu\mu}(q,p,r) = g_{\mu\nu}A_1 + q_\mu q_\nu A_2 + r_\mu r_\nu A_3 + q_\mu r_\nu A_4 + r_\mu q_\nu A_5$$



What about the constraint ?

Computed in the kinematic configuration where $p^2 = q^2$ and $r^2 = 3q^2$

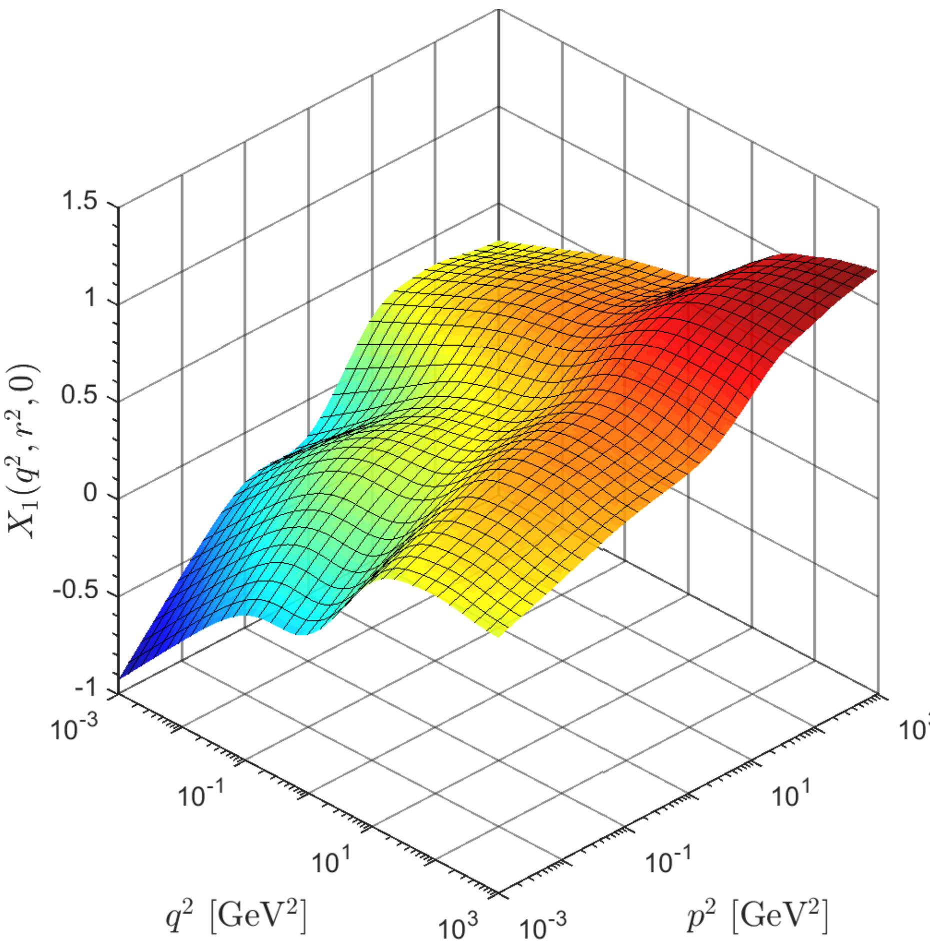


Looks rather "controlled"…

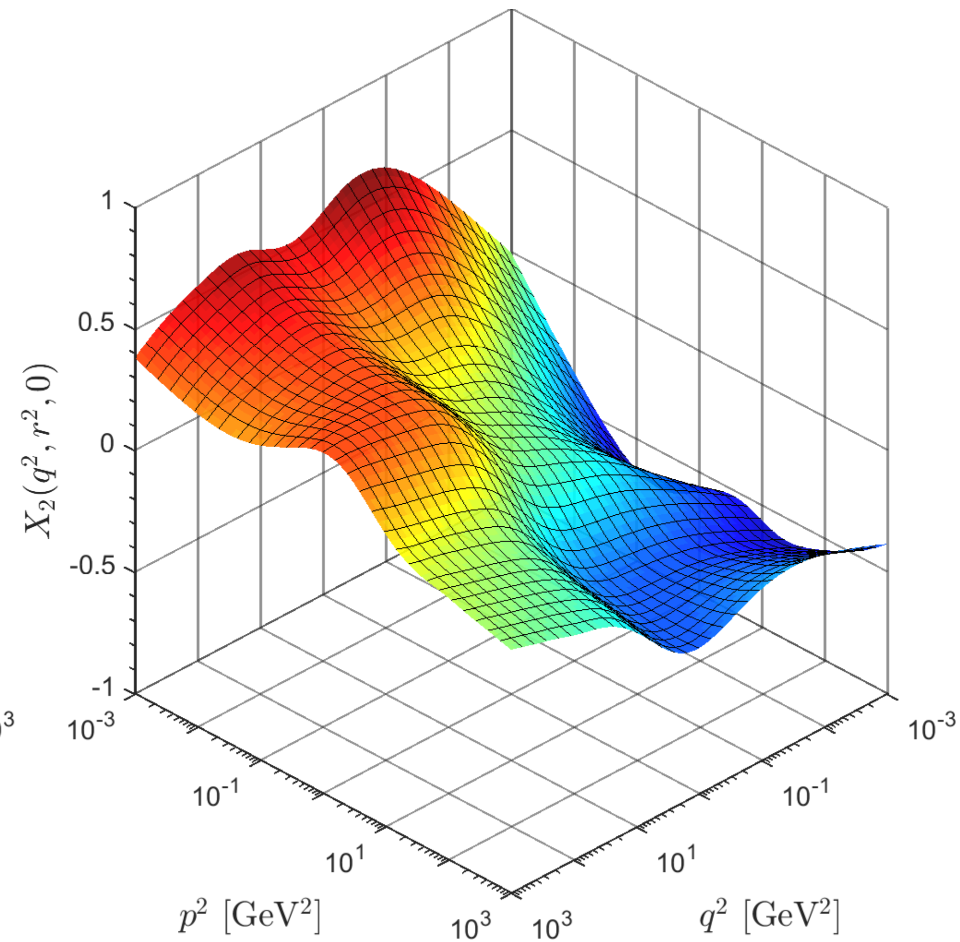
Numerical results
for the three gluon vertex

✓ The angular dependence is weak and barely visible in the 3D plots

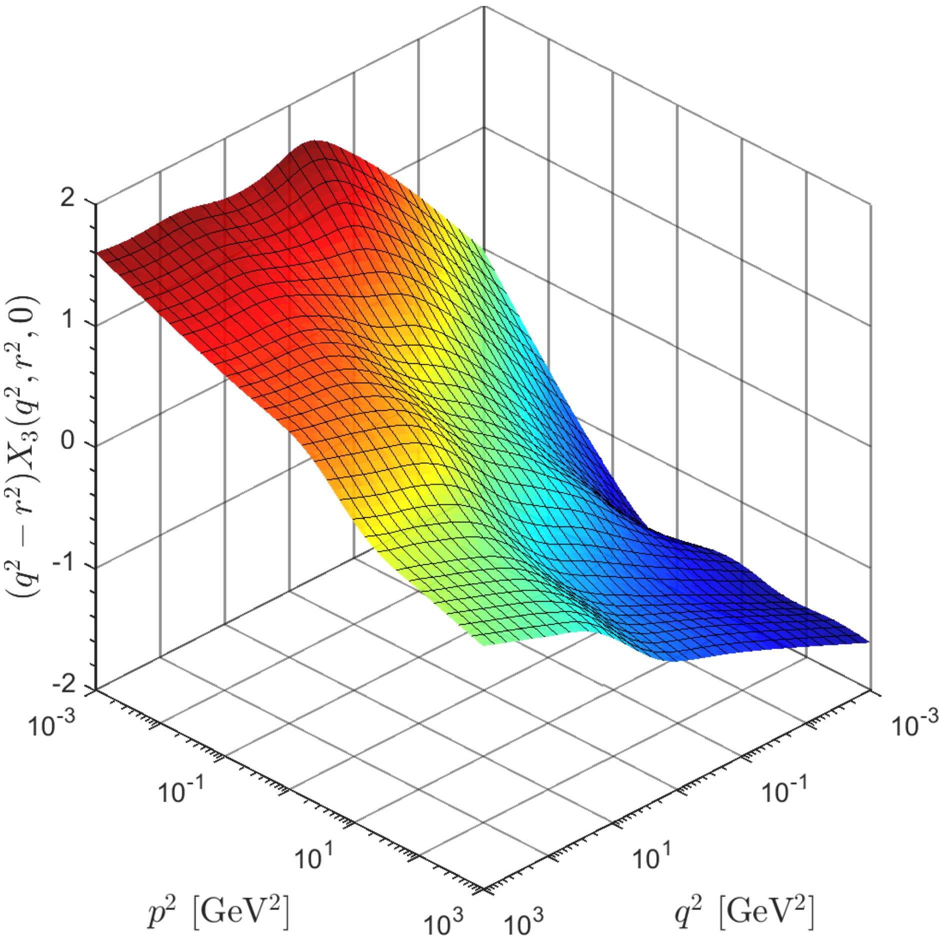
$X_1(q^2, r^2, \theta = 0^\circ)$



$X_2(q^2, r^2, \theta = 0^\circ)$



$$(q^2 - r^2)X_3(q^2, r^2, \theta = 0^\circ)$$

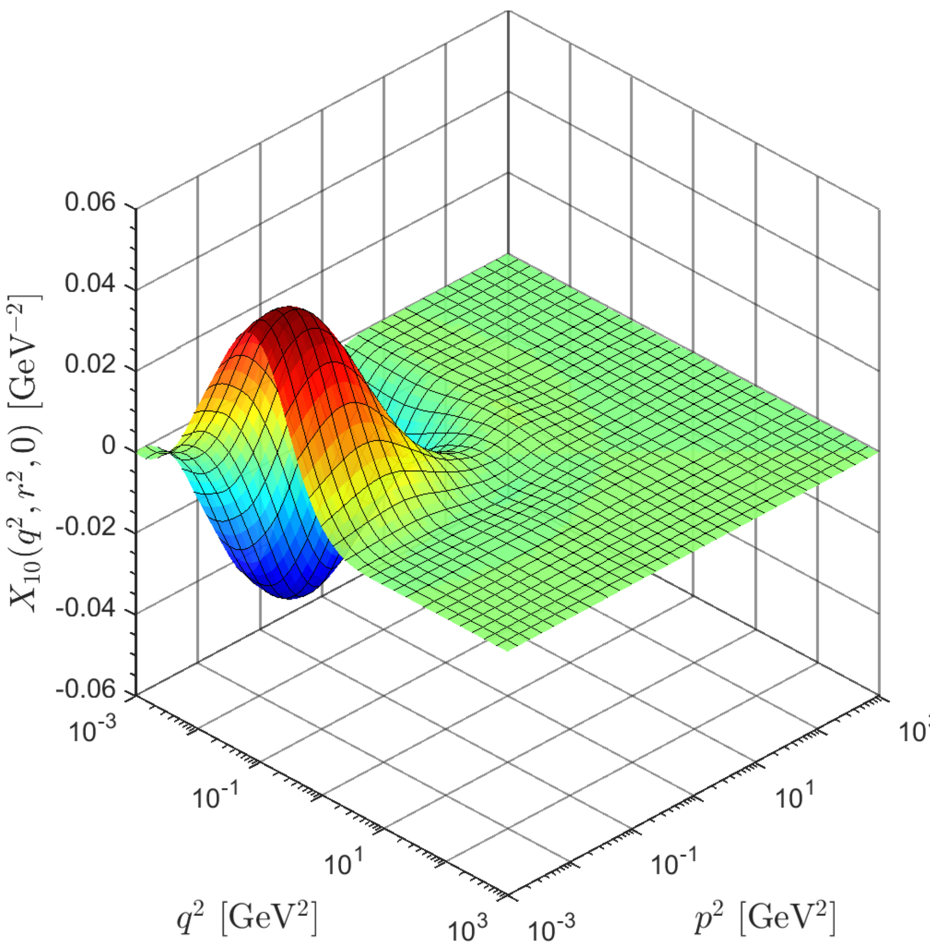


- ✓ Very suppressed
- ✓ Display the strong angular dependence
- ✓ Recovers the perturbative behavior
- ✓ X_{10} vanishes in both tree and one loop levels

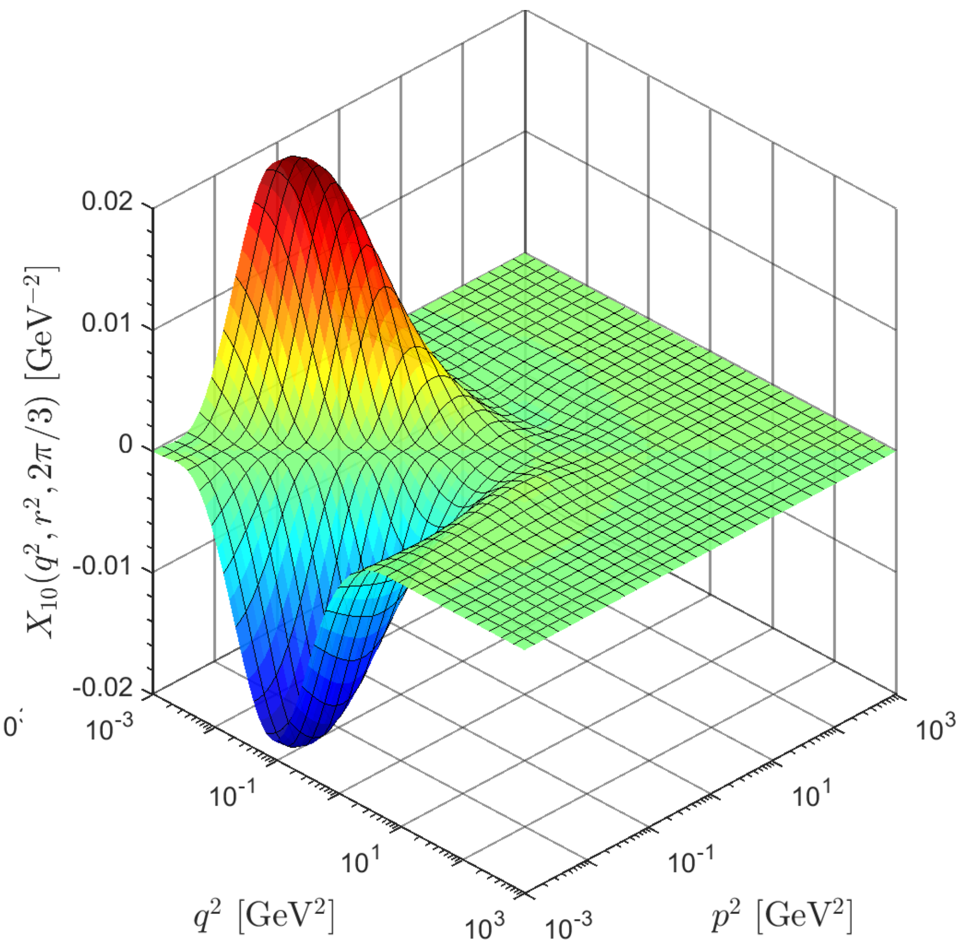
J. S. Ball and T. W. Chiu, Phys. Rev. D 22, 2550 (1980)

A. I. Davydychev, P. Osland, O. V. Tarasov, Phys. Rev. D54, 4087 (1996)

$$X_{10}(q^2, r^2, \theta = 0^\circ)$$



$$X_{10}(q^2, r^2, \theta = 120^\circ)$$



Comparison with the lattice

$$L(q,p,r) = \frac{W_{\alpha'\mu'\nu'}(q,r,p)P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,\mathrm{I\!\Gamma}_{\alpha\mu\nu}(q,r,p)}{W^{\alpha\mu\nu}(q,p,r)W_{\alpha\mu\nu}(q,p,r)}$$

$$= \frac{W_{\alpha'\mu'\nu'}(q,r,p)P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,[\Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) + \Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)]}{W^{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,p,r)}$$

But, the pole term of the vertex is longitudinally coupled

$$P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = 0$$

Comparison with the lattice

$$L(q,p,r) = \frac{W_{\alpha'\mu'\nu'}(q,r,p)P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,\mathrm{I}\!\Gamma_{\alpha\mu\nu}(q,r,p)}{W^{\alpha\mu\nu}(q,p,r)W_{\alpha\mu\nu}(q,p,r)}$$

$$= \frac{W_{\alpha'\mu'\nu'}(q,r,p)P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,[\Gamma^{\mathbf{np}}_{\alpha\mu\nu}(q,r,p) + \Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p)]}{W^{\alpha\mu\nu}(q,r,p)W_{\alpha\mu\nu}(q,p,r)} \qquad \Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) \to 0$$

But, the pole term of the vertex is longitudinally coupled

$$P^{\alpha\alpha'}(q)P^{\mu\mu'}(r)P^{\nu\nu'}(p)\,\Gamma^{\mathbf{p}}_{\alpha\mu\nu}(q,r,p) = 0$$

$L(q,p,r)$ depends only on the form factors of $\Gamma^{\mathbf{np}}_{\alpha\mu\nu}$

Comparison with the lattice

- In the totally symmetric configuration:

$$q^2 = p^2 = r^2 = Q^2 \qquad q \cdot p = q \cdot r = p \cdot r = -\frac{1}{2}Q^2$$

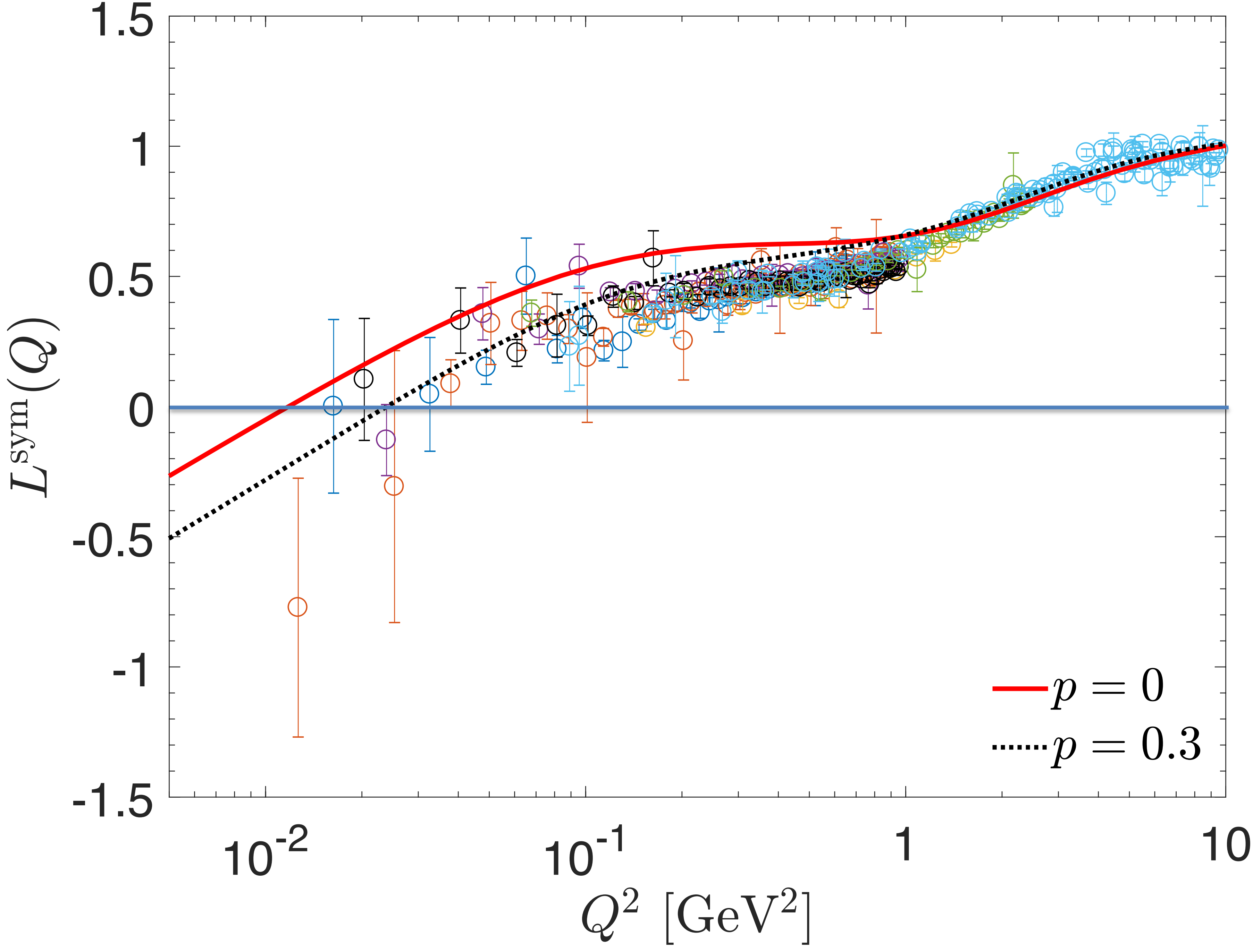
- Lattice has access to the following combination of form factors

$$L^{sym}(Q) = \frac{1}{2}\left[2X_1(Q) - Q^2 X_3(Q)\right] + \frac{Q^2}{4}\left[Q^2 Y_1(Q) + 2Y_4(Q)\right]$$

- **Our approach does *not* determine the transverse form factors**

We will set $Y_i(Q) = 0$

A. Athenodorou, D. Binosi, P. Boucaud, F. De Soto, J.P, J. Rodriguez-Quintero and S.Zafeiropoulos,
Phys. Lett. B761, 444 (2016)



1.5
1
0.5
0
-0.5
-1
-1.5
$L^{\rm sym}(Q)$
10^{-2}
10^{-1}
1
10
Q^2 [GeV^2]
$p = 0$
$p = 0.3$

What if we had applied the Ball-Chiu procedure naively ?

$$q^{\alpha}\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p) = F(q^2)\left[\Delta^{-1}(p^2)P_{\nu}^{\alpha}(p)H_{\alpha\mu}(p,q,r) - \Delta^{-1}(r^2)P_{\mu}^{\alpha}(r)H_{\alpha\nu}(r,q,p)\right]$$

Do not set $\Delta^{-1}(q^2) = q^2 J(q^2) + m^2(q^2)$

Instead, interpret Ball-Chiu literally

$$\Delta^{-1}(q^2) = q^2 J_{\mathrm{BC}}(q^2) \quad \Rightarrow \quad J_{\mathrm{BC}}(q^2) = \frac{\Delta^{-1}(q^2)}{q^2}$$

But if $\Delta^{-1}(0) = m^2(0)$

$\Rightarrow$ $J_{\mathrm{BC}}(0) \to \infty$

$\Downarrow$

$\mathrm{I\Gamma}_{\alpha\mu\nu}(q,r,p)$ has massless poles

but

of the wrong kind !