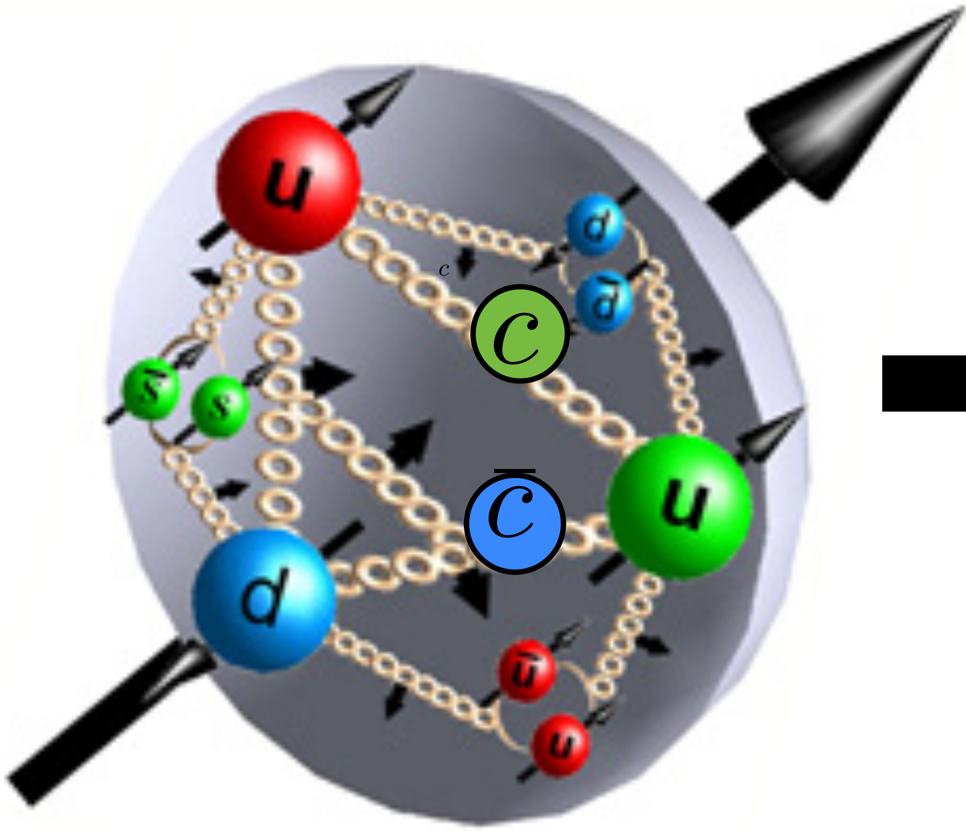
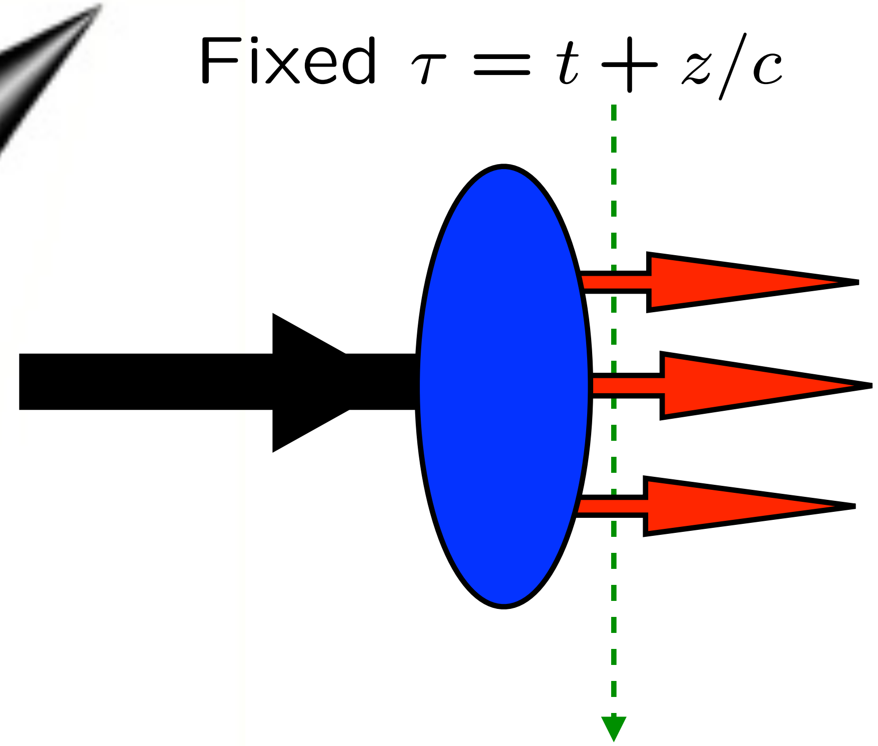
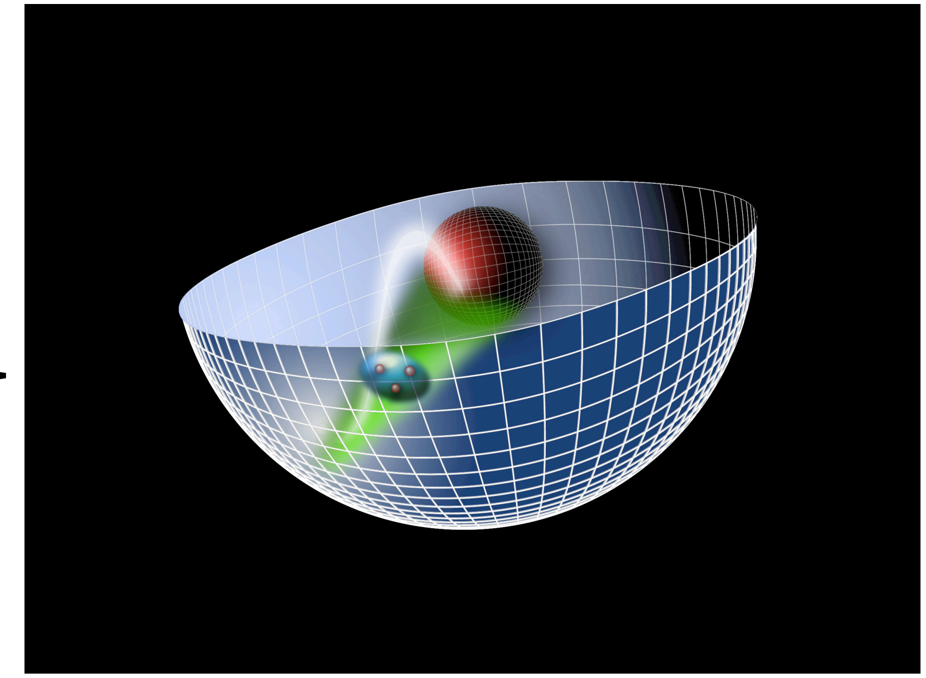
# *Emergent Hadron Mass from Light-Front Holography and Superconformal Quantum Mechanics*




Stan Brodsky

SLAC
NATIONAL ACCELERATOR LABORATORY

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17-21 September 2018

EUROPEAN CENTRE FOR THEORETICAL STUDIES
IN NUCLEAR PHYSICS AND RELATED AREAS


**with Guy de Tèramond, Hans Günter Dosch, C. Lorce, K. Chiu, R. S. Sufian, A.Deur, C. Roberts**

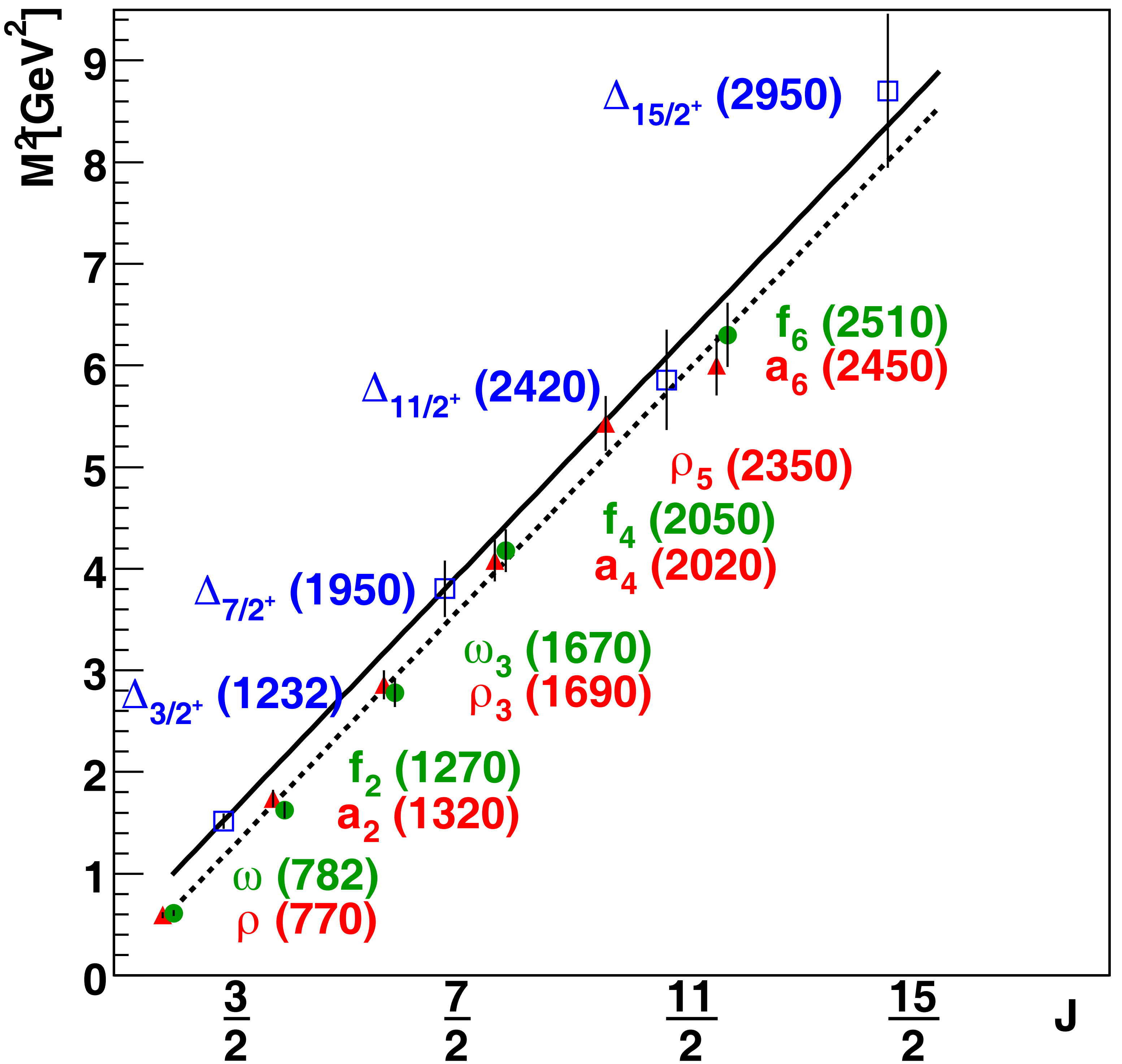

**Emergent Mass and its Consequences in the Standard Model**

## A Profound Question for Hadron Physics

- *Origin of the QCD Mass Scale*
- *Color Confinement*
- *Spectroscopy: Tetraquarks, Pentaquarks, Gluonium, Exotic States*
- *Universal Regge Slopes: n, L, both Mesons and Baryons*
- *Massless Pion: Bound State*
- *Dynamics and Spectroscopy*
- *QCD Coupling at all Scales*
- *QCD Vacuum —Do QCD Condensates Exist?*

Mesons and Baryons: Same Regge Slope $M^2 \propto J$ !

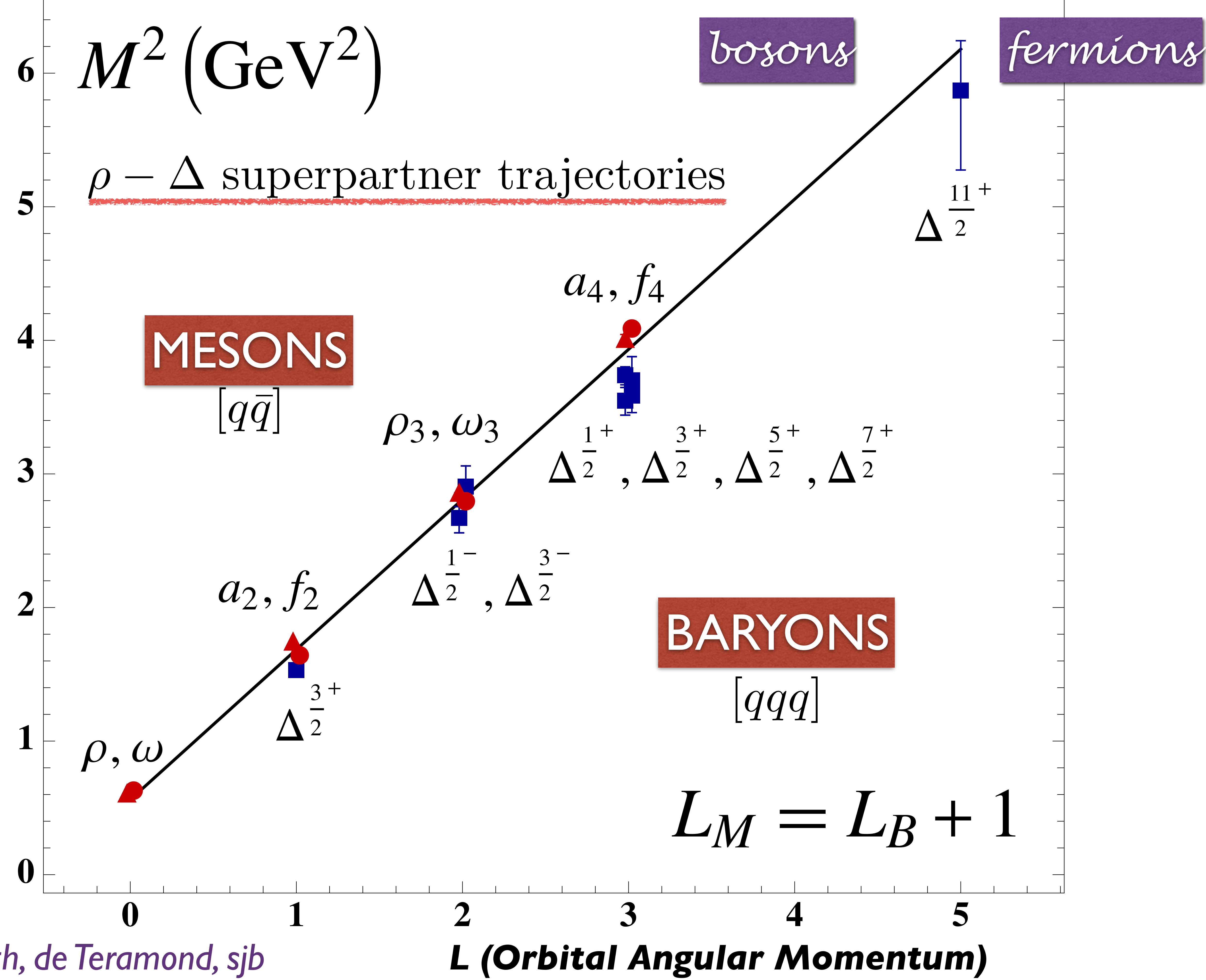
M²[GeV²]




***The leading Regge trajectory: Δ resonances with maximal J in a given mass range. Also shown is the Regge trajectory for mesons with J = L+S.***

*E. Klempt and B. Ch. Metsch*

2012

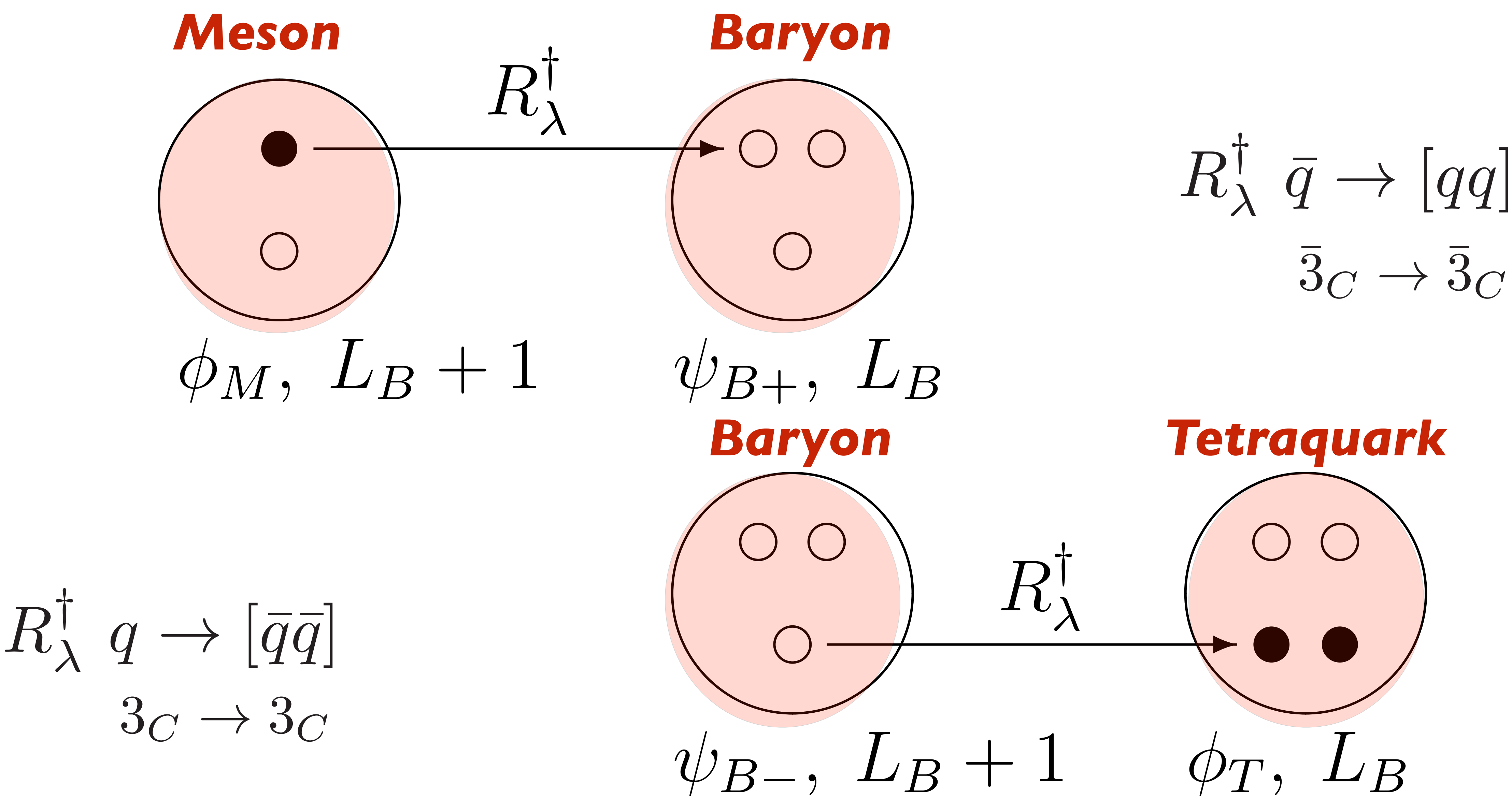



Dosch, de Teramond, sjb

# Superconformal Algebra

## 2X2 Hadronic Multiplets

### Bosons, Fermions with Equal Mass!

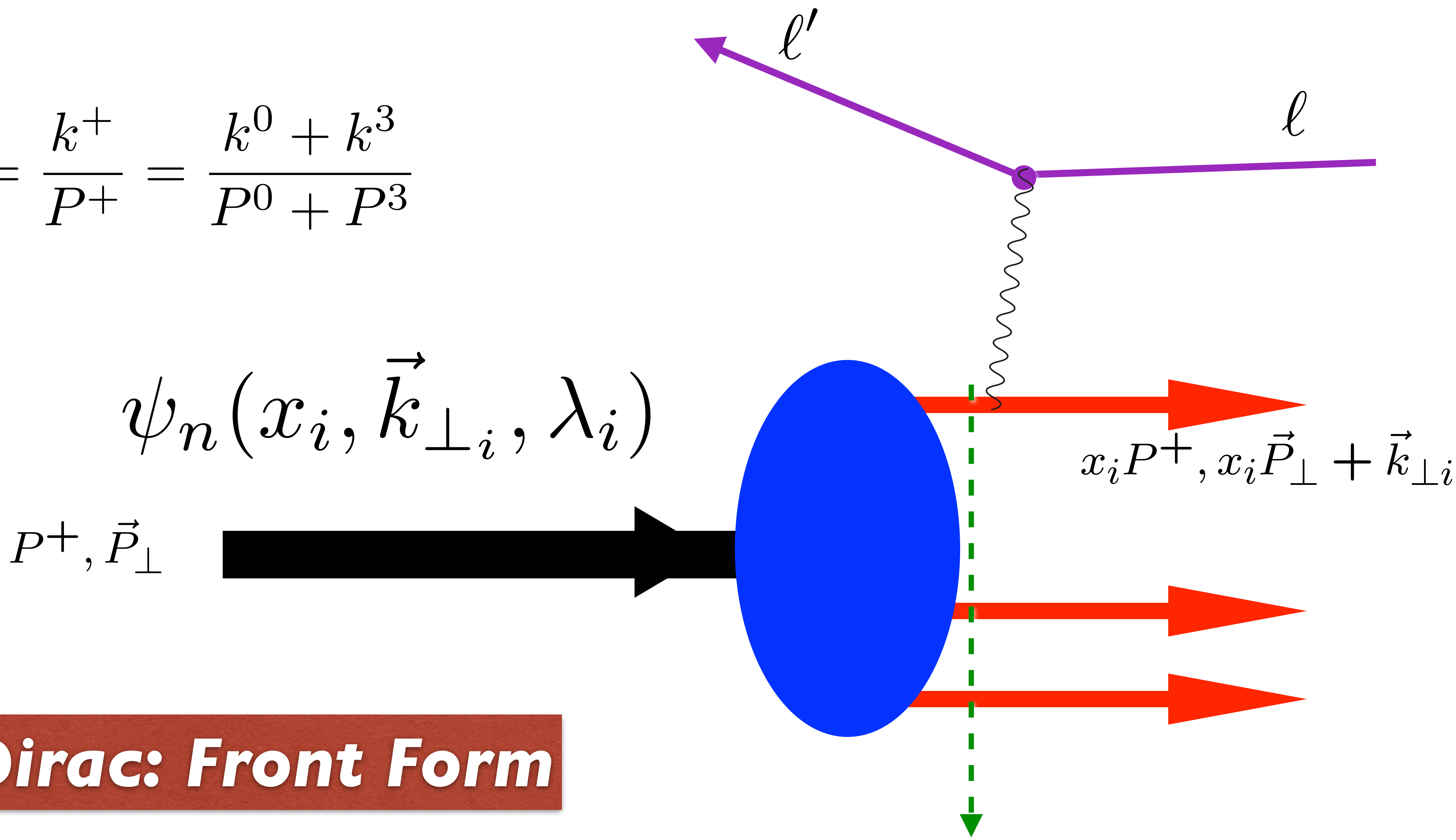



Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

# Supersymmetry in QCD

- A hidden symmetry of Color $SU(3)_C$ in hadron physics
- QCD: No squarks or gluinos!
- Emerges from Light-Front Holography and Super-Conformal Algebra
- Color Confinement
- Massless Pion in Chiral Limit

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$




## Dirac: Front Form

***Measurements of hadron LF wavefunction are at fixed LF time***

Fixed $\tau = t + z/c$

***Like a flash photograph***

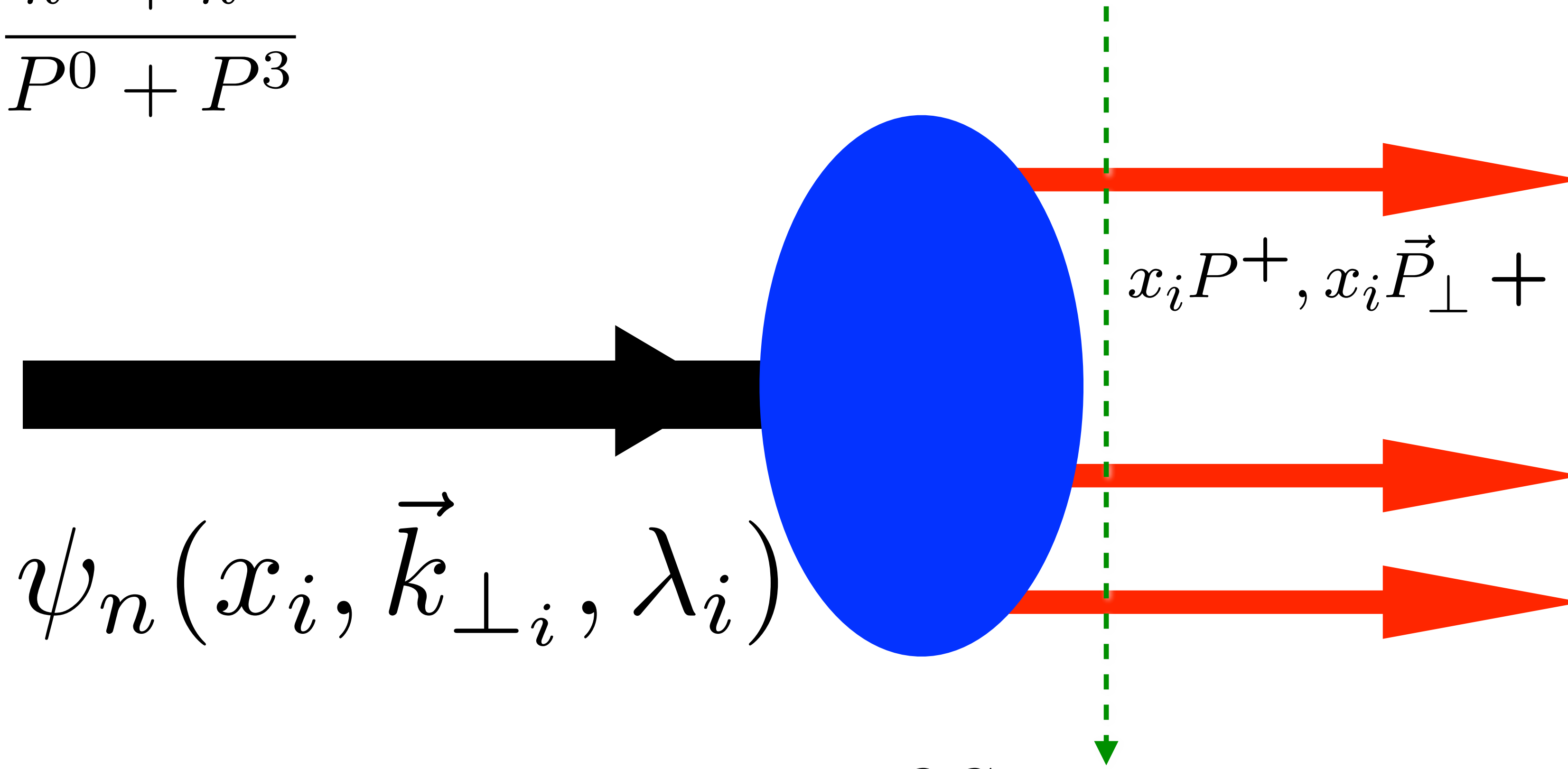
$$x_{bj} = x = \frac{k^+}{P^+}$$

*Invariant under boosts! Independent of $P^\mu$*

# Light-Front Wavefunctions: rigorous representation of composite systems in quantum field theory

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

Fixed $\tau = t + z/c$



$P^+, \vec{P}_\perp$

$x_i P^+, x_i \vec{P}_\perp + \vec{k}_{\perp i}$

$\sum_i^n x_i = 1$

$\sum_i^n \vec{k}_{\perp i} = \vec{0}_\perp$

$$\psi_n(x_i, \vec{k}_{\perp_i}, \lambda_i)$$

*Eigenstate of LF Hamiltonian*

$$H_{LF}^{QCD} |\Psi_h> = \mathcal{M}_h^2 |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

*Invariant under boosts! Independent of $P^\mu$*

***Causal, Frame-independent. Creation Operators on Simple Vacuum, Current Matrix Elements are Overlaps of LFWFS***

# Light-Front QCD

*Physical gauge:* $A^+ = 0$

*Exact frame-independent formulation of nonperturbative QCD!*

$$L^{QCD} \to H^{QCD}_{LF}$$

$$H^{QCD}_{LF} = \sum_i [\frac{m^2 + k^2_\perp}{x}]_i + H^{int}_{LF}$$

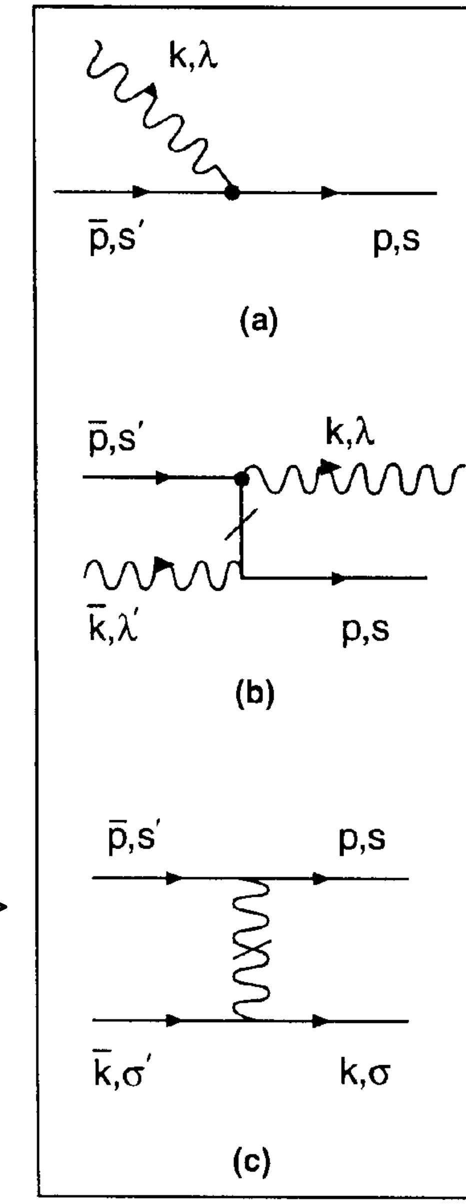
$H^{int}_{LF}$: Matrix in Fock Space

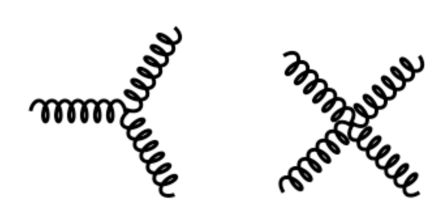
$$H^{QCD}_{LF} |\Psi_h> = \mathcal{M}^2_h |\Psi_h>$$

$$|p, J_z> = \sum_{n=3} \psi_n(x_i, \vec{k}_{\perp i}, \lambda_i) |n; x_i, \vec{k}_{\perp i}, \lambda_i>$$

*Eigenvalues and Eigensolutions give Hadronic Spectrum and Light-Front wavefunctions*

**LFWFs: Off-shell in P- and invariant mass**



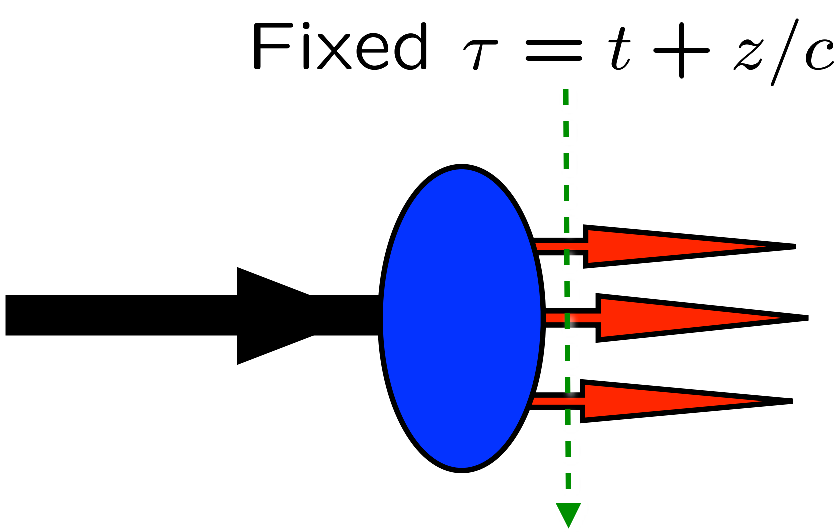



$H^{int}_{LF}$

# Bound States in Relativistic Quantum Field Theory:

## *Light-Front Wavefunctions*

Dirac's Front Form: Fixed $\tau = t + z/c$




$$\psi(x_i, \vec{k}_{\perp i}, \lambda_i)$$

$$x = \frac{k^+}{P^+} = \frac{k^0 + k^3}{P^0 + P^3}$$

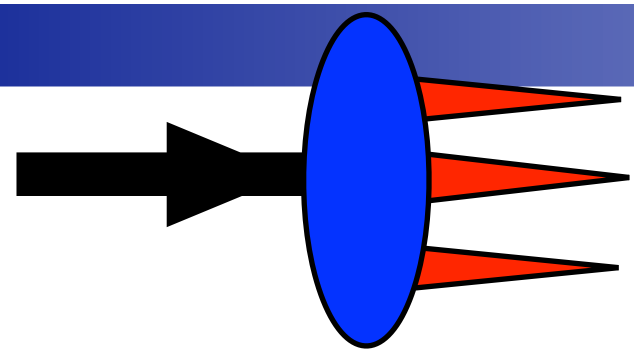
***Invariant under boosts. Independent of $P^\mu$***

$$\mathrm{H}^{QCD}_{LF} |\psi> = M^2 |\psi>$$

**Direct connection to QCD Lagrangian**

**LF Wavefunction: off-shell in invariant mass**

*Remarkable new insights from AdS/CFT, the duality between conformal field theory and Anti-de Sitter Space*



$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$

## Light-Front Wavefunctions underly hadronic observables

*Lorce, Pasquini*

Momentum space $\vec{k}_\perp \leftrightarrow \vec{z}_\perp$ Position space

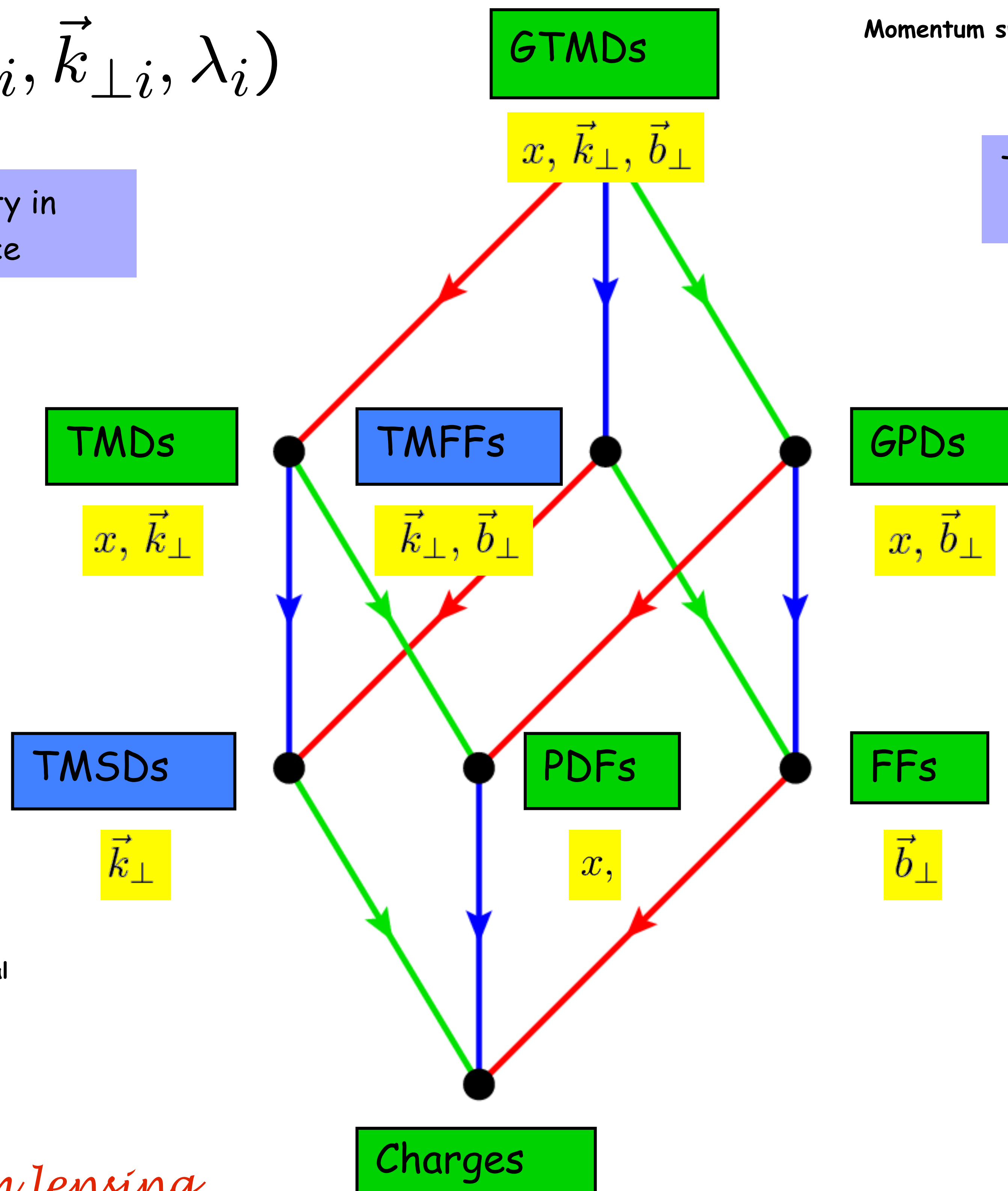
$\vec{\Delta}_\perp \leftrightarrow \vec{b}_\perp$

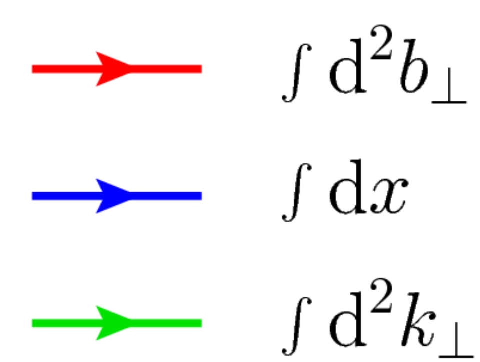
Transverse density in momentum space

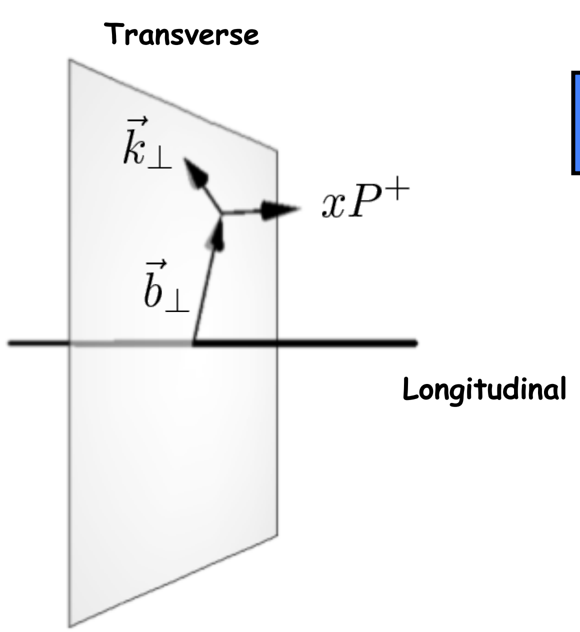
Transverse density in position space

*Weak transition form factors*

*DGLAP, ERBL Evolution Factorization Theorems*




*Sivers, T-odd from lensing*

*Single-spin asymmetries*

**Leading Twist Sivers Effect**

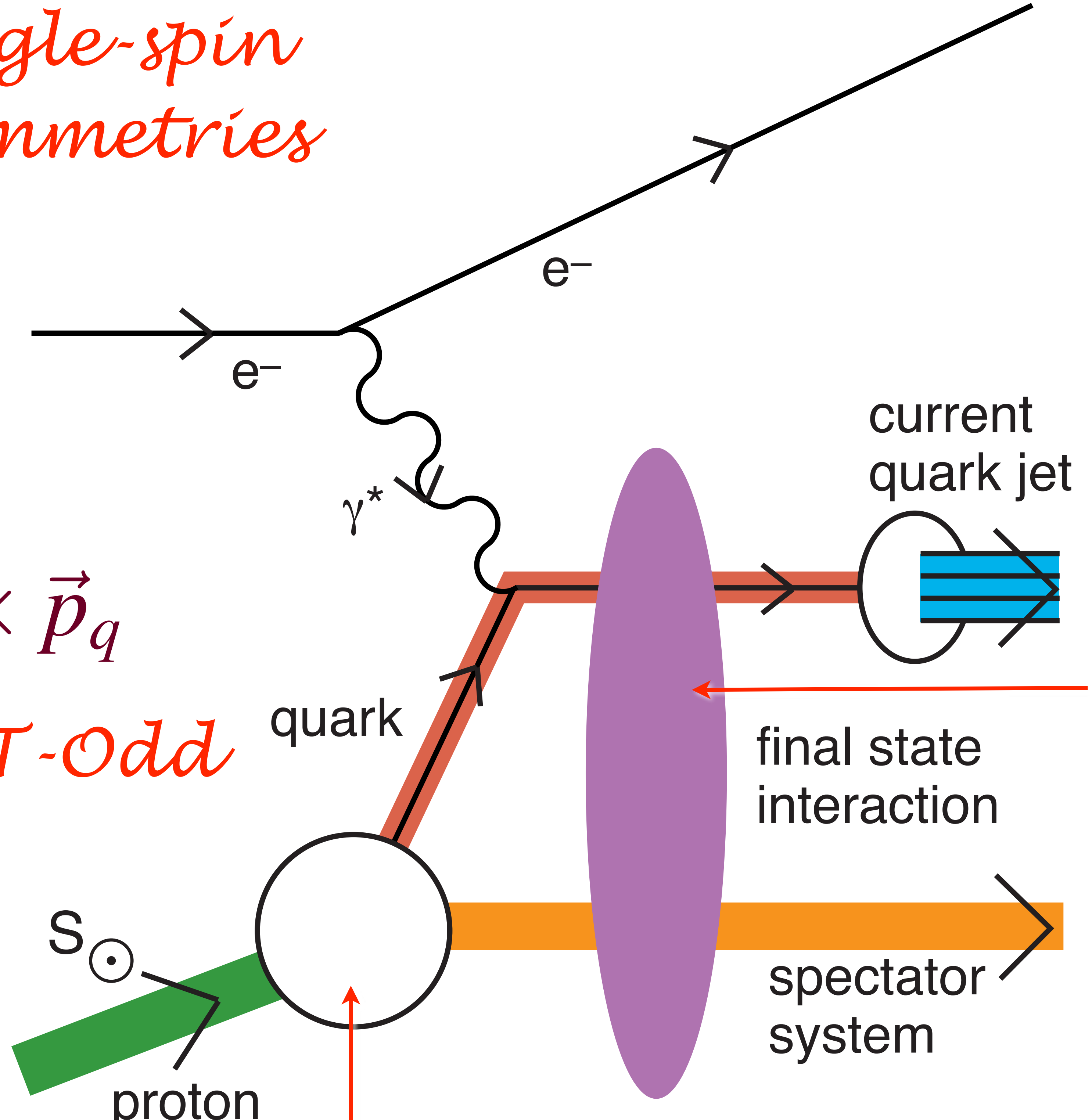
**Hwang, Schmidt, sjb**

**Collins, Burkardt, Ji, Yuan. Pasquini, ...**

$$i \ \vec{S}_p \cdot \vec{q} \times \vec{p}_q$$

*Pseudo- T-Odd*

**"Lensing" involves soft scales**




***Light-Front Wavefunction S and P- Waves!***

*QCD S- and P- Coulomb Phases --Wilson Line*

**"Lensing Effect"**

*Leading-Twist Rescattering Violates pQCD Factorization!*

*Sign reversal in DY!*

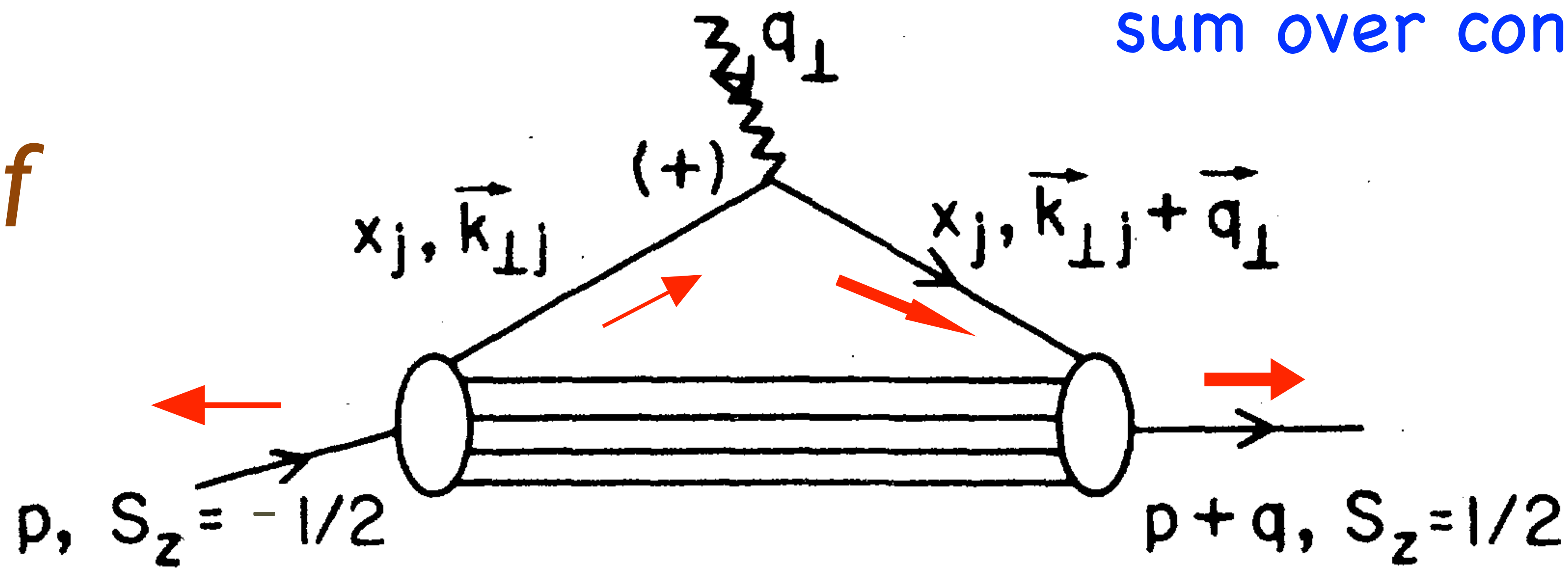
P. Lowdon, K. Chiu, Dae Sung Hwang, Bo-Qiang Ma, Ivan Schmidt, sjb

**Terayev, Okun:** *B(0) Must vanish because of Equivalence Theorem*

*graviton*

sum over constituents

*LF Proof*




B(0) = 0

*Each Fock State*

*Vanishing Anomalous gravitomagnetic moment B(0)*

$$H_{QED}$$

*QED atoms: positronium and muonium*

$$(H_0 + H_{int})\ |\Psi >= E\ |\Psi >$$

*Coupled Fock states*

$$[-\frac{\Delta^2}{2m_{\rm red}} + V_{\rm eff}(\vec{S}, \vec{r})]\ \psi(\vec{r}) = E\ \psi(\vec{r})$$

*Effective two-particle equation*

**Includes Lamb Shift, quantum corrections**

$$[-\frac{1}{2m_{\rm red}}\frac{d^2}{dr^2} + \frac{1}{2m_{\rm red}}\frac{\ell(\ell+1)}{r^2} + V_{\rm eff}(r, S, \ell)]\ \psi(r) = E\ \psi(r)$$

*Spherical Basis* $r, \theta, \phi$

$$V_{eff} \to V_C(r) = -\frac{\alpha}{r}$$

*Coulomb potential*

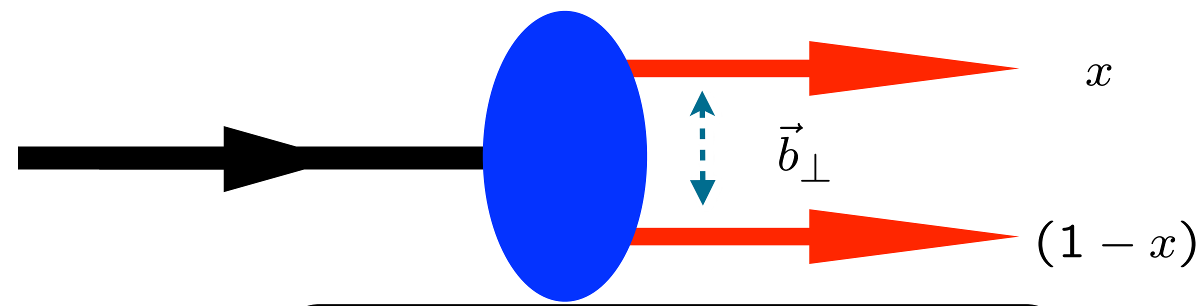
**Bohr Spectrum**

*Semiclassical first approximation to QED*

*Schrödinger Eq.*

# Light-Front QCD

Fixed $\tau = t + z/c$




$$\zeta^2 = x(1-x)b_\perp^2$$

$\mathcal{L}_{QCD} \rightarrow H^{LF}_{QCD}$

$$(H^0_{LF} + H^I_{LF})|\Psi> = M^2|\Psi>$$

Coupled Fock states

Eliminate higher Fock states and retarded interactions

$$[\frac{\vec{k}_\perp^2 + m^2}{x(1-x)} + V^{LF}_{\rm eff}]\ \psi_{LF}(x, \vec{k}_\perp) = M^2\ \psi_{LF}(x, \vec{k}_\perp)$$

Effective two-particle equation

$$[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

Azimuthal Basis $\zeta, \phi$

**Single variable Equation**

$m_q = 0$

**AdS/QCD:**

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

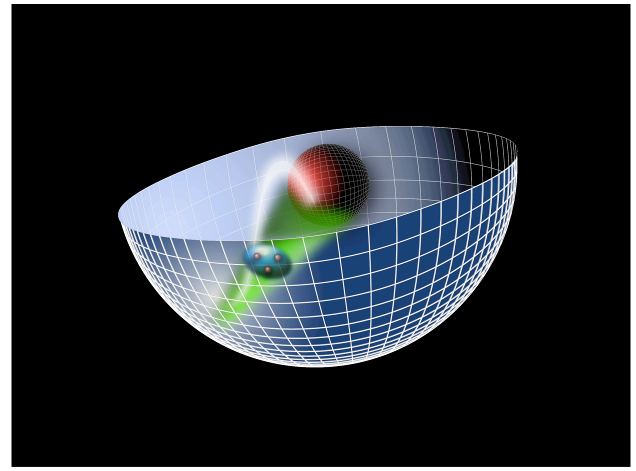
Confining AdS/QCD potential!

***Sums an infinite # diagrams***

Semiclassical first approximation to QCD *de Tèramond, Dosch, sjb*

AdS/QCD
Soft-Wall Model

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$



$$\zeta^2 = x(1-x)\mathrm{b}_\perp^2.$$

Light-Front Holography

$$\left[-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = M^2\psi(\zeta)$$

***Light-Front Schrödinger Equation***

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L+S-1)$$

Single variable $\zeta$

***Unique Confinement Potential!***

Conformal Symmetry of the action

***Confinement scale:*** $\kappa \simeq 0.5~GeV$

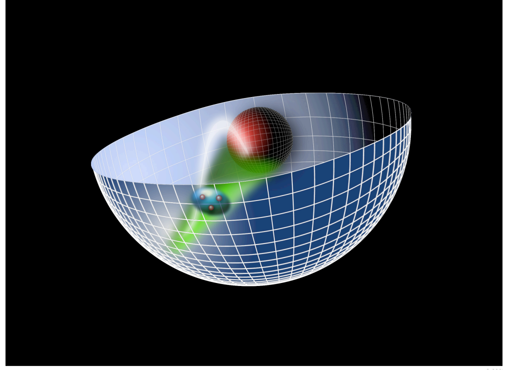
- **de Alfaro, Fubini, Furlan:**
- **Fubini, Rabinovici:**

***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

GeV units external to QCD: Only Ratios of Masses Determined

Maldacena

# AdS5



- Isomorphism of $SO(4,2)$ of conformal QCD with the group of isometries of AdS space

$$ds^2 = \frac{R^2}{z^2}(\eta_{\mu\nu}dx^\mu dx^\nu - dz^2),$$

invariant measure

$x^\mu \to \lambda x^\mu, \ z \to \lambda z$, maps scale transformations into the holographic coordinate $z$.

- AdS mode in $z$ is the extension of the hadron wf into the fifth dimension.

- Different values of $z$ correspond to different scales at which the hadron is examined.

$$x^2 \to \lambda^2 x^2, \quad z \to \lambda z.$$

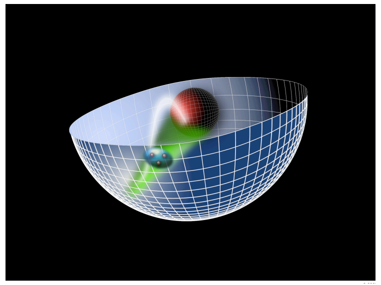
$x^2 = x_\mu x^\mu$: invariant separation between quarks

- The AdS boundary at $z \to 0$ correspond to the $Q \to \infty$, UV zero separation limit.

*AdS/CFT*

# Dilaton-Modified AdS

$$ds^2 = e^{\varphi(z)} \frac{R^2}{z^2} (\eta_{\mu\nu} x^\mu x^\nu - dz^2)$$



- Soft-wall dilaton profile breaks conformal invariance $e^{\varphi(z)} = e^{+\kappa^2 z^2}$
- Color Confinement in z
- Introduces confinement scale κ
- Uses $AdS_5$ as template for conformal theory

# Introduce "Dilaton" to simulate confinement analytically

- Nonconformal metric dual to a confining gauge theory

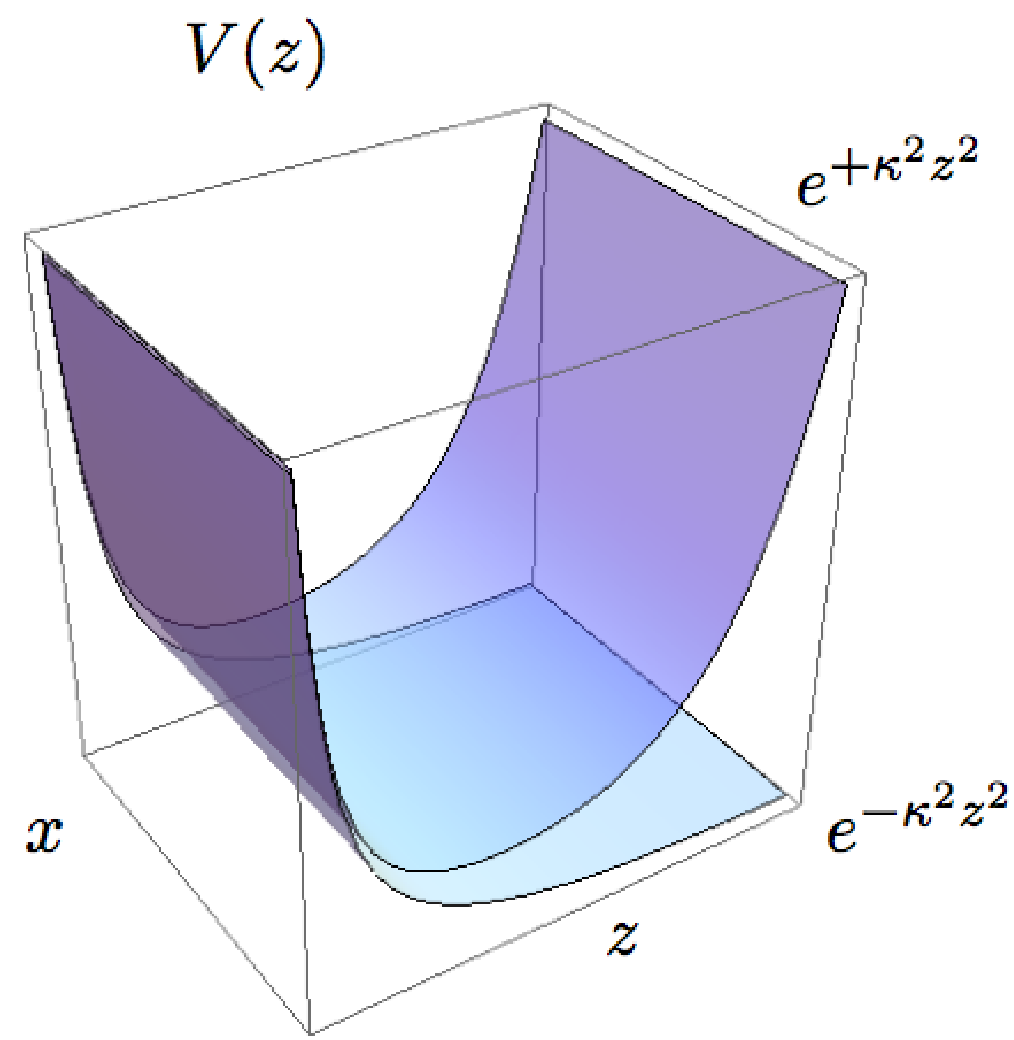
$$ds^2 = \frac{R^2}{z^2} e^{\varphi(z)} \left(\eta_{\mu\nu} dx^\mu dx^\nu - dz^2\right)$$

where $\varphi(z) \to 0$ at small $z$ for geometries which are asymptotically $AdS_5$

- Gravitational potential energy for object of mass $m$

$$V = mc^2 \sqrt{g_{00}} = mc^2 R \frac{e^{\varphi(z)/2}}{z}$$

- Consider warp factor $\exp(\pm \kappa^2 z^2)$
- Plus solution: $V(z)$ increases exponentially confining any object in modified AdS metrics to distances $\langle z \rangle \sim 1/\kappa$




Klebanov and Maldacena

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

- de Teramond, sjb

# Bosonic Solutions: Hard Wall Model

- Conformal metric: $ds^2 = g_{\ell m} dx^\ell dx^m$. $x^\ell = (x^\mu, z)$, $g_{\ell m} \to \left(R^2/z^2\right) \eta_{\ell m}$ .

- Action for massive scalar modes on $AdS_{d+1}$:

$$S[\Phi] = \frac{1}{2} \int d^{d+1}x \sqrt{g} \, \tfrac{1}{2} \left[ g^{\ell m} \partial_\ell \Phi \partial_m \Phi - \mu^2 \Phi^2 \right], \quad \sqrt{g} \to (R/z)^{d+1}.$$

- Equation of motion

$$\frac{1}{\sqrt{g}} \frac{\partial}{\partial x^\ell} \left( \sqrt{g} \, g^{\ell m} \frac{\partial}{\partial x^m} \Phi \right) + \mu^2 \Phi = 0.$$

- Factor out dependence along $x^\mu$-coordinates , $\Phi_P(x, z) = e^{-iP \cdot x} \, \Phi(z)$, $P_\mu P^\mu = \mathcal{M}^2$ :

$$\left[ z^2 \partial_z^2 - (d-1) z \, \partial_z + z^2 \mathcal{M}^2 - (\mu R)^2 \right] \Phi(z) = 0.$$

- Solution: $\Phi(z) \to z^\Delta$ as $z \to 0$,

$$\Phi(z) = C z^{d/2} J_{\Delta - d/2}(z\mathcal{M}) \qquad \Delta = \tfrac{1}{2} \left( d + \sqrt{d^2 + 4\mu^2 R^2} \right).$$

$$\Delta = 2 + L \qquad d = 4 \qquad (\mu R)^2 = L^2 - 4$$

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

**Positive-sign dilaton**

- de Teramond, sjb

# AdS Soft-Wall Schrödinger Equation for bound state of two scalar constituents:

$$\left[-\frac{d^2}{dz^2} - \frac{1-4L^2}{4z^2} + U(z)\right]\Phi(z) = \mathcal{M}^2\Phi(z)$$

$$U(z) = \kappa^4 z^2 + 2\kappa^2(L+S-1)$$

*Derived from variation of Action for Dilaton-Modified $AdS_5$*

***Identical to Single-Variable Light-Front Bound State Equation in $\zeta$!***

$$z \longleftrightarrow \zeta = \sqrt{x(1-x)\vec{b}_\perp^2}$$

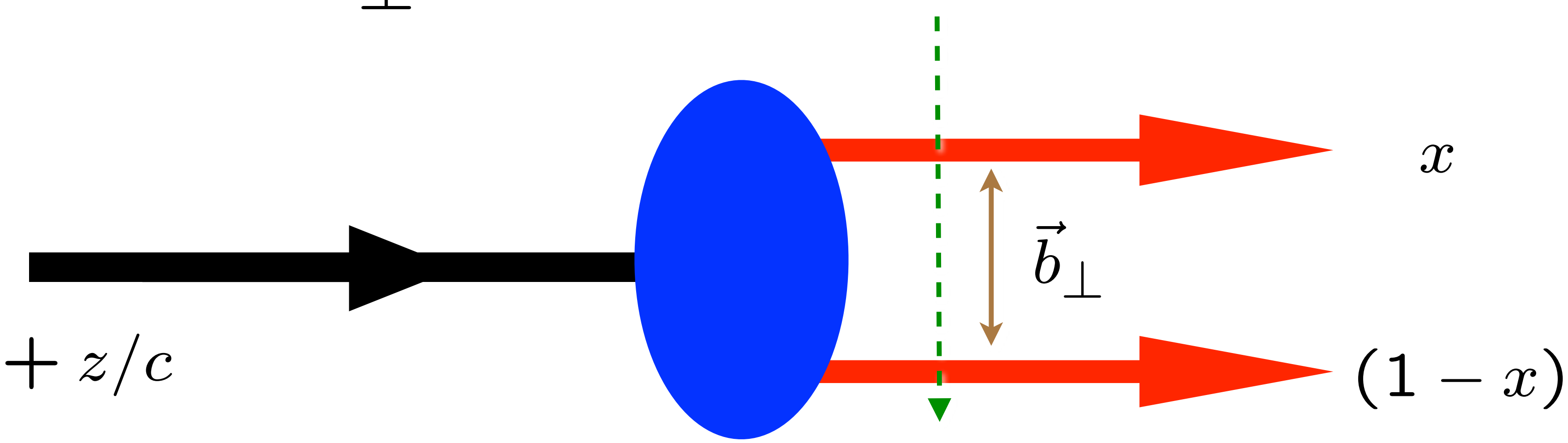
de Teramond, sjb

LF(3+1) ⟷ $AdS_5$

# *Light-Front Holographic Dictionary*

$$\psi(x,\vec{b}_\perp) \longleftrightarrow \phi(z)$$

$$\zeta = \sqrt{x(1-x)\vec{b}_\perp^2} \longleftrightarrow z$$




Fixed $\tau = t + z/c$

$$\psi(x,\zeta) = \sqrt{x(1-x)}\,\zeta^{-1/2}\phi(\zeta)$$

$$(\mu R)^2 = L^2 - (J-2)^2$$

***Light-Front Holography**: Unique mapping derived from equality of LF and AdS formula for EM and gravitational current matrix elements and identical equations of motion*

Massless pion!

## Meson Spectrum in Soft Wall Model

$$m_\pi = 0 \text{ if } m_q = 0$$

*Pion: Negative term for J=0 cancels positive terms from LFKE and potential*

- Effective potential: $U(\zeta^2) = \kappa^4 \zeta^2 + 2\kappa^2 (J-1)$
- LF WE

$$\left(-\frac{d^2}{d\zeta^2} - \frac{1-4L^2}{4\zeta^2} + \kappa^4\zeta^2 + 2\kappa^2(J-1)\right)\phi_J(\zeta) = M^2\phi_J(\zeta)$$

- Normalized eigenfunctions $\langle\phi|\phi\rangle = \int d\zeta\, \phi^2(z)^2 = 1$

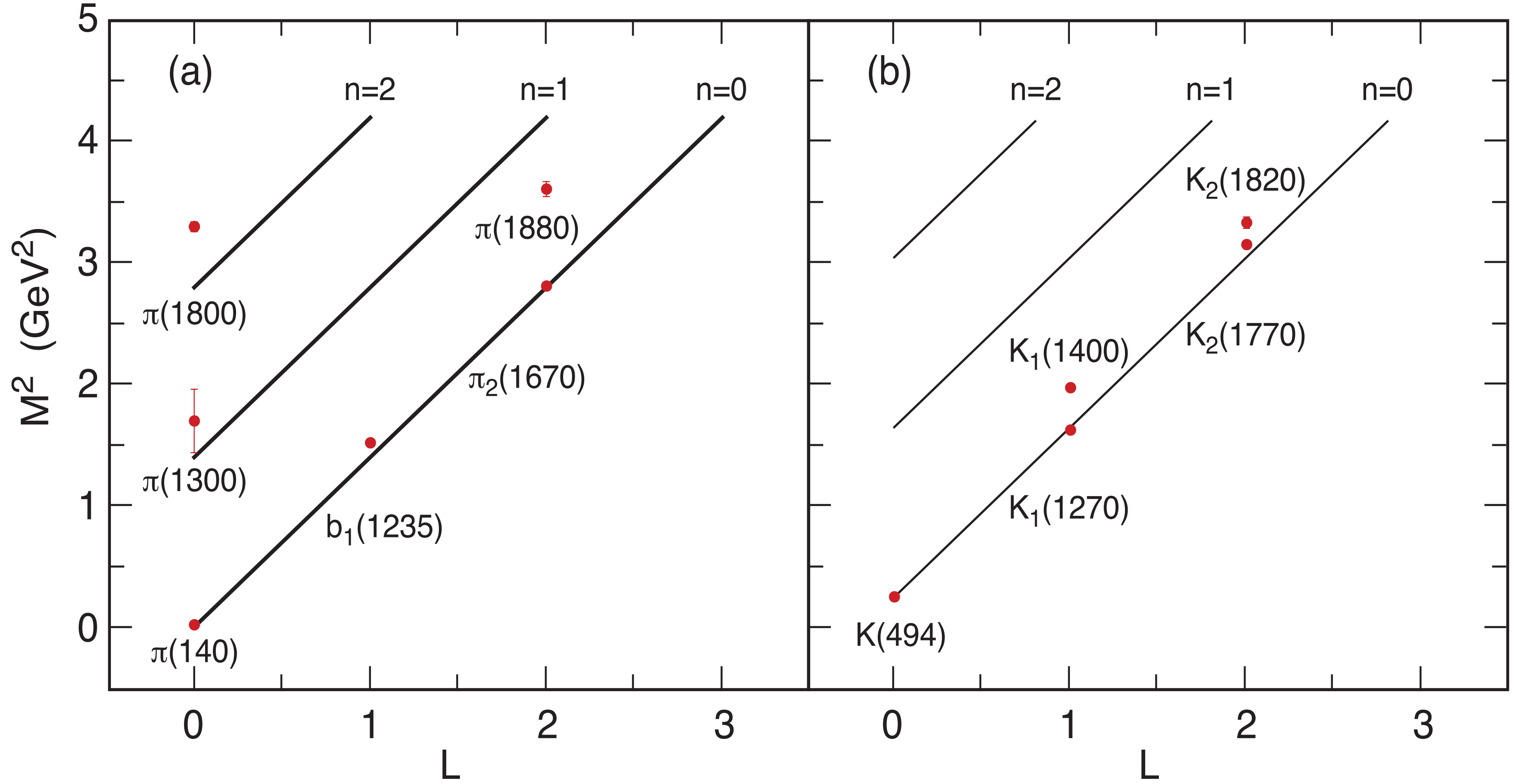
$$\phi_{n,L}(\zeta) = \kappa^{1+L}\sqrt{\frac{2n!}{(n+L)!}}\,\zeta^{1/2+L}e^{-\kappa^2\zeta^2/2}L_n^L(\kappa^2\zeta^2)$$

- Eigenvalues

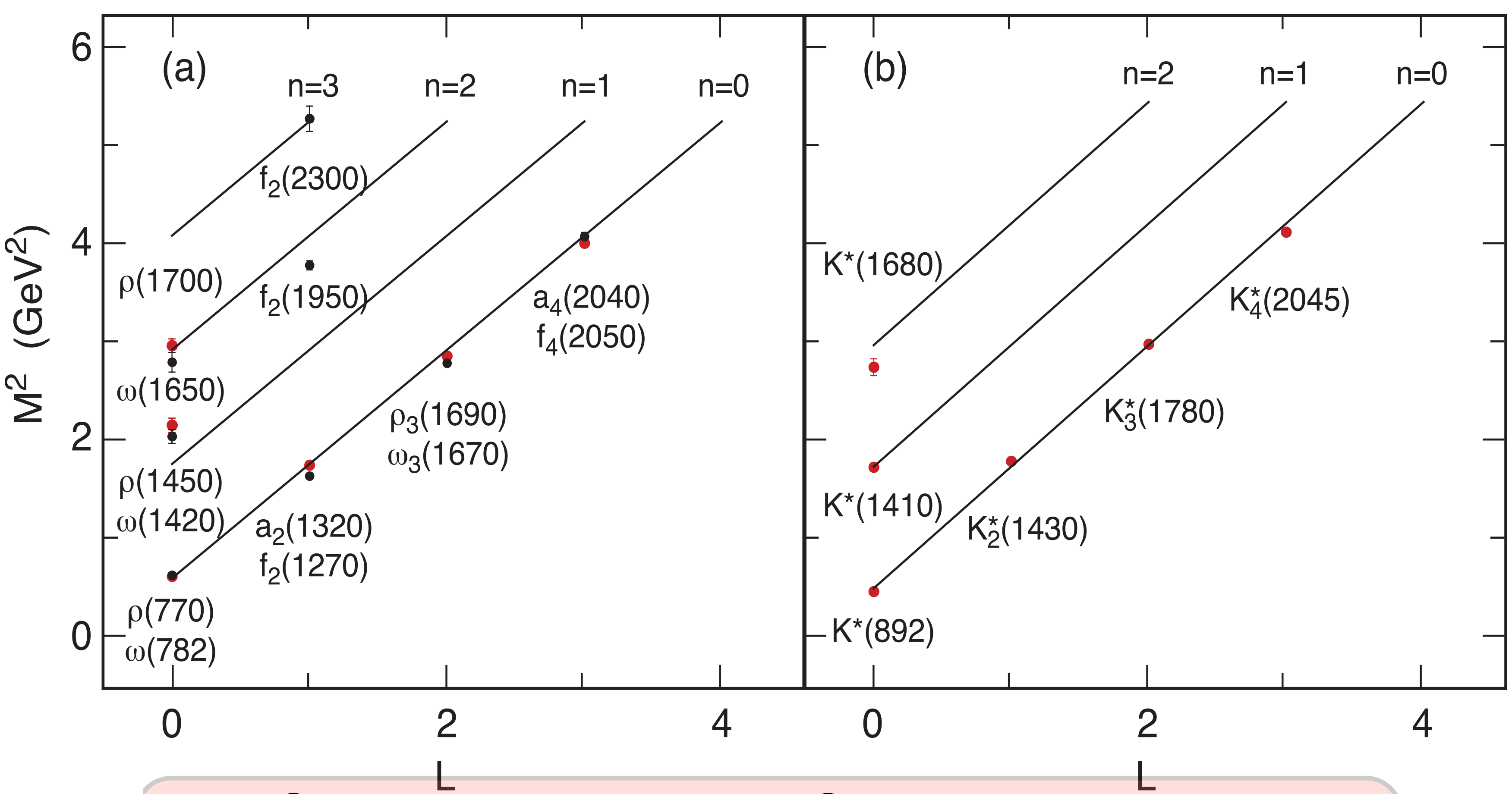
$$\mathcal{M}^2_{n,J,L} = 4\kappa^2\left(n + \frac{J+L}{2}\right)$$

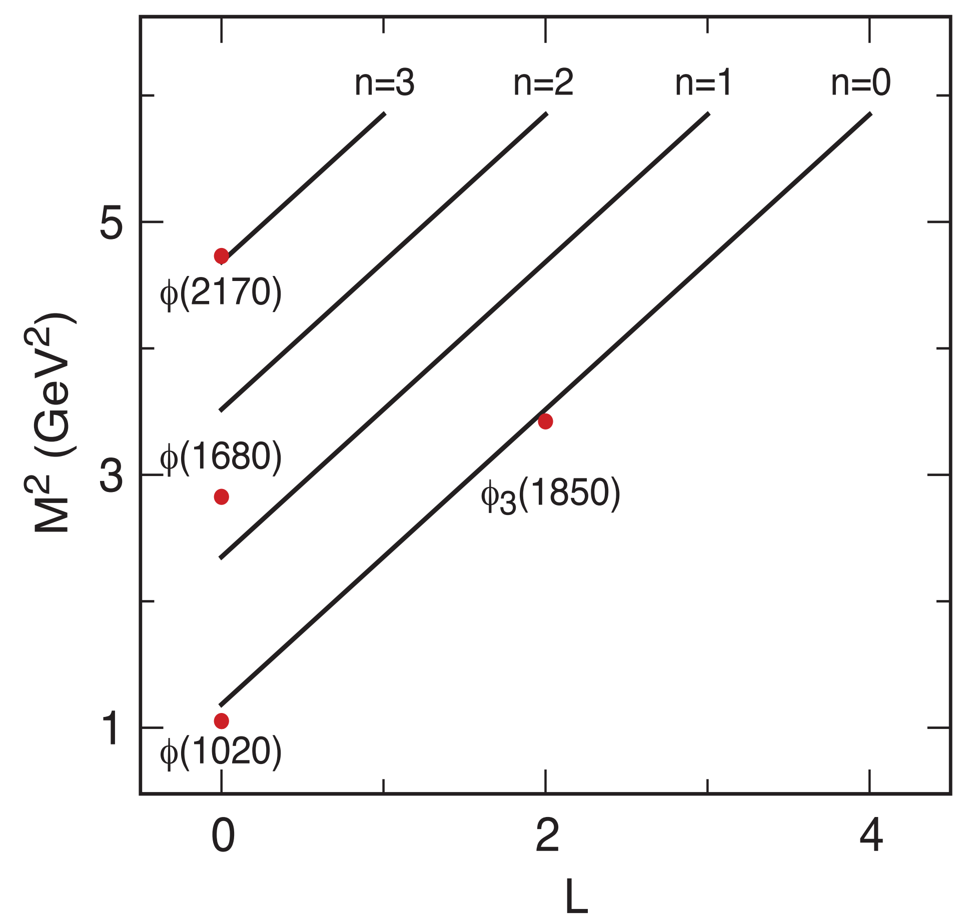
$$\vec{\zeta}^2 = \vec{b}^2_\perp x(1-x)$$

**G. de Teramond, H. G. Dosch, sjb**




$$m_q = 0$$




$$M^2(n, L, S) = 4\kappa^2(n + L + S/2)$$

*Equal Slope in n and L*

Quark separation increases with L

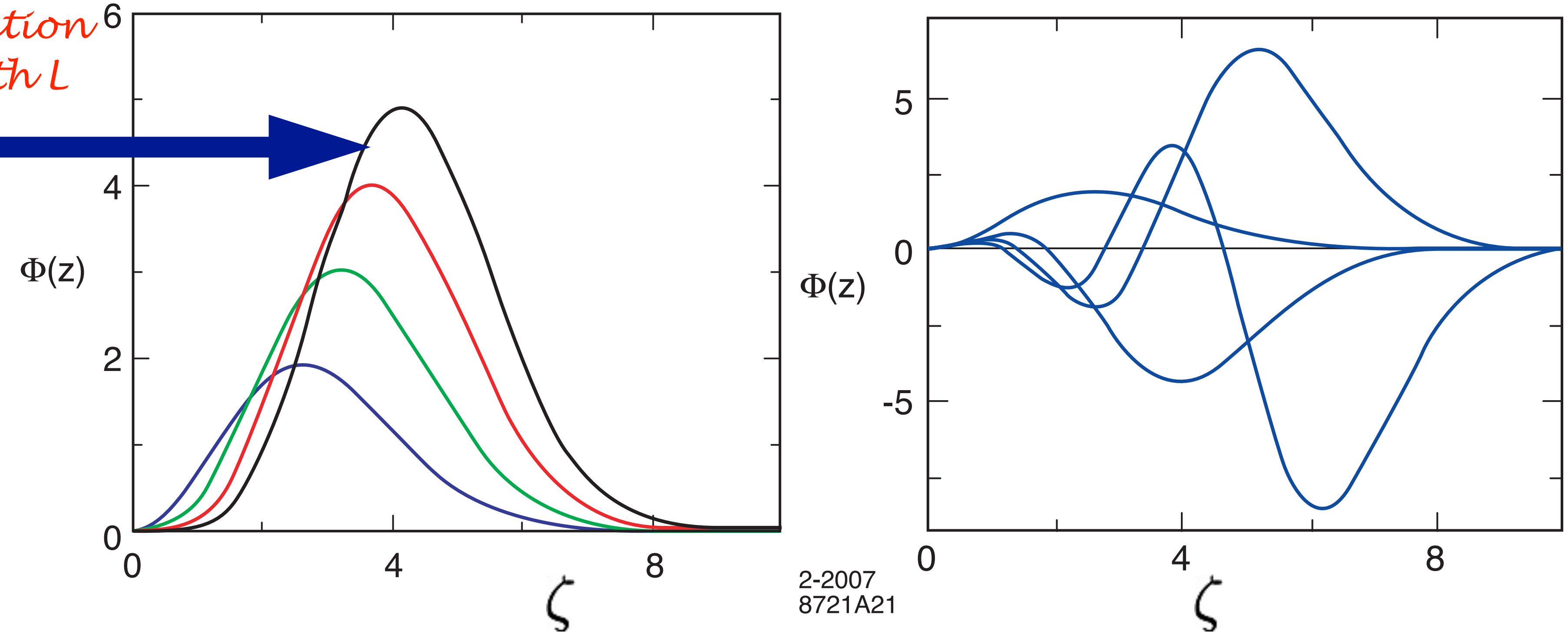
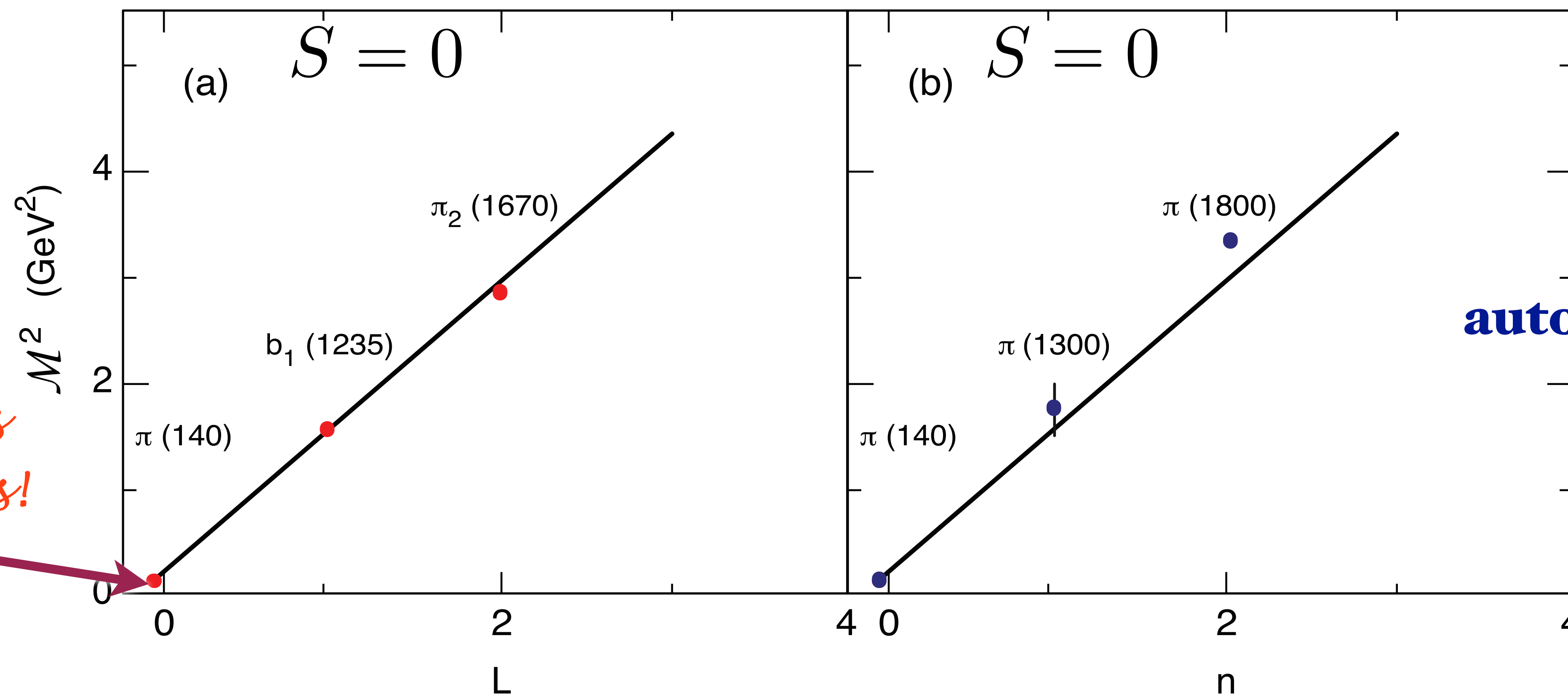



Fig: Orbital and radial AdS modes in the soft wall model for $\kappa$ = 0.6 GeV .

# Soft Wall Model

***Same slope in n and L!***




Pion has zero mass!

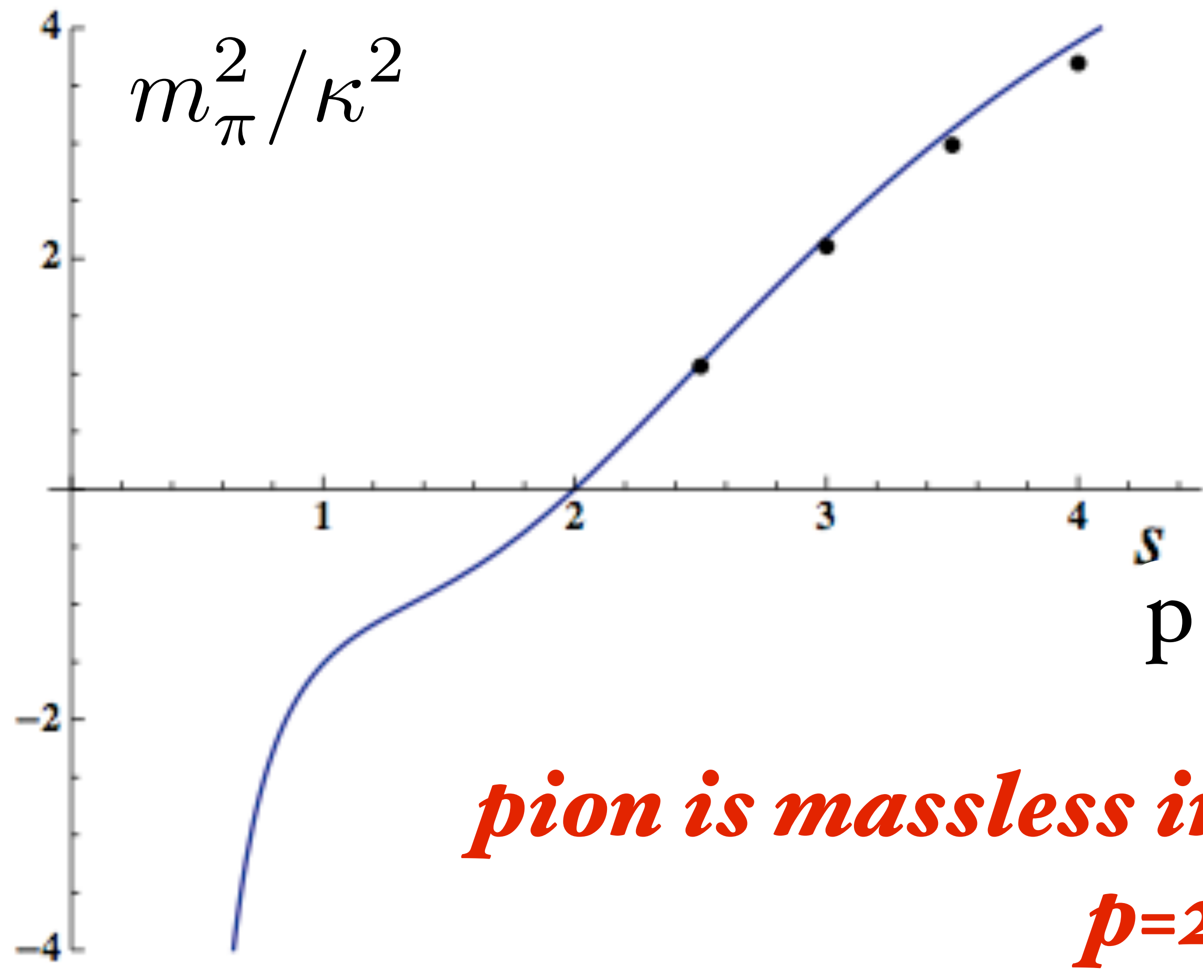
**Pion mass automatically zero!**

$$m_q = 0$$

Light meson orbital (a) and radial (b) spectrum for $\kappa = 0.6$ GeV.

# Uniqueness of Dilaton

$$\varphi_p(z) = \kappa^p z^p$$




$m_\pi^2/\kappa^2$

***pion is massless in chiral limit iff p=2!***

$$e^{\varphi(z)} = e^{+\kappa^2 z^2}$$

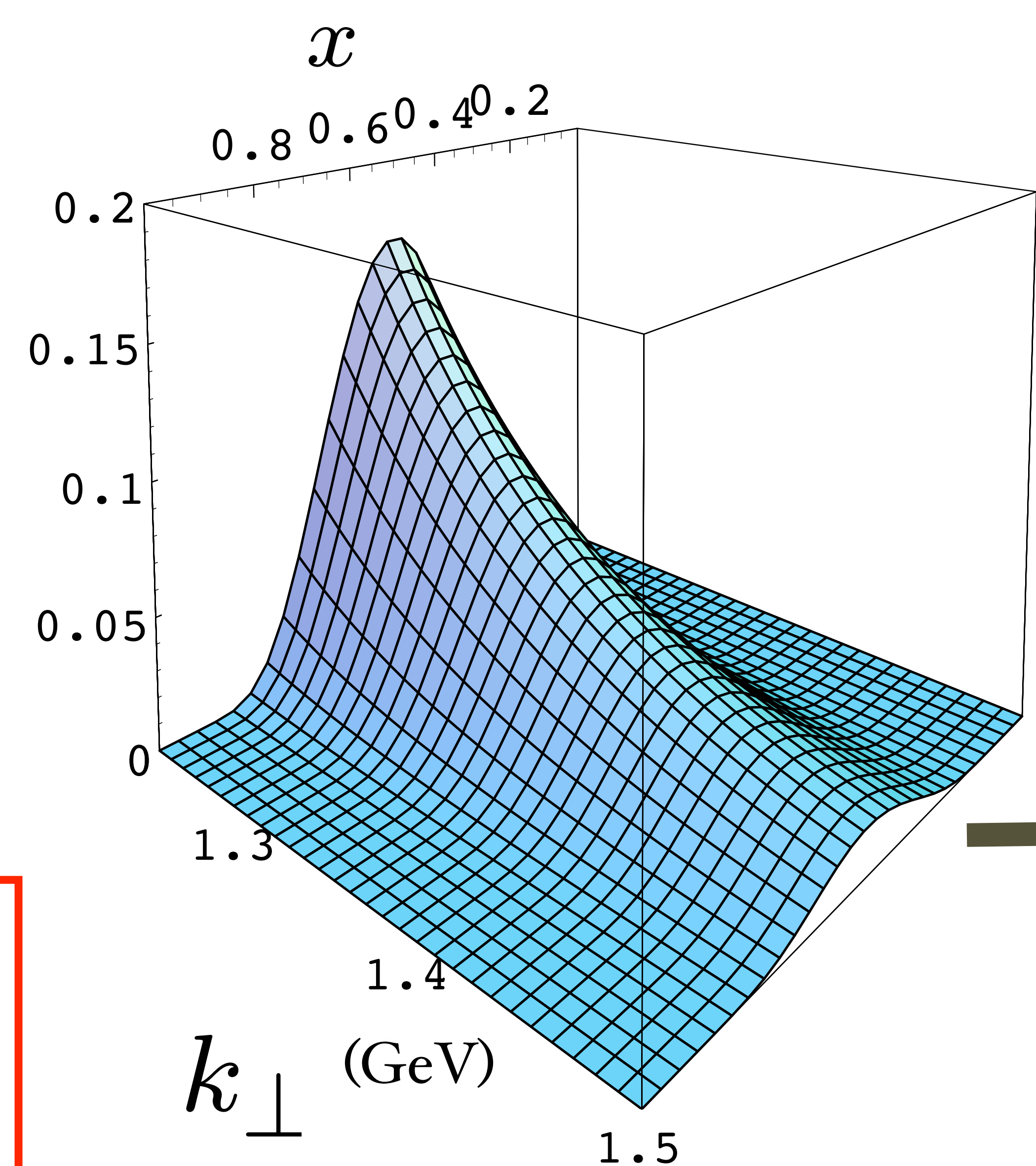
- **Dosch, de Tèramond, sjb**

# Prediction from AdS/QCD: Meson LFWF

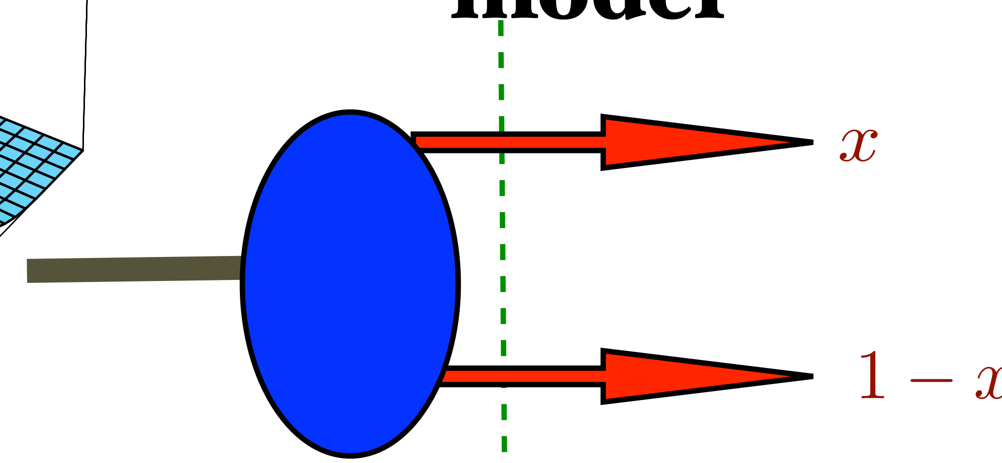
$$e^{\varphi(z)} = e^{+\kappa^2 z}$$

**de Teramond, Cao, sjb**

$$\psi_M(x, k_\perp^2)$$




**"Soft Wall" model**




massless quarks

**Note coupling**

$k_\perp^2, x$

$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

$$\phi_\pi(x) = \frac{4}{\sqrt{3}\pi} f_\pi \sqrt{x(1-x)}$$

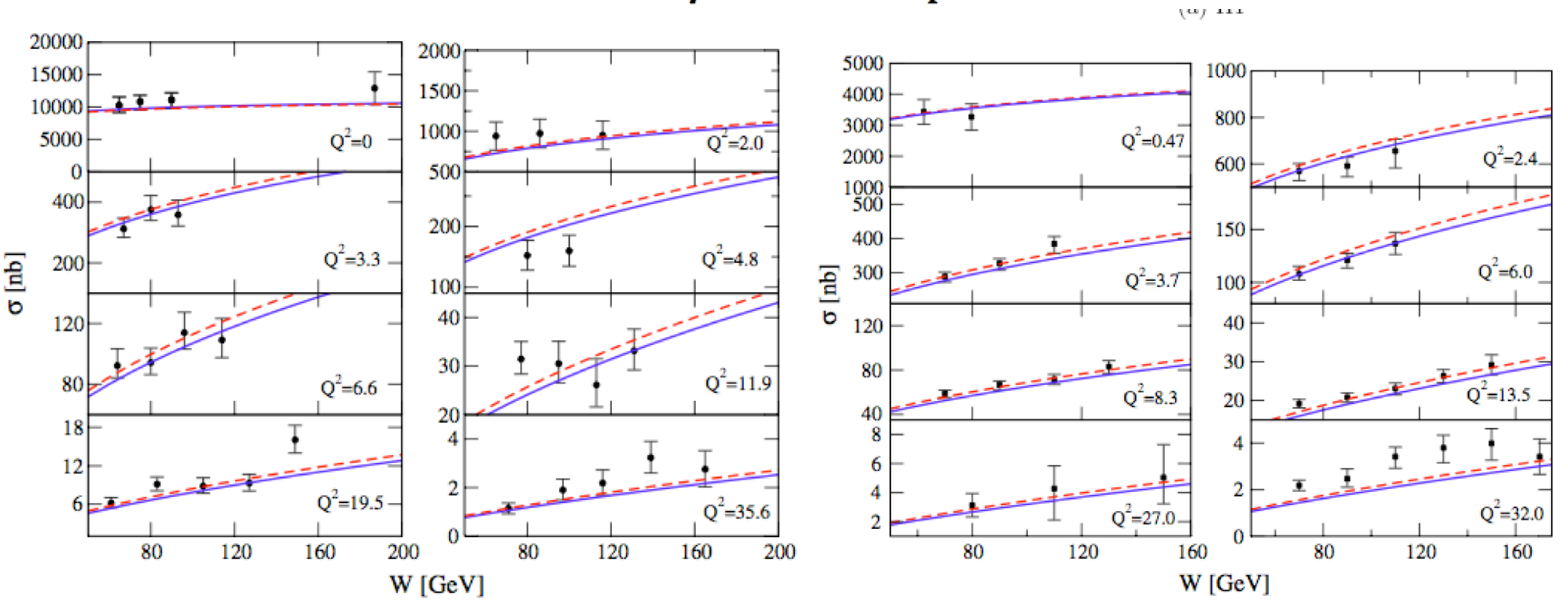
$$f_\pi = \sqrt{P_{q\bar{q}}}\frac{\sqrt{3}}{8}\kappa = 92.4 \text{ MeV}$$

**Same as DSE!** **C. D. Roberts et al.**

*Provides Connection of Confinement to Hadron Structure*

PRL 109, 081601 (2012) PHYSICAL REVIEW LETTERS week ending 24 AUGUST 2012

# AdS/QCD Holographic Wave Function for the $\rho$ Meson and Diffractive $\rho$ Meson Electroproduction

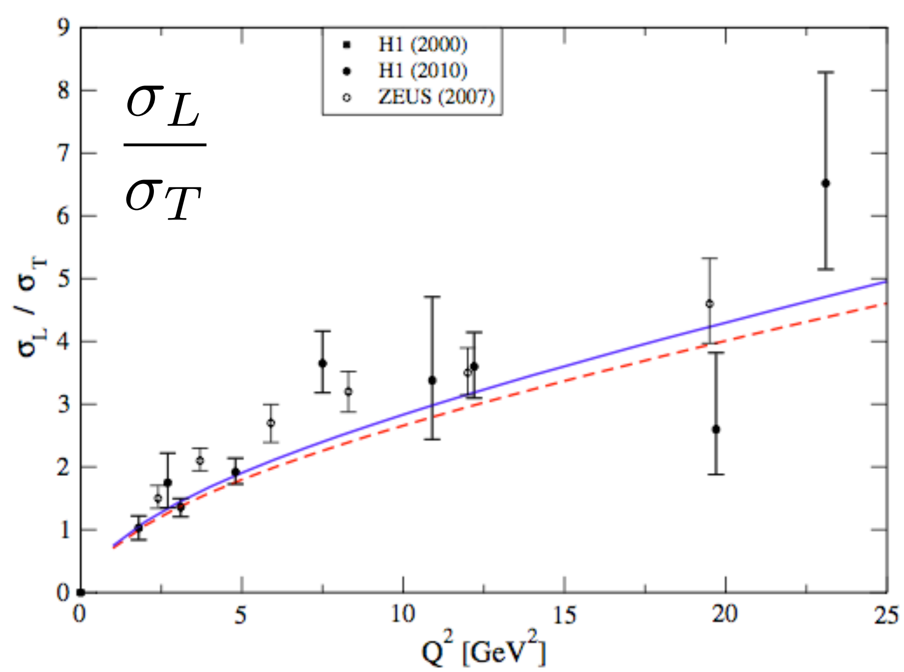



(a) H1

(b) ZEUS

**J. R. Forshaw,**
**R. Sandapen**

$$\gamma^* p \to \rho^0 p'$$




$$\psi_M(x, k_\perp) = \frac{4\pi}{\kappa\sqrt{x(1-x)}} e^{-\frac{k_\perp^2}{2\kappa^2 x(1-x)}}$$

# Light-Front Perturbation Theory for pQCD

$$T = H_I + H_I \frac{1}{\mathcal{M}^2_{initial} - \mathcal{M}^2_{intermediate} + i\epsilon} H_I + cdots$$

- "History" : Compute any subgraph only once since the LFPth numerator does not depend on the process — only the denominator changes!

- Wick Theorem applies, but few amplitudes since all $k^+ > 0$.

- $J_z$ Conservation at every vertex $\left| \sum_{initial} S^z - \sum_{final} S_z \right| \leq n$ at order $g^n$

- Unitarity is explicit

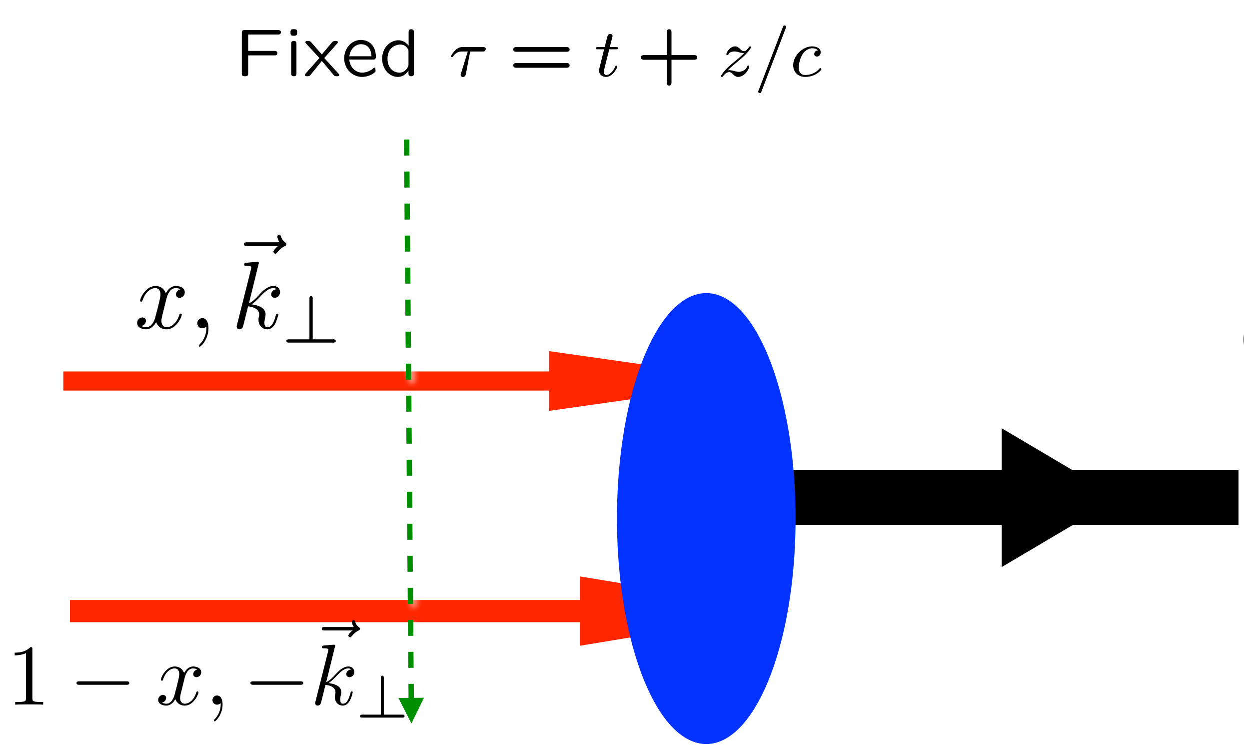
- Loop Integrals are 3-dimensional $\int_0^1 dx \int d^2 k_\perp$

- hadronization: coalesce comoving quarks and gluons to hadrons using light-front wavefunctions $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

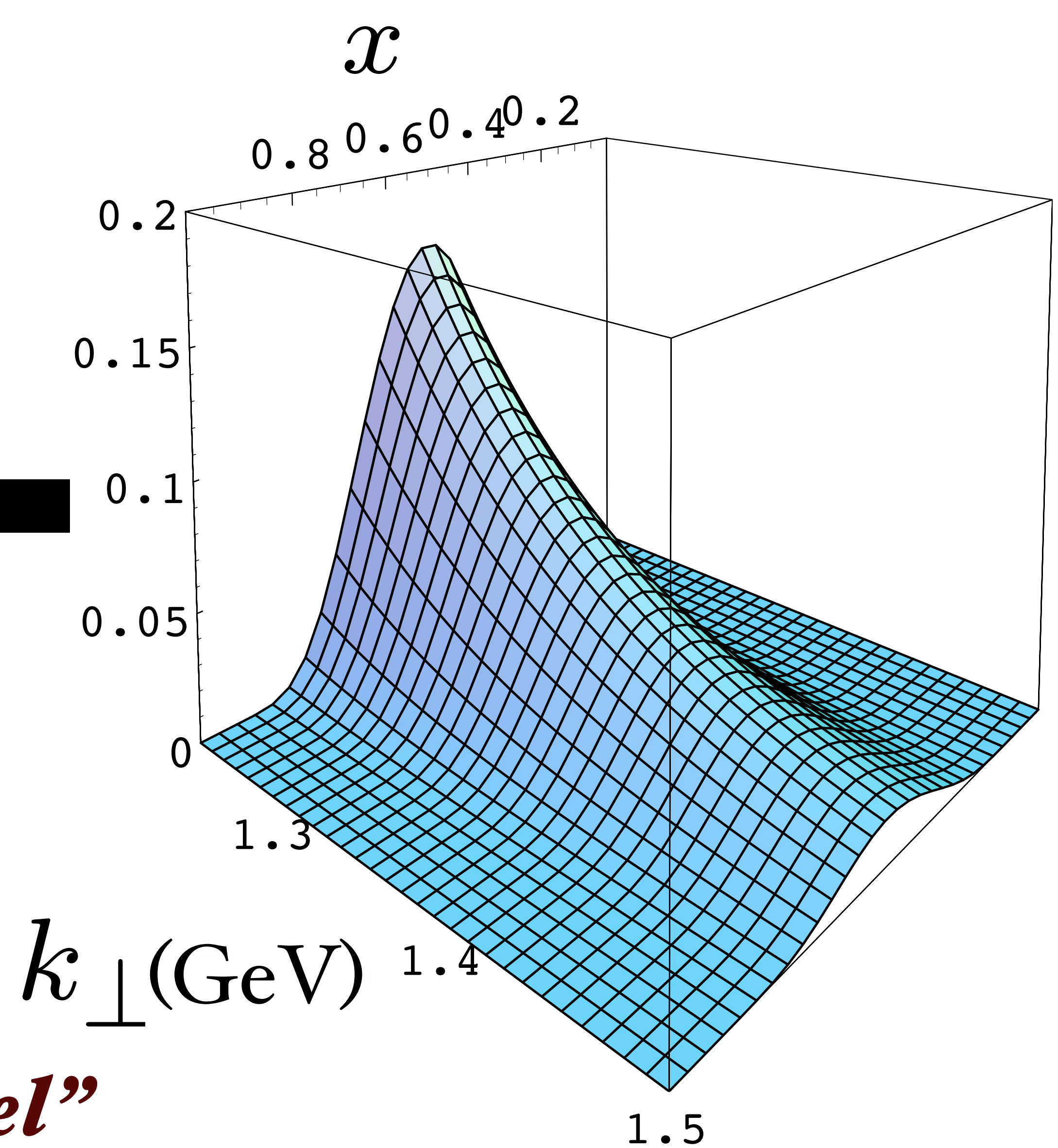
K. Chiu, sjb

- *Light Front Wavefunctions:* $\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$

off-shell in $P^-$ and invariant mass $\mathcal{M}^2_{q\bar{q}}$




$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$




***"Hadronization at the Amplitude Level"***

**Boost-invariant LFWF connects confined quarks and gluons to hadrons**

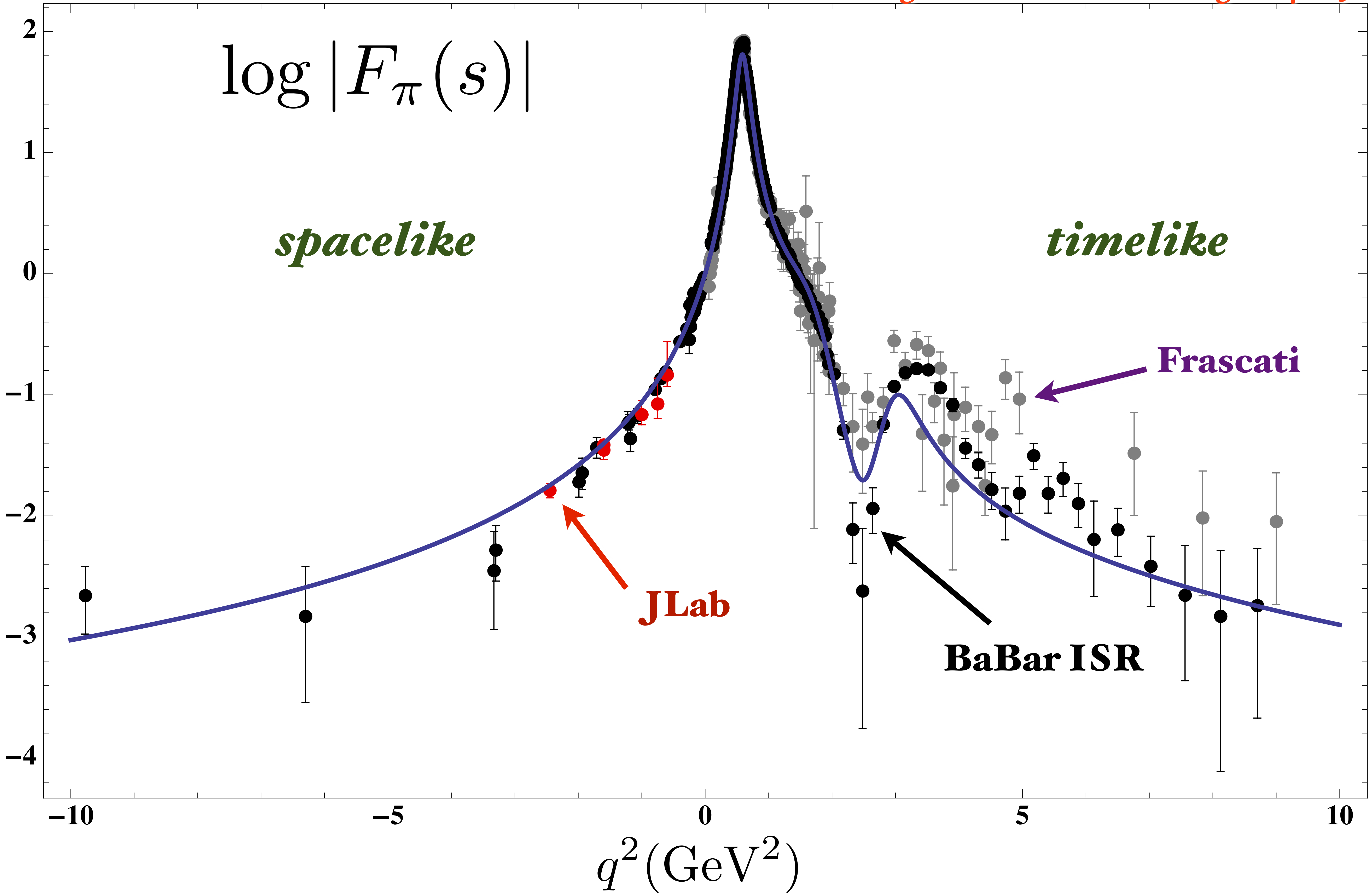
# Connection to the Linear Instant-Form Potential

Linear instant nonrelativistic form $V(r) = Cr$ for heavy quarks

Harmonic Oscillator $U(\zeta) = \kappa^4\zeta^2$ LF Potential for relativistic light quarks

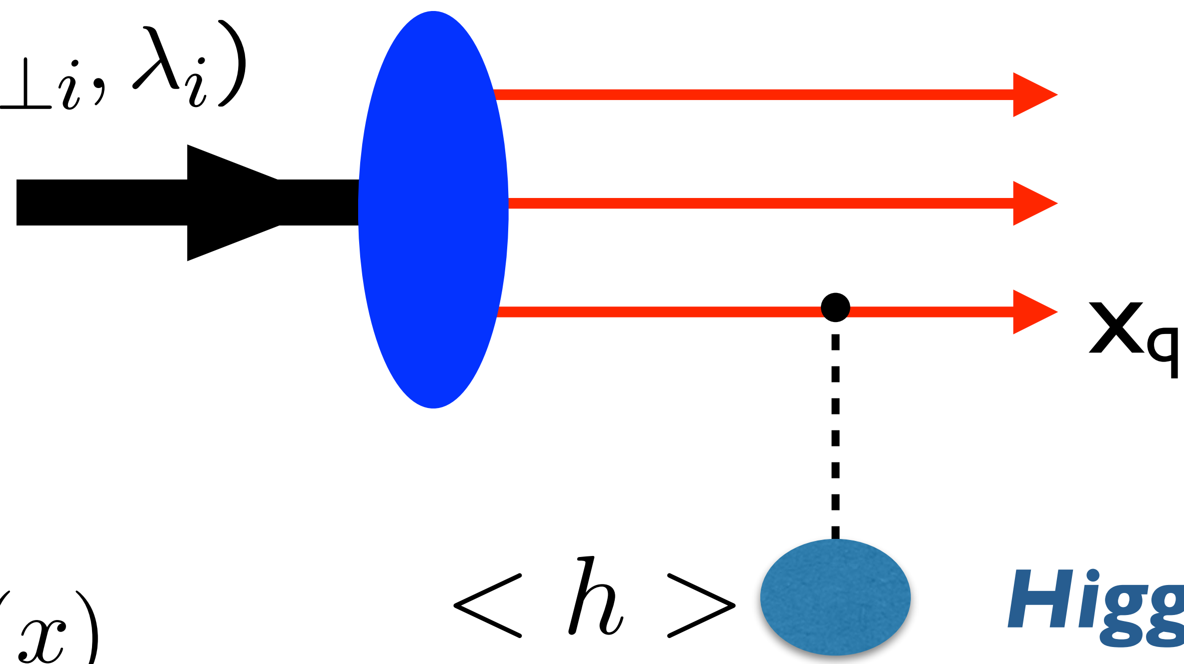
A.P. Trawinski, S.D. Glazek, H. D. Dosch, G. de Teramond, sjb

# Pion Form Factor from AdS/QCD and Light-Front Holography



# Coupling of confined quarks to Higgs Zero Mode <h>

$$\Psi_n(x_i, \vec{k}_{\perp i}, \lambda_i)$$




$$< h >$$

**Higgs Zero Mode!**

$$g_q \bar{\psi}_q(x) \psi_q(x) h(x)$$

*Yukawa Higgs coupling of confined quark to Higgs zero mode gives*

$$\bar{u}u \; g_q < h > = \frac{m_q}{x_q} m_q = \frac{m_q^2}{x_q}$$

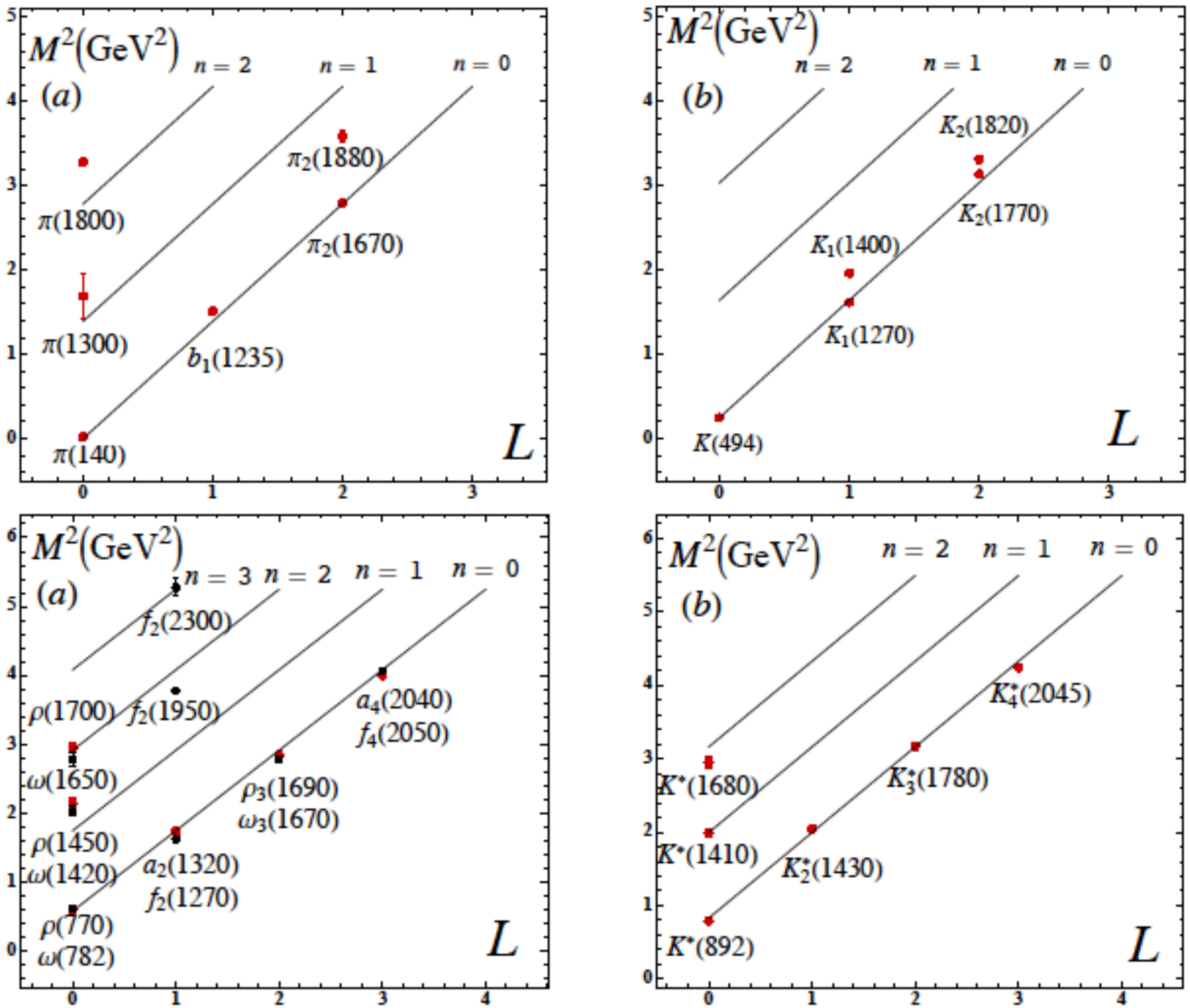
$$H_{LFKE} = \sum_i \left[\frac{\vec{k}_\perp^2 + m_q^2}{x_q}\right]_i = \mathcal{M}^2 = \left[\sum_i k_q^\mu\right]^2$$

De Tèramond, Dosch, sjb

$$m_u = m_d = 46 \text{ MeV}, \quad m_s = 357 \text{ MeV}$$

$$M^2 = M_0^2 + \left\langle X \left| \frac{m_q^2}{x} \right| X \right\rangle + \left\langle X \left| \frac{m_{\bar{q}}^2}{1-x} \right| X \right\rangle$$

from LF Higgs mechanism




Effective mass from $m(p^2)$

Tandy, Roberts, et al

# Remarkable Features of Light-Front Schrödinger Equation

**Dynamics + Spectroscopy!**

- ***Relativistic, frame-independent***
- ***QCD scale appears - unique LF potential***
- ***Reproduces spectroscopy and dynamics of light-quark hadrons with one parameter***
- ***Zero-mass pion for zero mass quarks!***
- ***Regge slope same for n and L -- not usual HO***
- ***Splitting in L persists to high mass -- contradicts conventional wisdom based on breakdown of chiral symmetry***
- ***Phenomenology: LFWFs, Form factors, electroproduction***
- ***Extension to heavy quarks***

$$U(\zeta) = \kappa^4 \zeta^2 + 2\kappa^2(L + S - 1)$$

# QCD Lagrangian

$$\mathcal{L}_{QCD} = -\frac{1}{4}Tr(G^{\mu\nu}G_{\mu\nu}) + \sum_{f=1}^{n_f} i\bar{\Psi}_f D_\mu \gamma^\mu \Psi_f + \sum_{f=1}^{n_f} m_f \bar{\Psi}_f \Psi_f$$

$$iD^\mu = i\partial^\mu - gA^\mu \qquad G^{\mu\nu} = \partial^\mu A^\mu - \partial^\nu A^\mu - g[A^\mu, A^\nu]$$

***Classical Chiral Lagrangian is Conformally Invariant***

**Where does the QCD Mass Scale come from?**

**QCD does not know what MeV units mean!**
**Only Ratios of Masses Determined**

- **de Alfaro, Fubini, Furlan:** ***Scale can appear in Hamiltonian and EQM without affecting conformal invariance of action!***

***Unique confinement potential!***

- **de Alfaro, Fubini, Furlan** (dAFF)

$$G|\psi(\tau)>= i\frac{\partial}{\partial\tau}|\psi(\tau)>$$

**New term**

$$G = uH + vD + wK$$

$$G = H_\tau = \frac{1}{2}\left(-\frac{d^2}{dx^2} + \frac{g}{x^2} + \frac{4uw - v^2}{4}x^2\right)$$

*Retains conformal invariance of action despite mass scale!*

$$4uw - v^2 = \kappa^4 = [M]^4$$

*Identical to LF Hamiltonian with unique potential and dilaton!*

$$\left[-\frac{d^2}{d\zeta^2} + \frac{1-4L^2}{4\zeta^2} + U(\zeta)\right]\psi(\zeta) = \mathcal{M}^2\psi(\zeta)$$

$$U(\zeta) = \kappa^4\zeta^2 + 2\kappa^2(L + S - 1)$$

- **Dosch, de Teramond, sjb**

# dAFF: New Time Variable

$$\tau = \frac{2}{\sqrt{4uw - v^2}} \arctan\left(\frac{2tw + v}{\sqrt{4uw - v^2}}\right),$$

- **Identify with difference of LF time $\Delta x^+/P^+$ between constituents**
- **Finite range**
- **Measure in Double-Parton Processes**

*Retains conformal invariance of action despite mass scale!*

*Haag, Lopuszanski, Sohnius (1974)*

## Superconformal Quantum Mechanics

$$\{\psi, \psi^+\} = 1 \qquad B = \frac{1}{2}[\psi^+, \psi] = \frac{1}{2}\sigma_3$$

$$\psi = \frac{1}{2}(\sigma_1 - i\sigma_2), \quad \psi^+ = \frac{1}{2}(\sigma_1 + i\sigma_2)$$

$$Q = \psi^+[-\partial_x + \frac{f}{x}], \quad Q^+ = \psi[\partial_x + \frac{f}{x}], \qquad S = \psi^+ x, \quad S^+ = \psi x$$

$$\{Q, Q^+\} = 2H, \quad \{S, S^+\} = 2K$$

$$\{Q, S^+\} = f - B + 2iD, \quad \{Q^+, S\} = f - B - 2iD$$

generates conformal algebra

[H,D]= i H, [H, K] =2 i D, [K, D] = - i K

$$Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$$

# Supersymmetric Superconformal QM

(Fubini & Rabinovici, NPB245 (84) 17)

graded algebra of two fermionic operators (super charges) $Q, \; Q^\dagger$

$\{Q,Q\}=0, \; \{Q^\dagger,Q^\dagger\}=0$ with $H=\{Q,Q^\dagger\}$ → $[Q,H]=0, \; [Q^\dagger,H]=0$

minimum conformal realization -> particle with 2 degrees of freedom with:

$$Q=\psi^\dagger\left(-\frac{\partial}{\partial x}+\frac{f}{x}\right), \; Q^\dagger=\psi\left(\frac{\partial}{\partial x}+\frac{f}{x}\right)$$

$\psi, \; \psi^\dagger$ spinor operators with

$\{\psi^\dagger,\psi\}=I, [\psi^\dagger,\psi]=\sigma_3$

in matrix notation

$$Q=\begin{pmatrix} 0 & -\partial_x+\frac{f}{x} \\ 0 & 0 \end{pmatrix}, \; Q^\dagger=\begin{pmatrix} 0 & 0 \\ \partial_x+\frac{f}{x} & 0 \end{pmatrix}$$ →

$$H=\begin{pmatrix} -\partial_x^2+\frac{f^2+f}{x^2} & 0 \\ 0 & -\partial_x^2+\frac{f^2-f}{x^2} \end{pmatrix}$$

H operates on two component states

$$|\phi\rangle=\begin{pmatrix} \phi_M \\ \phi_B \end{pmatrix}$$

with same eigenvalue

# Superconformal Quantum Mechanics

**Baryon Equation** $Q \simeq \sqrt{H}, \quad S \simeq \sqrt{K}$

Consider $R_w = Q + wS;$

$w$: dimensions of mass squared

$$G = \{R_w, R_w^+\} = 2H + 2w^2 K + 2wfI - 2wB \qquad 2B = \sigma_3$$

Retains Conformal Invariance of Action

***Fubini and Rabinovici***

*New Extended Hamiltonian G is diagonal:*

$$G_{11} = \left( -\partial_x^2 + w^2 x^2 + 2wf - w + \frac{4(f+\frac{1}{2})^2 - 1}{4x^2} \right)$$

$$G_{22} = \left( -\partial_x^2 + w^2 x^2 + 2wf + w + \frac{4(f-\frac{1}{2})^2 - 1}{4x^2} \right)$$

Identify $f - \frac{1}{2} = L_B \, , \quad w = \kappa^2$ $\qquad \lambda = \kappa^2$

Eigenvalue of $G$: $M^2(n, L) = 4\kappa^2(n + L_B + 1)$

*LF Holography*

**Baryon Equation**

Superconformal Quantum Mechanics

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(L_B+1) + \frac{4L_B^2-1}{4\zeta^2}\right)\psi_J^+ = M^2\psi_J^+$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2 L_B + \frac{4(L_B+1)^2-1}{4\zeta^2}\right)\psi_J^- = M^2\psi_J^-$$

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

**S=1/2, P=+**

**Meson Equation**

$$\lambda = \kappa^2$$

$$\left(-\partial_\zeta^2 + \kappa^4\zeta^2 + 2\kappa^2(J-1) + \frac{4L_M^2-1}{4\zeta^2}\right)\phi_J = M^2\phi_J$$

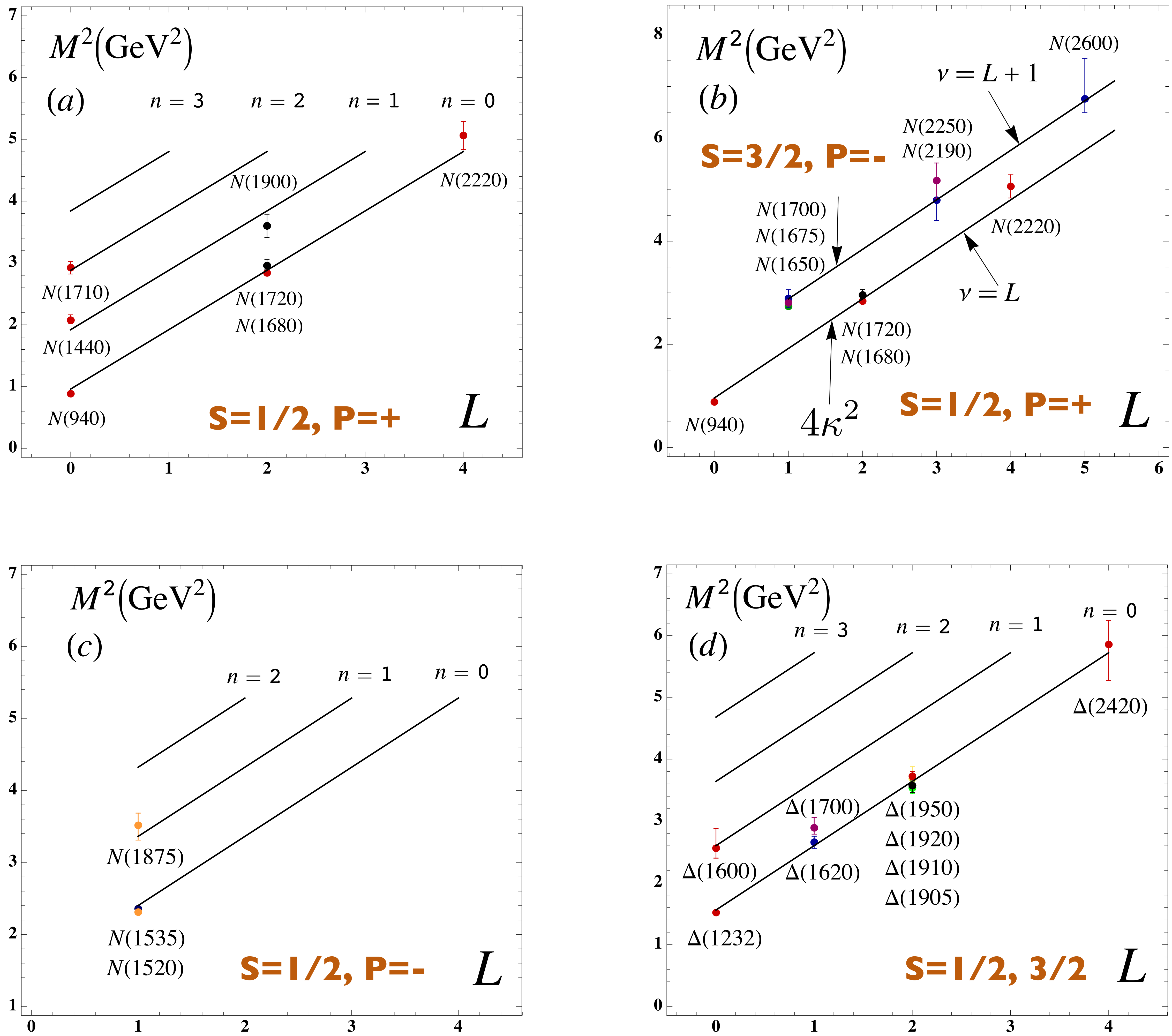
**S=0, P=+**

$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

*Same* $\kappa$ *!*

***S=0, I=1 Meson is superpartner of S=1/2, I=1 Baryon***

***Meson-Baryon Degeneracy for $L_M=L_B+1$***



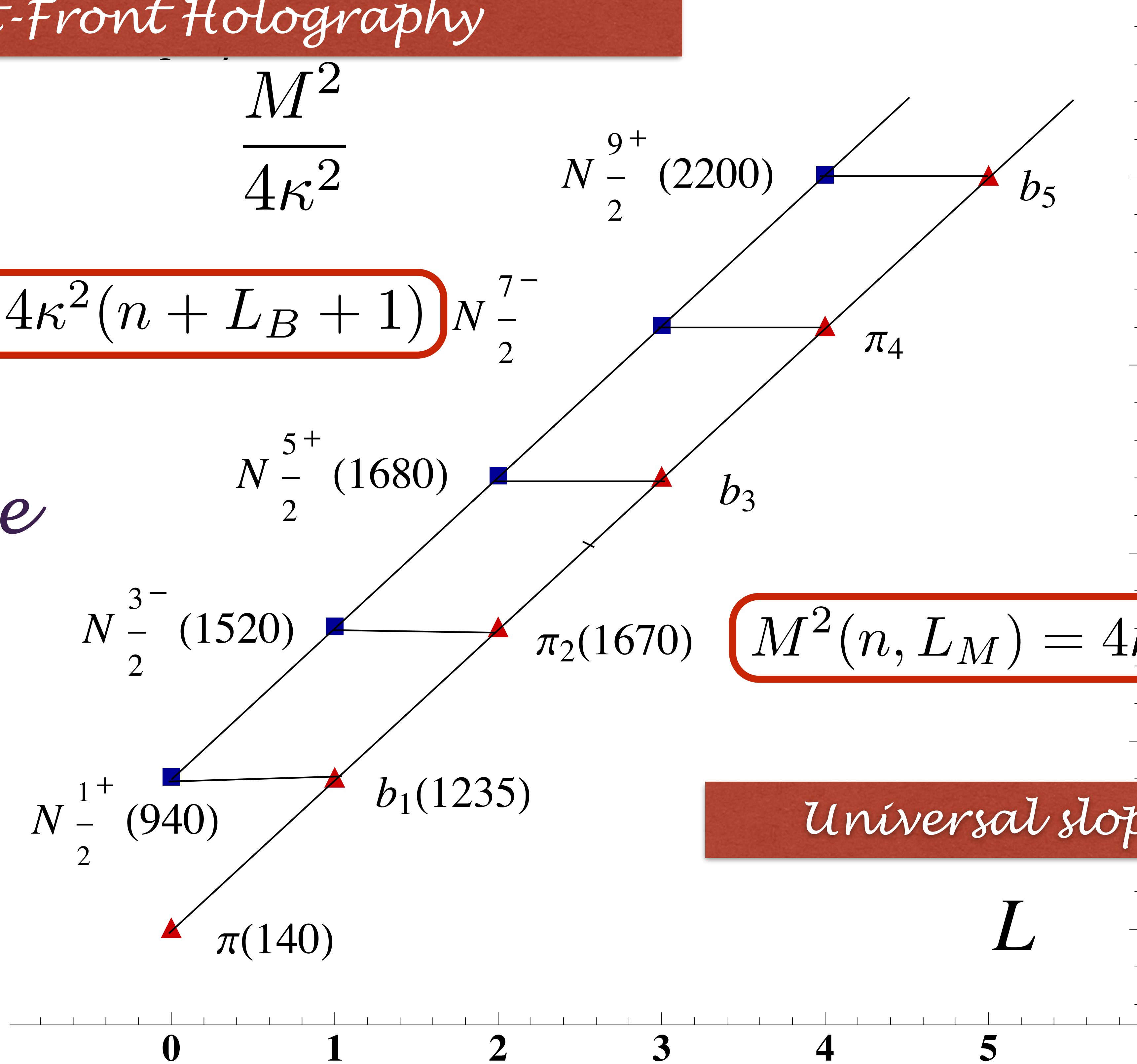
$M^2(\mathrm{GeV}^2)$
(a)
n = 3
n = 2
n = 1
n = 0
N(1900)
N(2220)
N(1710)
N(1440)
N(1720)
N(1680)
N(940)
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(b)
N(2600)
$\nu = L + 1$
N(2250)
N(2190)
S=3/2, P=-
N(1700)
N(1675)
N(1650)
N(2220)
$\nu = L$
N(1720)
N(1680)
N(940)
$4\kappa^2$
S=1/2, P=+
L
$M^2(\mathrm{GeV}^2)$
(c)
n = 2
n = 1
n = 0
N(1875)
N(1535)
N(1520)
S=1/2, P=-
L
$M^2(\mathrm{GeV}^2)$
(d)
n = 0
n = 3
n = 2
n = 1
Δ(2420)
Δ(1700)
Δ(1950)
Δ(1920)
Δ(1910)
Δ(1905)
Δ(1600)
Δ(1620)
Δ(1232)
S=1/2, 3/2
L

## Superconformal Quantum Mechanics Light-Front Holography

de Tèramond, Dosch, Lorcè, sjb

$$M^2(n, L_B) = 4\kappa^2(n + L_B + 1)$$

*Same slope*

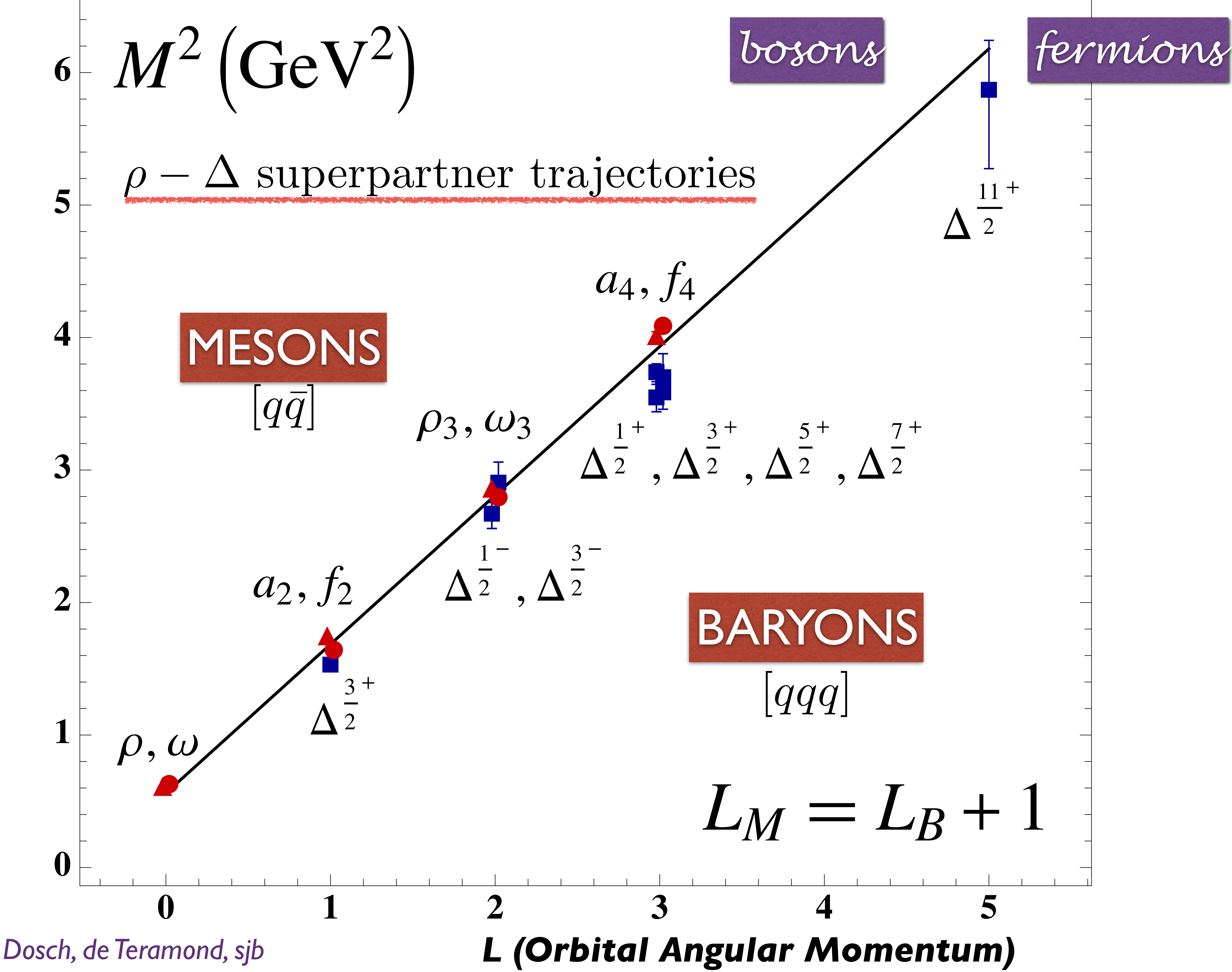



$$M^2(n, L_M) = 4\kappa^2(n + L_M)$$

*Universal slopes in n, L*

$$\frac{M^2_{meson}}{M^2_{nucleon}} = \frac{n + L_M}{n + L_B + 1}$$

**Meson-Baryon Mass Degeneracy for $L_M$=$L_B$+1**

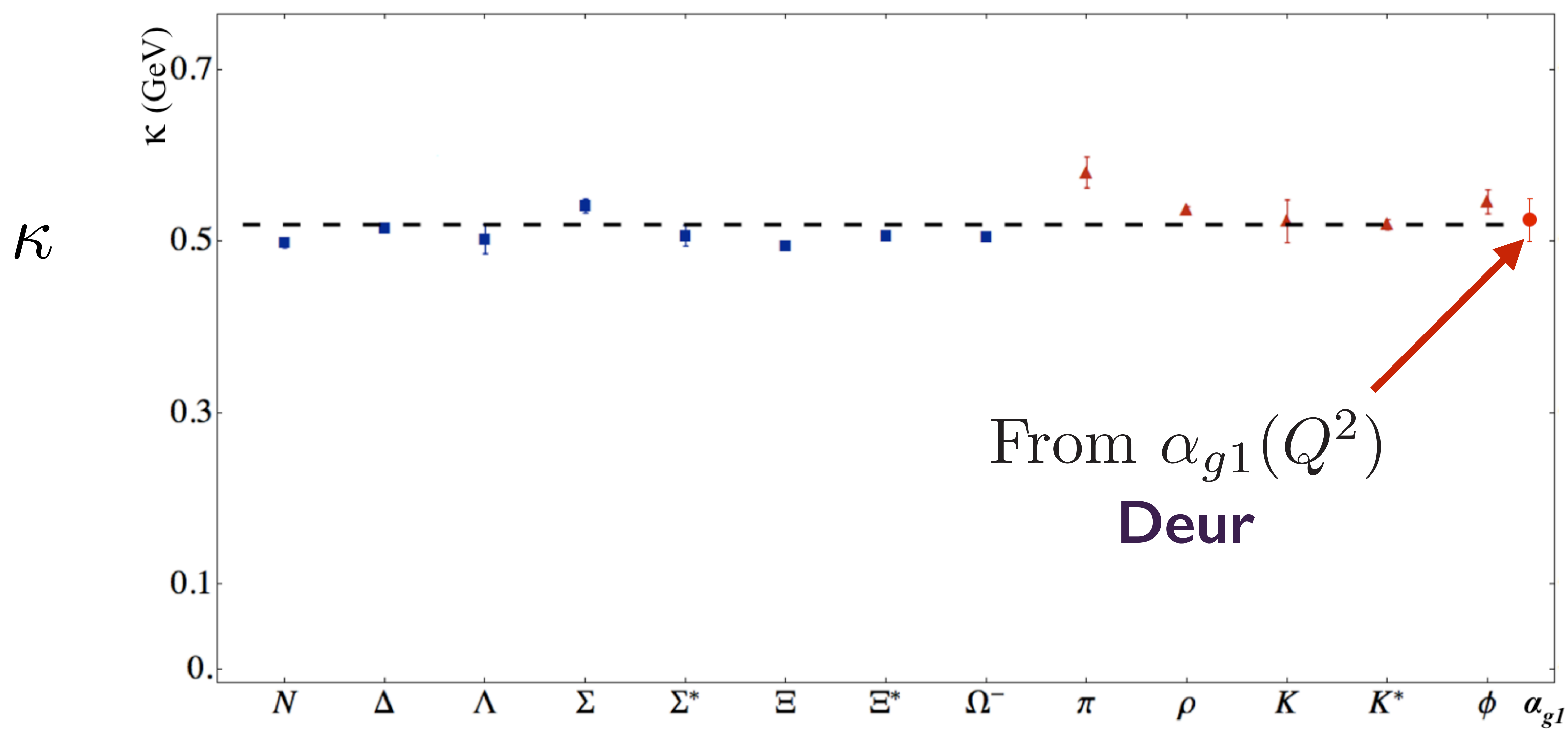


$M^2\,(\mathrm{GeV}^2)$
bosons
fermions
$\rho - \Delta$ superpartner trajectories
MESONS
$[q\bar{q}]$
BARYONS
$[qqq]$
$\rho, \omega$
$a_2, f_2$
$\rho_3, \omega_3$
$a_4, f_4$
$\Delta \frac{3}{2}^+$
$\Delta \frac{1}{2}^-, \Delta \frac{3}{2}^-$
$\Delta \frac{1}{2}^+, \Delta \frac{3}{2}^+, \Delta \frac{5}{2}^+, \Delta \frac{7}{2}^+$
$\Delta \frac{11}{2}^+$
$L_M = L_B + 1$
0
1
2
3
4
5
6
L (Orbital Angular Momentum)
Dosch, de Teramond, sjb

$\lambda = \kappa^2$

*de Tèramond, Dosch, Lorce', sjb*

$$m_u = m_d = 46 \text{ MeV}, \; m_s = 357 \text{ MeV}$$




**Fit to the slope of Regge trajectories, including radial excitations**

*Same Regge Slope for Meson, Baryons: Supersymmetric feature of hadron physics*

- Compute Dirac proton form factor using SU(6) flavor symmetry

$$F_1^p(Q^2) = R^4 \int \frac{dz}{z^4}\, V(Q,z) \Psi_+^2(z)$$

- Nucleon AdS wave function

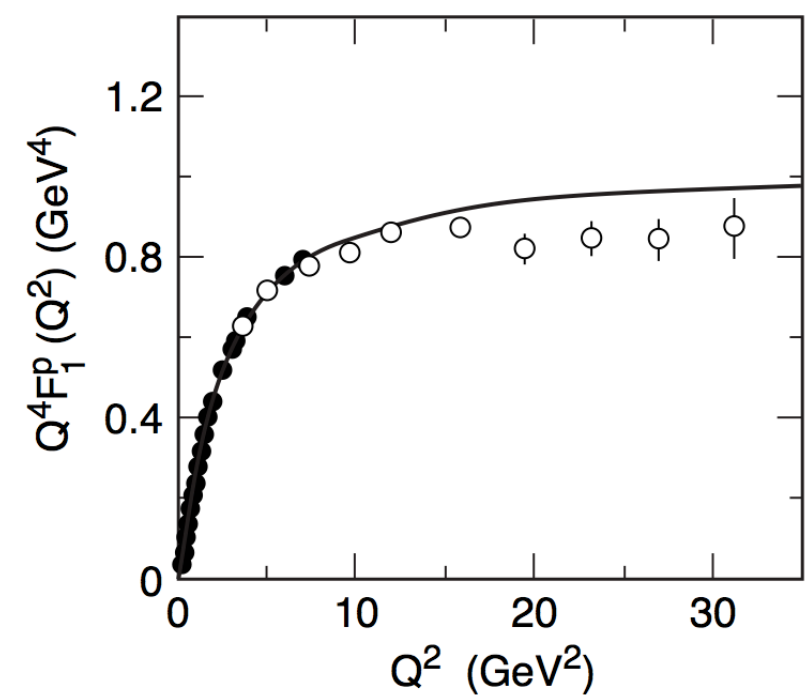
$$\Psi_+(z) = \frac{\kappa^{2+L}}{R^2} \sqrt{\frac{2n!}{(n+L)!}}\, z^{7/2+L} L_n^{L+1}\left(\kappa^2 z^2\right) e^{-\kappa^2 z^2/2}$$

- Normalization $\quad (F_1{}^p(0) = 1, \quad V(Q=0,z) = 1)$

$$R^4 \int \frac{dz}{z^4}\, \Psi_+^2(z) = 1$$

- Bulk-to-boundary propagator [Grigoryan and Radyushkin (2007)]

$$V(Q,z) = \kappa^2 z^2 \int_0^1 \frac{dx}{(1-x)^2}\, x^{\frac{Q^2}{4\kappa^2}}\, e^{-\kappa^2 z^2 x/(1-x)}$$




- Find

$$F_1^p(Q^2) = \frac{1}{\left(1 + \frac{Q^2}{\mathcal{M}_\rho^2}\right)\left(1 + \frac{Q^2}{\mathcal{M}_{\rho'}^2}\right)}$$

with $\mathcal{M}_{\rho_n}{}^2 \to 4\kappa^2(n + 1/2)$

## Space-Like Dirac Proton Form Factor

- Consider the spin non-flip form factors

$$F_+(Q^2) = g_+ \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_-(Q^2) = g_- \int d\zeta \, J(Q,\zeta) |\psi_-(\zeta)|^2,$$

where the effective charges $g_+$ and $g_-$ are determined from the spin-flavor structure of the theory.

- Choose the struck quark to have $S^z = +1/2$. The two AdS solutions $\psi_+(\zeta)$ and $\psi_-(\zeta)$ correspond to nucleons with $J^z = +1/2$ and $-1/2$.

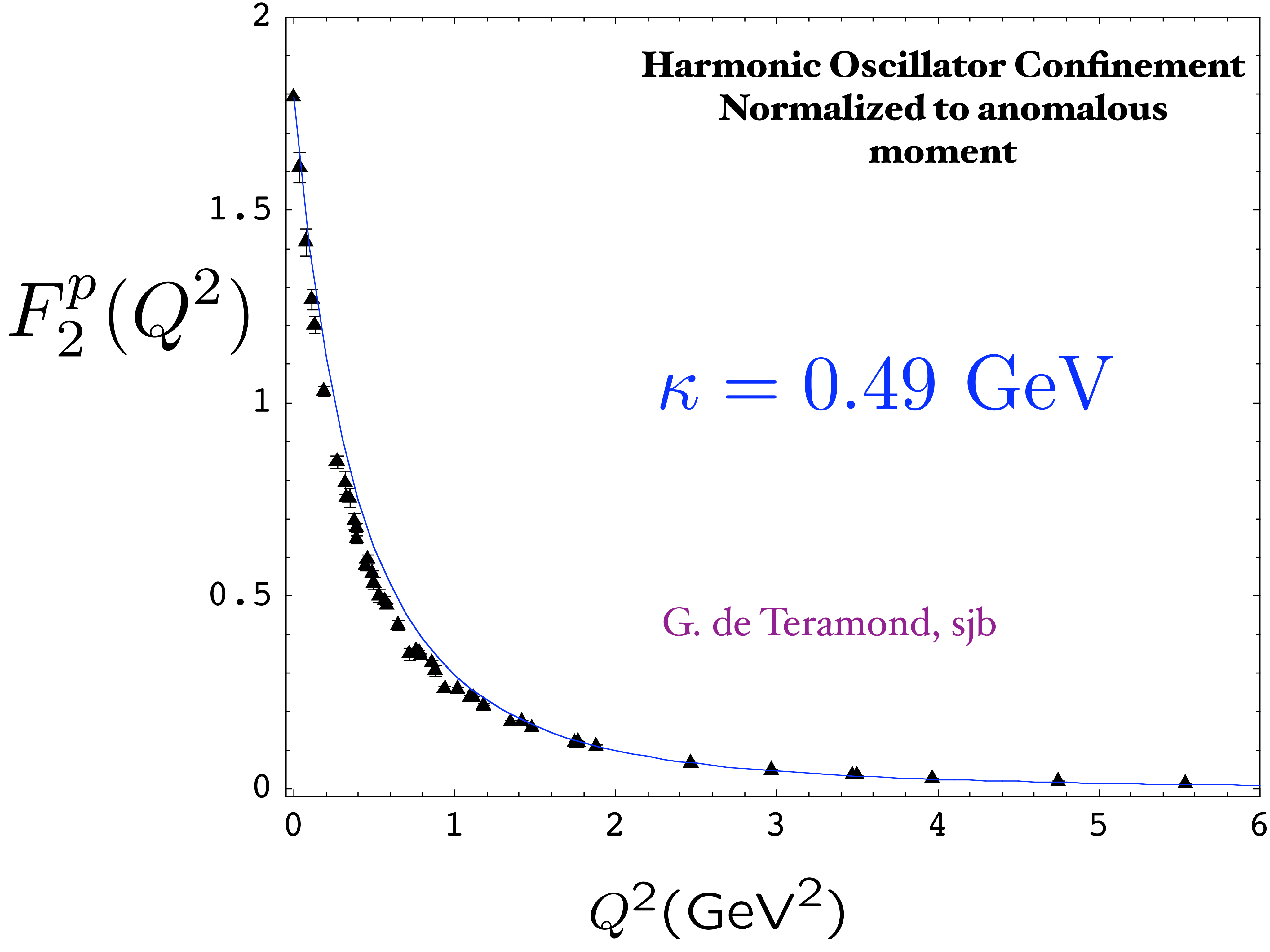
- For $SU(6)$ spin-flavor symmetry

$$F_1^p(Q^2) = \int d\zeta \, J(Q,\zeta) |\psi_+(\zeta)|^2,$$
$$F_1^n(Q^2) = -\frac{1}{3} \int d\zeta \, J(Q,\zeta) \left[ |\psi_+(\zeta)|^2 - |\psi_-(\zeta)|^2 \right],$$

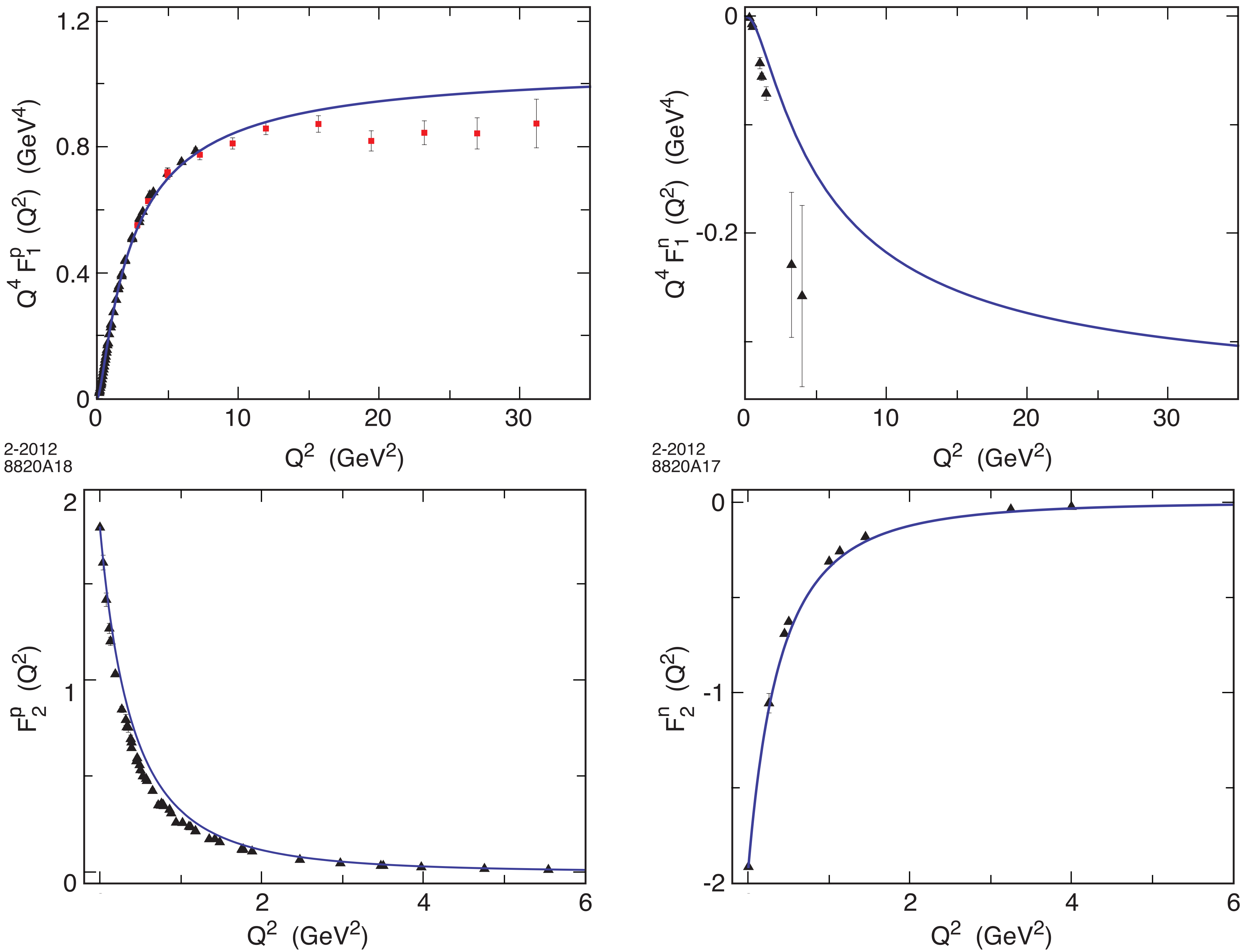
where $F_1^p(0) = 1, \; F_1^n(0) = 0$.

# Spacelike Pauli Form Factor

From overlap of $L = 1$ and $L = 0$ LFWFs



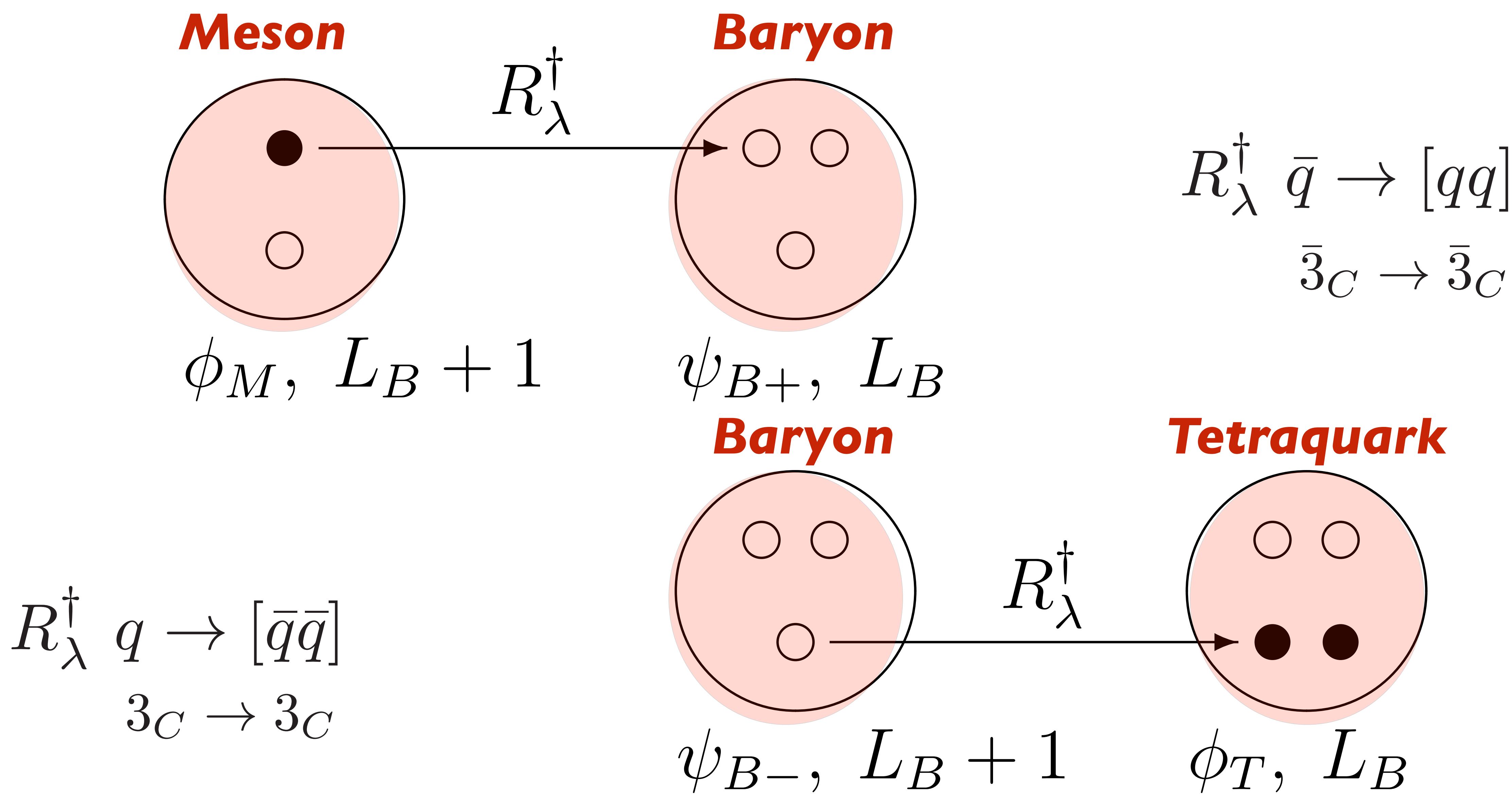
Using $SU(6)$ flavor symmetry and normalization to static quantities



# Superconformal Algebra

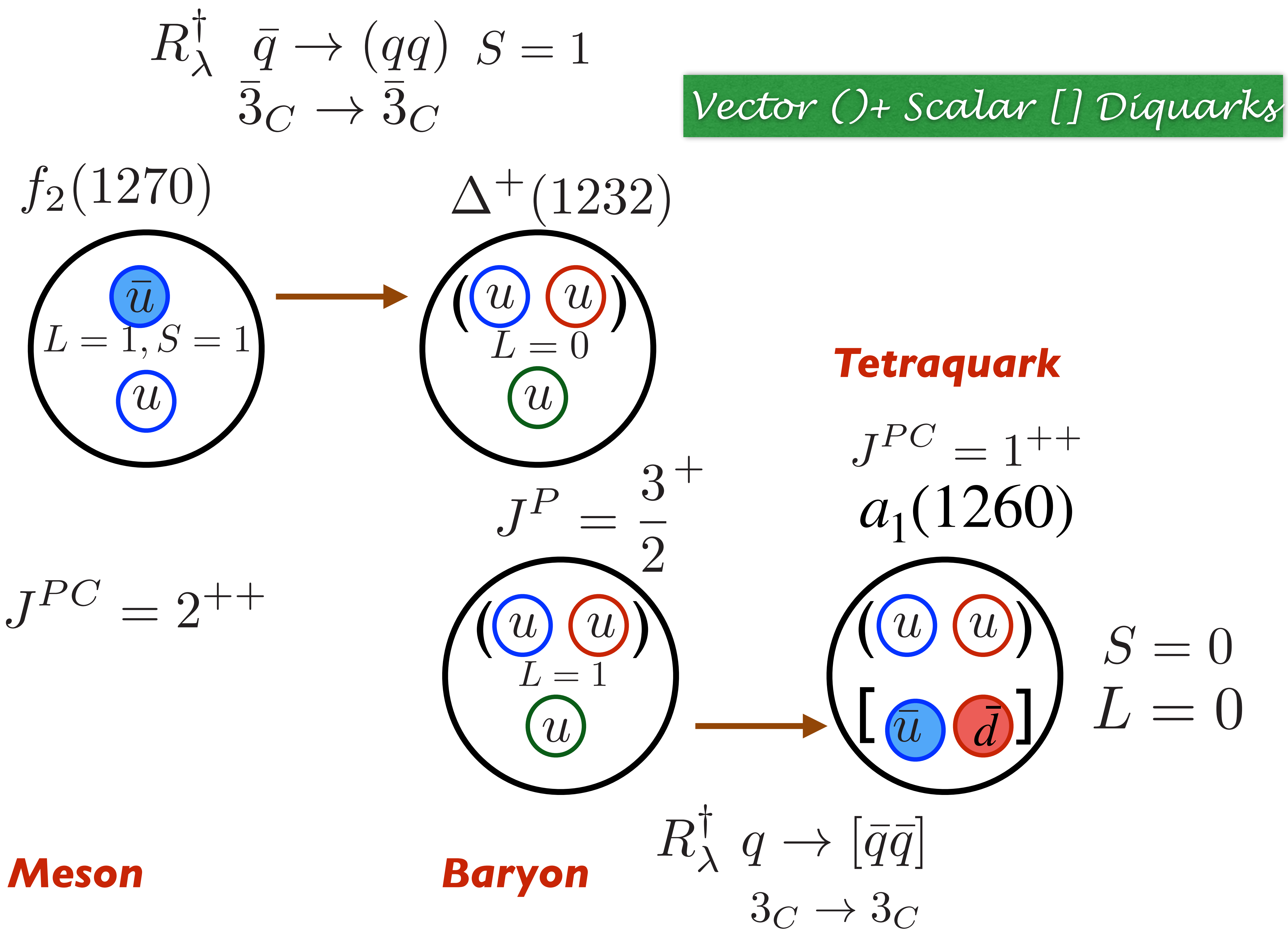
## 2X2 Hadronic Multiplets

### Bosons, Fermions with Equal Mass!




Proton: |u[ud]> Quark + Scalar Diquark
Equal Weight: L=0, L=1

# Superconformal Algebra 4-Plet



| Meson | | | Baryon | | | Tetraquark | | |
|---|---|---|---|---|---|---|---|---|
| q-cont | $J^{P(C)}$ | Name | q-cont | $J^P$ | Name | q-cont | $J^{P(C)}$ | Name |
| $\bar{q}q$ | $0^{-+}$ | $\pi(140)$ | — | — | — | — | — | — |
| $\bar{q}q$ | $1^{+-}$ | $h_1(1170)$ | $[ud]q$ | $(1/2)^+$ | $N(940)$ | $[ud][\bar{u}\bar{d}]$ | $0^{++}$ | $\sigma(500)$ |
| $\bar{q}q$ | $2^{-+}$ | $\eta_2(1645)$ | $[ud]q$ | $(3/2)^-$ | $N_{\frac{3}{2}^-}(1520)$ | $[ud][\bar{u}\bar{d}]$ | $1^{-+}$ | — |
| $\bar{q}q$ | $1^{--}$ | $\rho(770),\ \omega(780)$ | — | — | — | — | — | — |
| $\bar{q}q$ | $2^{++}$ | $a_2(1320),\ f_2(1270)$ | $(qq)q$ | $(3/2)^+$ | $\Delta(1232)$ | $(qq)[\bar{u}\bar{d}]$ | $1^{++}$ | $a_1(1260)$ |
| $\bar{q}q$ | $3^{--}$ | $\rho_3(1690),\ \omega_3(1670)$ | $(qq)q$ | $(3/2)^-$ | $\Delta_{\frac{3}{2}^-}(1700)$ | $(qq)[\bar{u}\bar{d}]$ | $1^{-+}$ | $\pi_1(1600)$ |
| $\bar{q}q$ | $4^{++}$ | $a_4(2040),\ f_4(2050)$ | $(qq)q$ | $(7/2)^+$ | $\Delta_{\frac{7}{2}^+}(1950)$ | $(qq)[\bar{u}\bar{d}]$ | — | — |
| $\bar{q}s$ | $0^-$ | $K(495)$ | — | — | — | — | — | — |
| $\bar{q}s$ | $1^+$ | $\bar{K}_1(1270)$ | $[ud]s$ | $(1/2)^+$ | $\Lambda(1115)$ | $[ud][\bar{s}\bar{q}]$ | $0^+$ | $K_0^*(1430)$ |
| $\bar{q}s$ | $2^-$ | $K_2(1770)$ | $[ud]s$ | $(3/2)^-$ | $\Lambda(1520)$ | $[ud][\bar{s}\bar{q}]$ | $1^-$ | — |
| $\bar{s}q$ | $0^-$ | $K(495)$ | — | — | — | — | — | — |
| $\bar{s}q$ | $1^+$ | $K_1(1270)$ | $[sq]q$ | $(1/2)^+$ | $\Sigma(1190)$ | $[sq][\bar{s}\bar{q}]$ | $0^{++}$ | $a_0(980)$<br>$f_0(980)$ |
| $\bar{s}q$ | $1^-$ | $K^*(890)$ | — | — | — | — | — | — |
| $\bar{s}q$ | $2^+$ | $K_2^*(1430)$ | $(sq)q$ | $(3/2)^+$ | $\Sigma(1385)$ | $(sq)[\bar{u}\bar{d}]$ | $1^+$ | $K_1(1400)$ |
| $\bar{s}q$ | $3^-$ | $K_3^*(1780)$ | $(sq)q$ | $(3/2)^-$ | $\Sigma(1670)$ | $(sq)[\bar{u}\bar{d}]$ | $2^-$ | $K_2(1820)$ |
| $\bar{s}q$ | $4^+$ | $K_4^*(2045)$ | $(sq)q$ | $(7/2)^+$ | $\Sigma(2030)$ | $(sq)[\bar{u}\bar{d}]$ | — | — |
| $\bar{s}s$ | $0^{-+}$ | $\eta'(958)$ | — | — | — | — | — | — |
| $\bar{s}s$ | $1^{+-}$ | $h_1(1380)$ | $[sq]s$ | $(1/2)^+$ | $\Xi(1320)$ | $[sq][\bar{s}\bar{q}]$ | $0^{++}$ | $f_0(1370)$<br>$a_0(1450)$ |
| $\bar{s}s$ | $2^{-+}$ | $\eta_2(1870)$ | $[sq]s$ | $(3/2)^-$ | $\Xi(1620)$ | $[sq][\bar{s}\bar{q}]$ | $1^{-+}$ | — |
| $\bar{s}s$ | $1^{--}$ | $\Phi(1020)$ | — | — | — | — | — | — |
| $\bar{s}s$ | $2^{++}$ | $f_2'(1525)$ | $(sq)s$ | $(3/2)^+$ | $\Xi^*(1530)$ | $(sq)[\bar{s}\bar{q}]$ | $1^{++}$ | $f_1(1420)$<br>$a_1(1420)$ |
| $\bar{s}s$ | $3^{--}$ | $\Phi_3(1850)$ | $(sq)s$ | $(3/2)^-$ | $\Xi(1820)$ | $(sq)[\bar{s}\bar{q}]$ | — | — |
| $\bar{s}s$ | $2^{++}$ | $f_2(1640)$ | $(ss)s$ | $(3/2)^+$ | $\Omega(1672)$ | $(ss)[\bar{s}\bar{q}]$ | $1^+$ | $K_1(1650)$ |

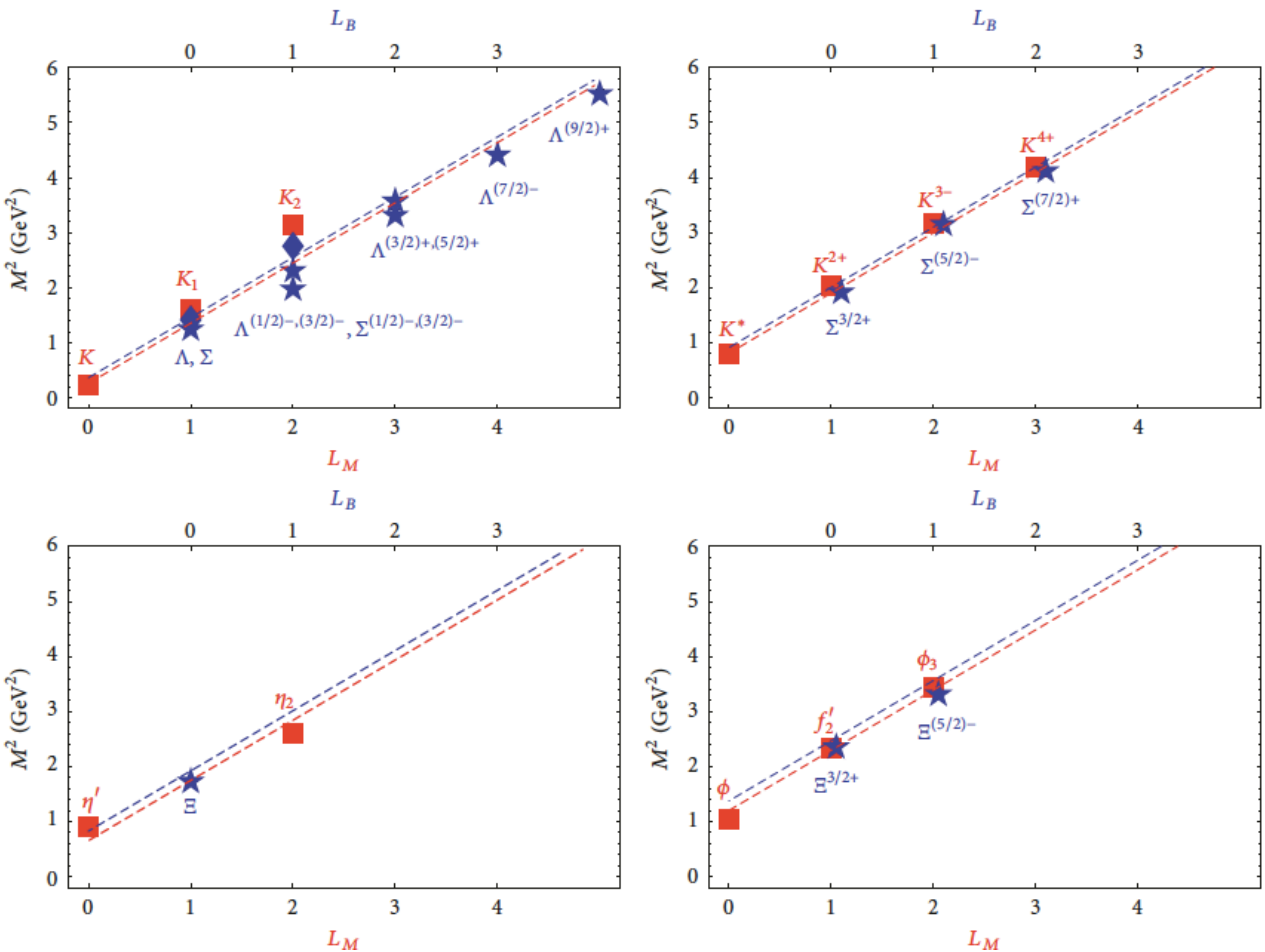
Meson Baryon Tetraquark

# New Organization of the Hadron Spectrum

M. Nielsen, sjb

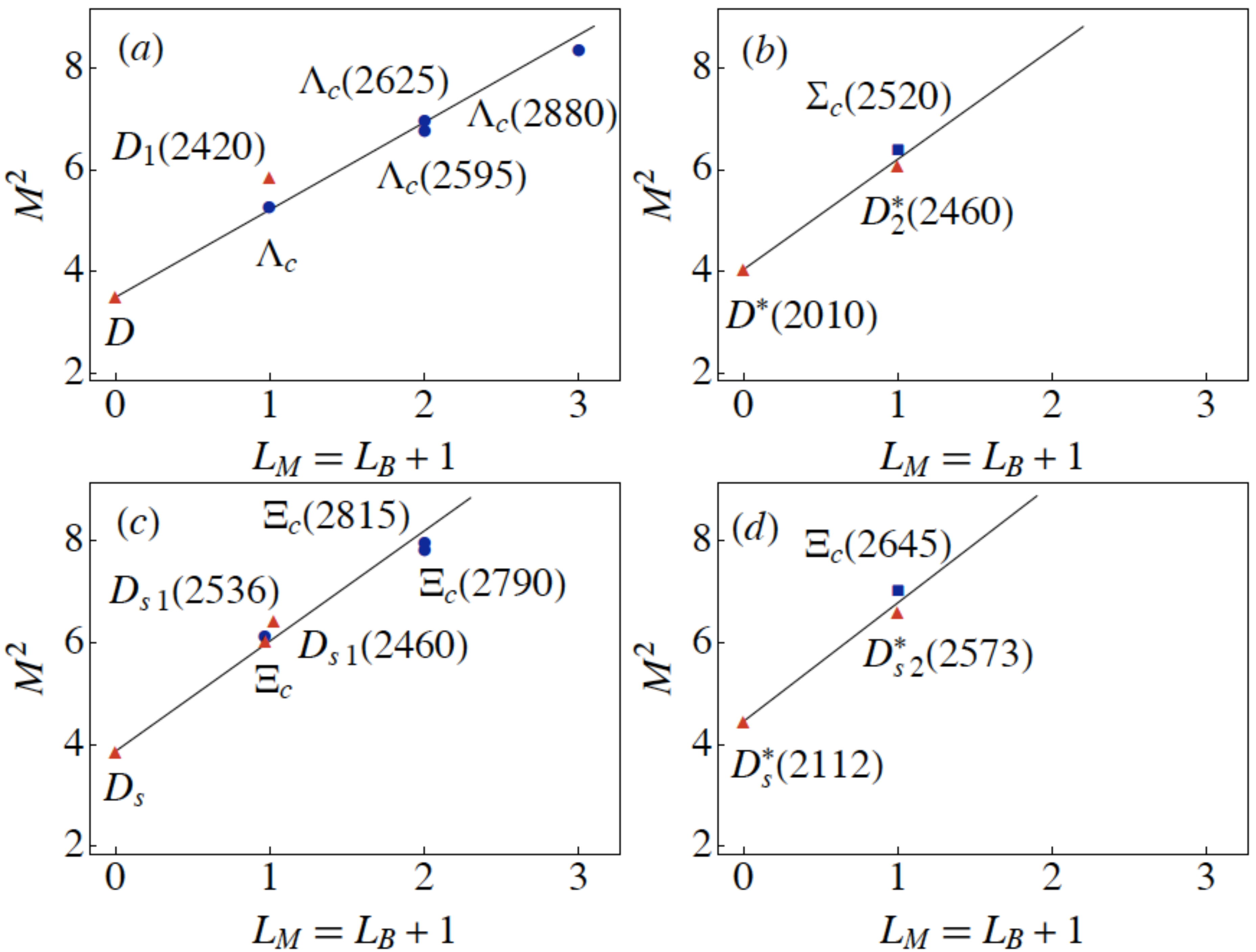
Dosch, de Teramond, sjb

# Supersymmetry across the light and heavy-light spectrum



Dosch, de Teramond, sjb

# Supersymmetry across the light and heavy-light spectrum




**Heavy charm quark mass does not break supersymmetry**

# Superpartners for states with one c quark

| Meson | | | Baryon | | | Tetraquark | | |
|---|---|---|---|---|---|---|---|---|
| q-cont | $J^{P(C)}$ | Name | q-cont | $J^P$ | Name | q-cont | $J^{P(C)}$ | Name |
| $\bar{q}c$ | $0^-$ | $D(1870)$ | — | — | — | — | — | — |
| $\bar{q}c$ | $1^+$ | $D_1(2420)$ | $[ud]c$ | $(1/2)^+$ | $\Lambda_c(2290)$ | $[ud][\bar{c}\bar{q}]$ | $0^+$ | $\bar{D}_0^*(2400)$ |
| $\bar{q}c$ | $2^-$ | $D_J(2600)$ | $[ud]c$ | $(3/2)^-$ | $\Lambda_c(2625)$ | $[ud][\bar{c}\bar{q}]$ | $1^-$ | — |
| $\bar{c}q$ | $0^-$ | $\bar{D}(1870)$ | — | — | — | — | — | — |
| $\bar{c}q$ | $1^+$ | $\bar{D}_1(2420)$ | $[cq]q$ | $(1/2)^+$ | $\Sigma_c(2455)$ | $[cq][\bar{u}\bar{d}]$ | $0^+$ | $D_0^*(2400)$ |
| $\bar{q}c$ | $1^-$ | $D^*(2010)$ | — | — | — | — | — | — |
| $\bar{q}c$ | $2^+$ | $D_2^*(2460)$ | $(qq)c$ | $(3/2)^+$ | $\Sigma_c^*(2520)$ | $(qq)[\bar{c}\bar{q}]$ | $1^+$ | $D(2550)$ |
| $\bar{q}c$ | $3^-$ | $D_3^*(2750)$ | $(qq)c$ | $(3/2)^-$ | $\Sigma_c(2800)$ | $(qq)[\bar{c}\bar{q}]$ | — | — |
| $\bar{s}c$ | $0^-$ | $D_s(1968)$ | — | — | — | — | — | — |
| $\bar{s}c$ | $1^+$ | $D_{s1}(2460)$ | $[qs]c$ | $(1/2)^+$ | $\Xi_c(2470)$ | $[qs][\bar{c}\bar{q}]$ | $0^+$ | $\bar{D}_{s0}^*(2317)$ |
| $\bar{s}c$ | $2^-$ | $D_{s2}(\sim 2860)?$ | $[qs]c$ | $(3/2)^-$ | $\Xi_c(2815)$ | $[sq][\bar{c}\bar{q}]$ | $1^-$ | — |
| $\bar{s}c$ | $1^-$ | $D_s^*(2110)$ | — | — | — | — | — | — |
| $\bar{s}c$ | $2^+$ | $D_{s2}^*(2573)$ | $(sq)c$ | $(3/2)^+$ | $\Xi_c^*(2645)$ | $(sq)[\bar{c}\bar{q}]$ | $1^+$ | $D_{s1}(2536)$ |
| $\bar{c}s$ | $1^+$ | $D_{s1}(\sim 2700)?$ | $[cs]s$ | $(1/2)^+$ | $\Omega_c(2695)$ | $[cs][\bar{s}\bar{q}]$ | $0^+$ | ?? |
| $\bar{s}c$ | $2^+$ | $D_{s2}^*(\sim 2750)?$ | $(ss)c$ | $(3/2)^+$ | $\Omega_c(2770)$ | $(ss)[\bar{c}\bar{s}]$ | $1^+$ | ?? |

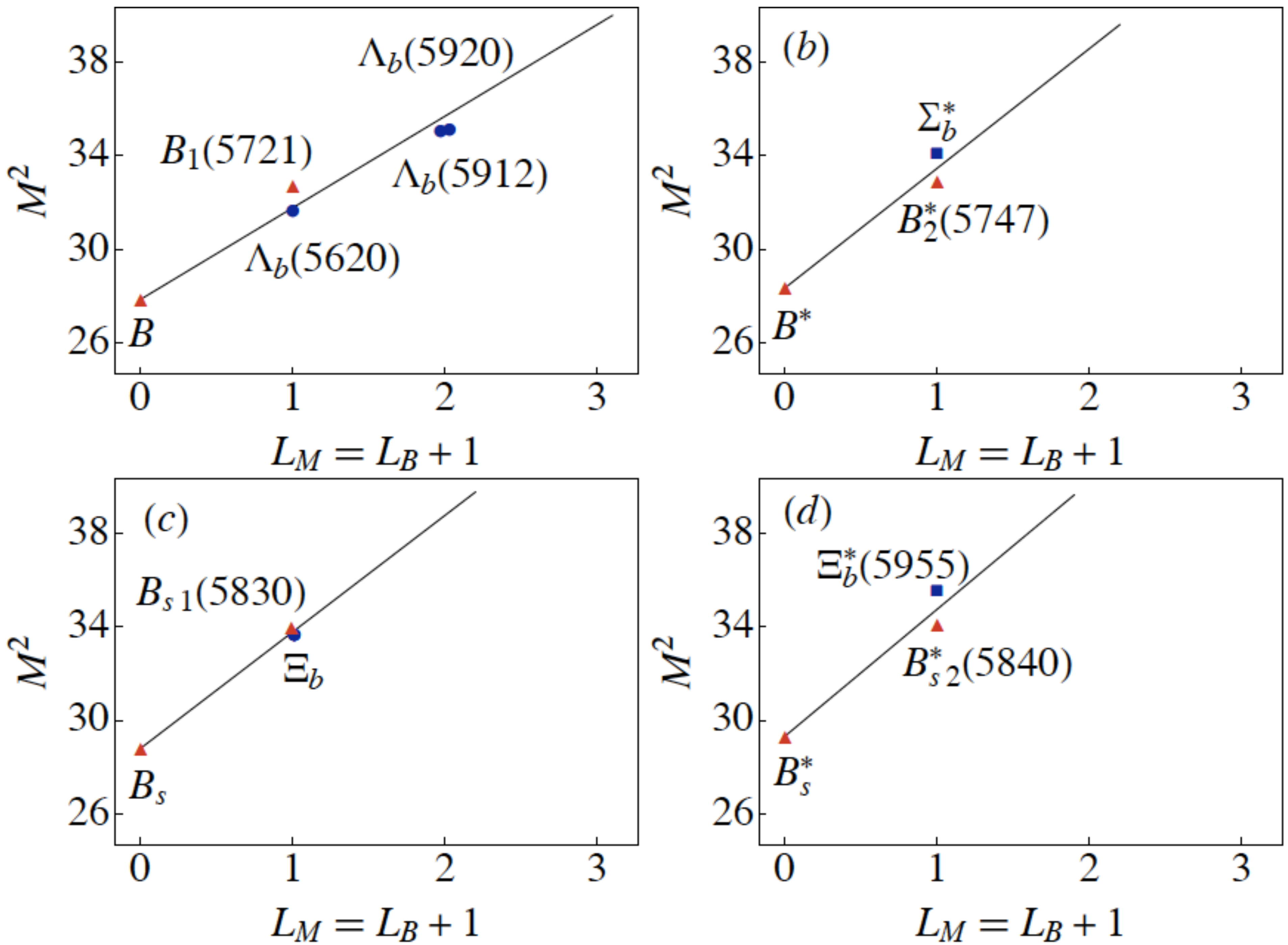
predictions

beautiful agreement!

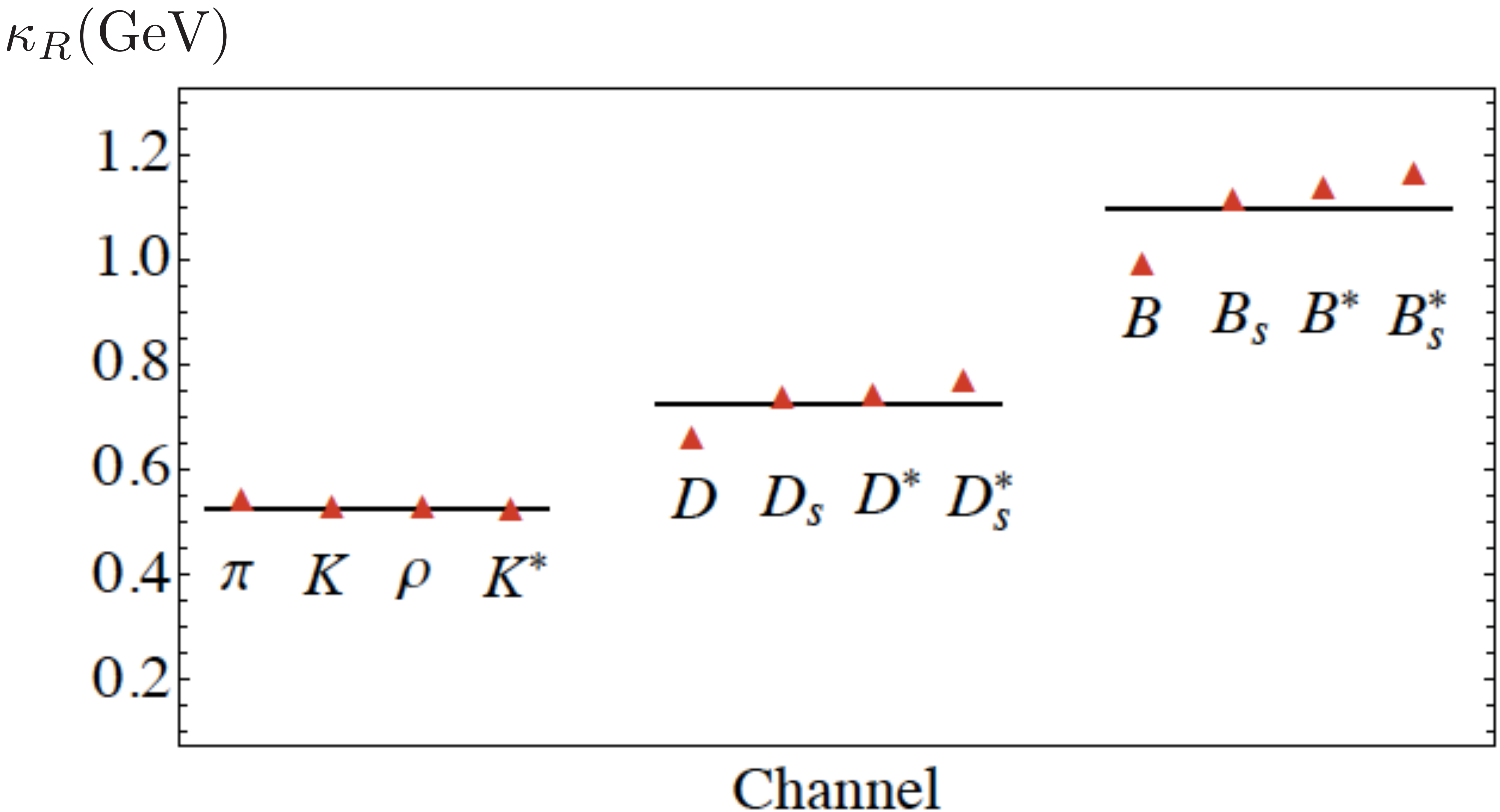
M. Nielsen, sjb

Dosch, de Teramond, sjb

# Supersymmetry across the light and heavy-light spectrum

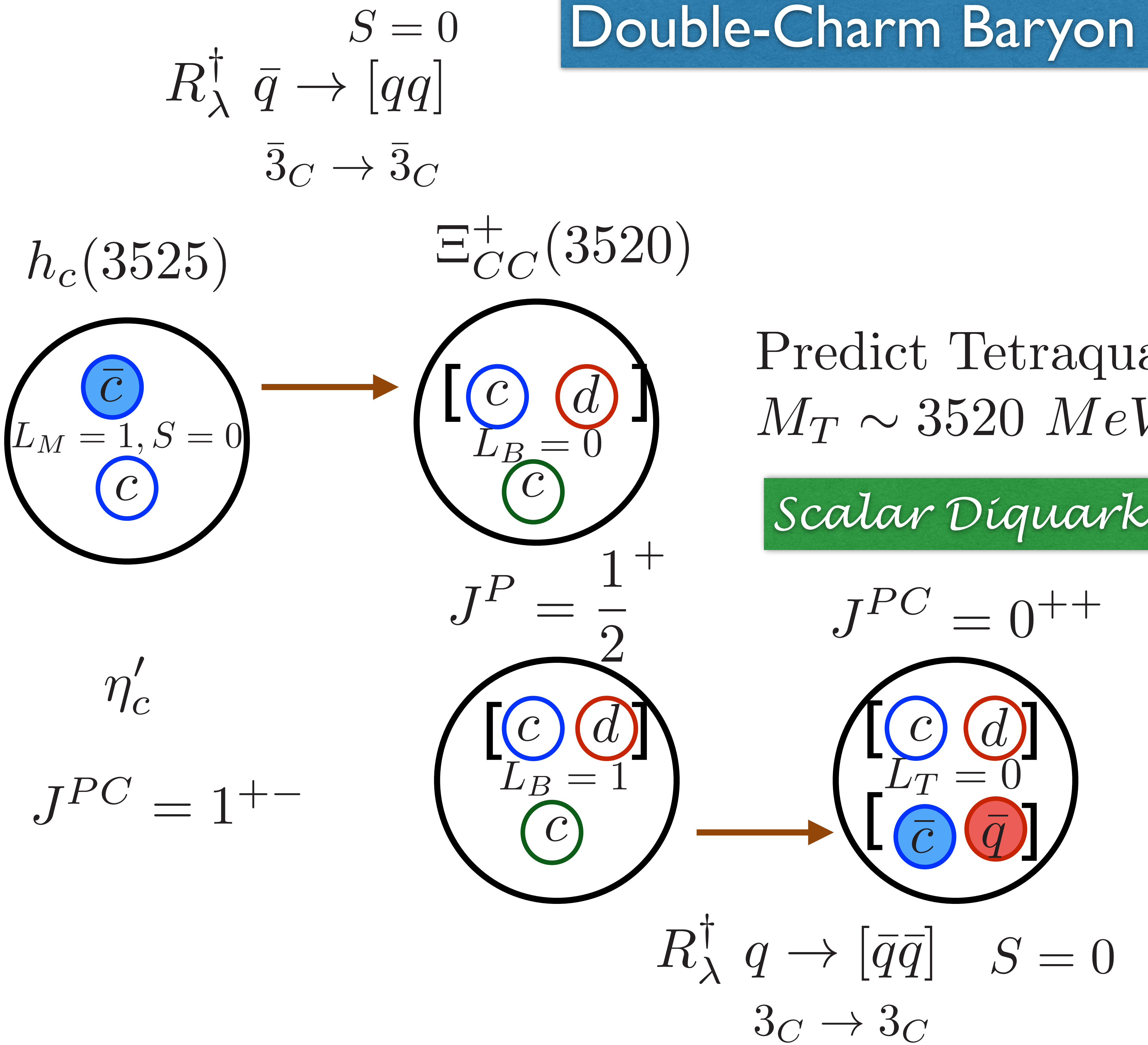



Heavy bottom quark mass does not break supersymmetry




***Regge slope for heavy-light mesons, baryons: increases with heavy quark mass***

# Double-Charm Baryon (SELEX)




Predict Tetraquark $T_{c\bar{c}q\bar{q}}$

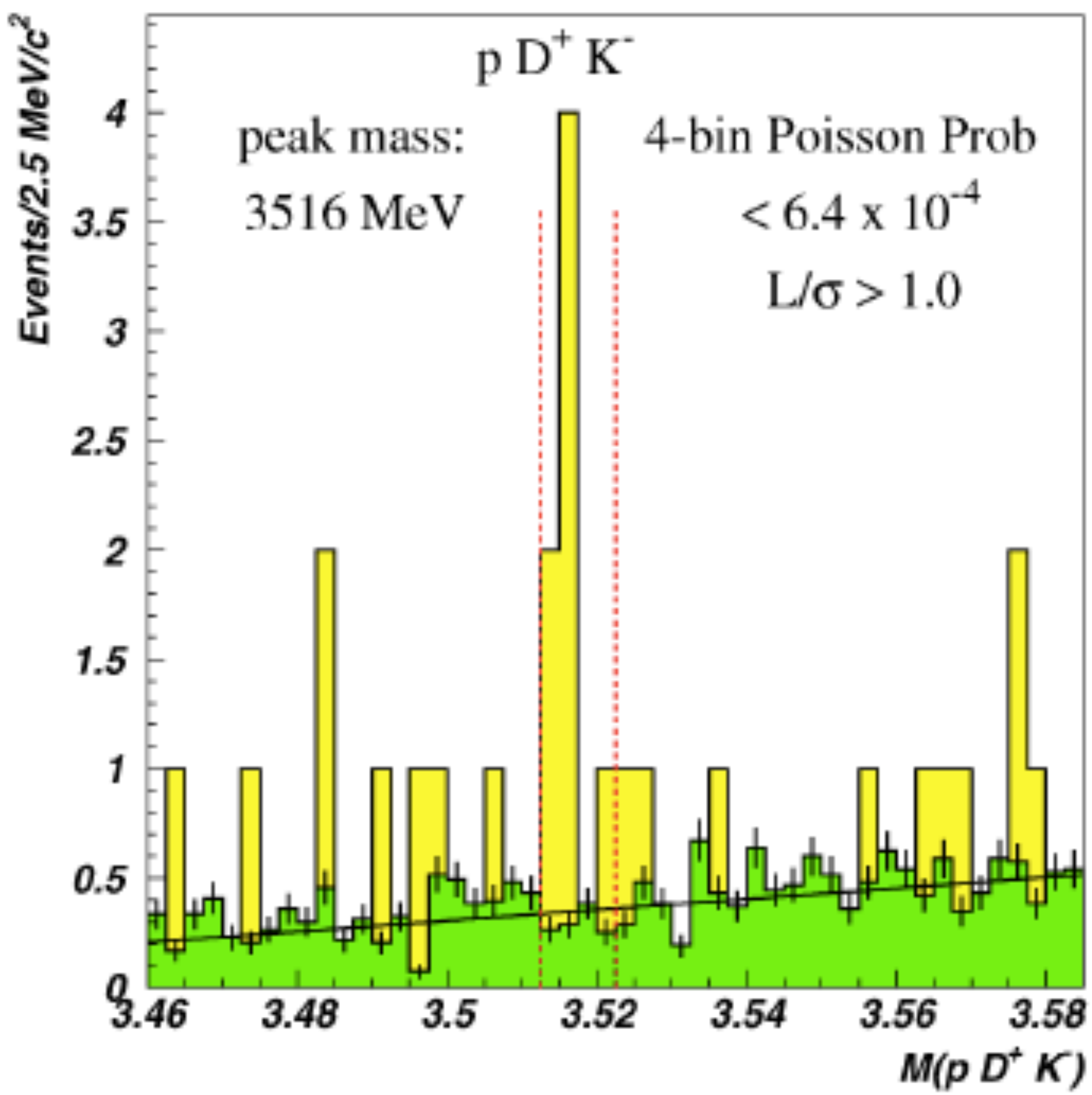
$M_T \sim 3520 \ MeV$

Scalar Diquarks

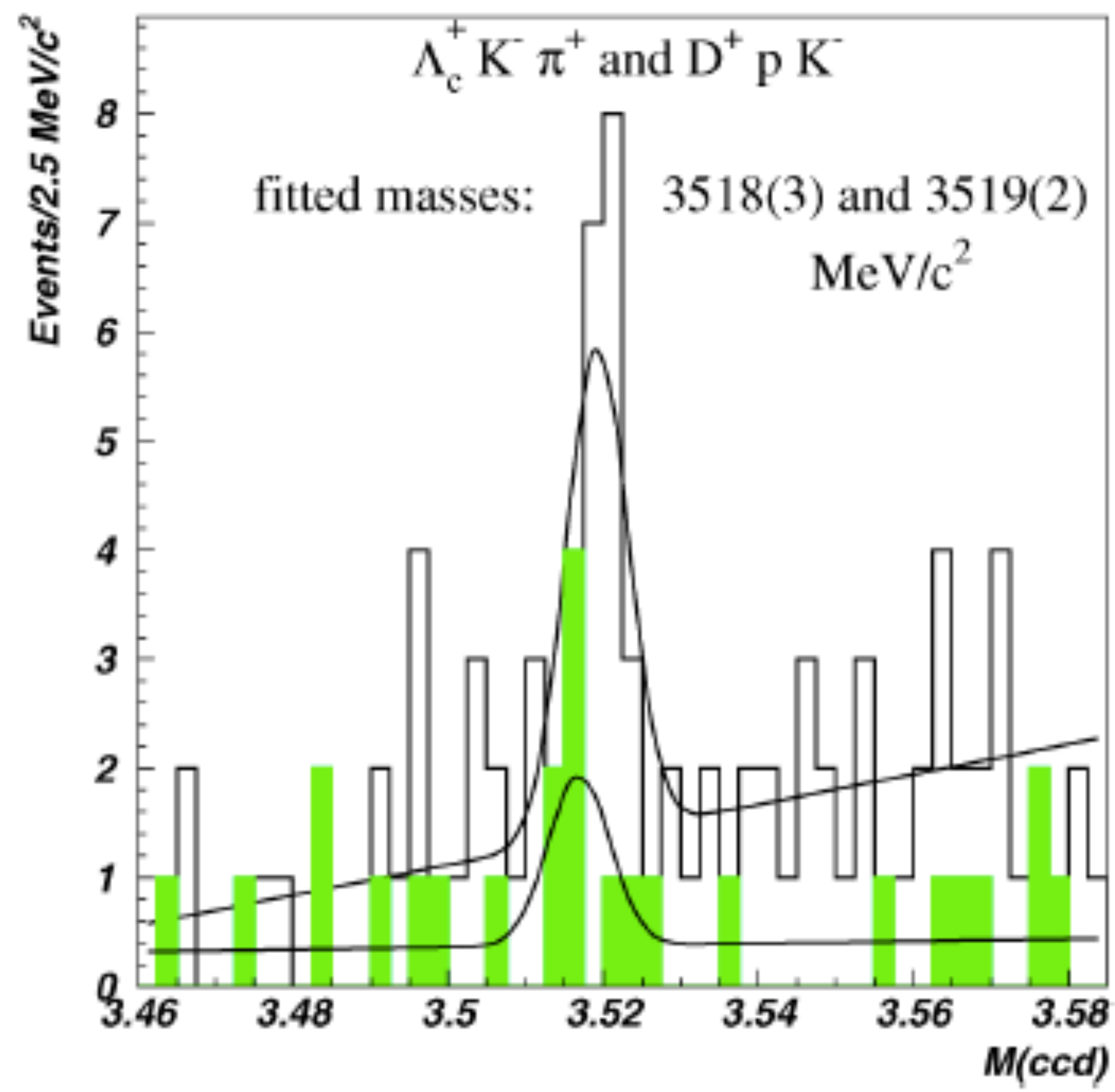
SELEX ($3520 \pm 1\ MeV$) $J^P = \frac{1}{2}^-$ $|[cd]c>$

Two decay channels: $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+, pD^+K^-$

SELEX Collaboration / Physics Letters B 628 (2005) 18–24

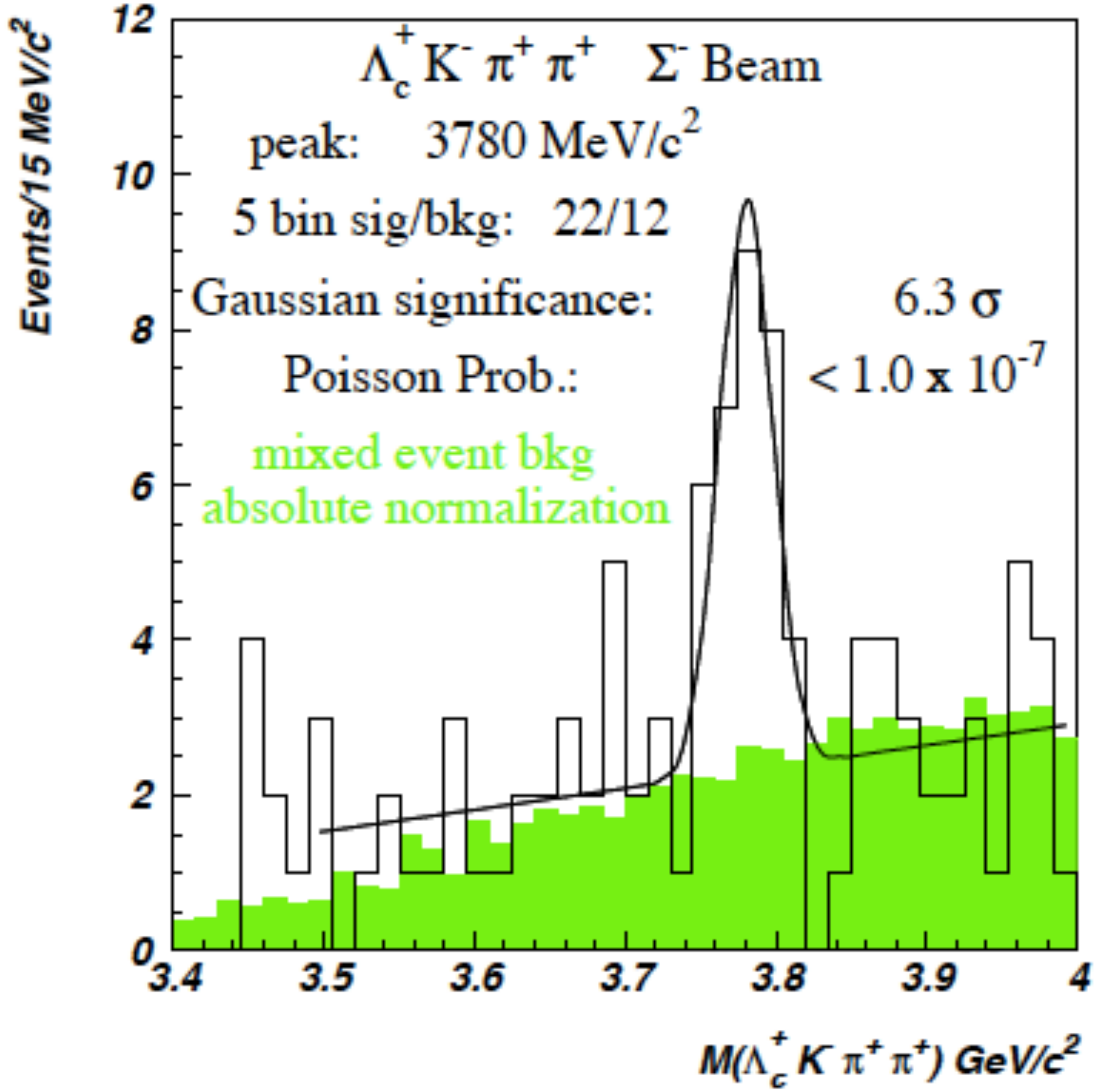



$\Xi_{cc}^+ \to pD^+K^-$ mass distribution from Fig. 2(a) with high-statistics measurement of random combinatoric background computed from event-mixing.

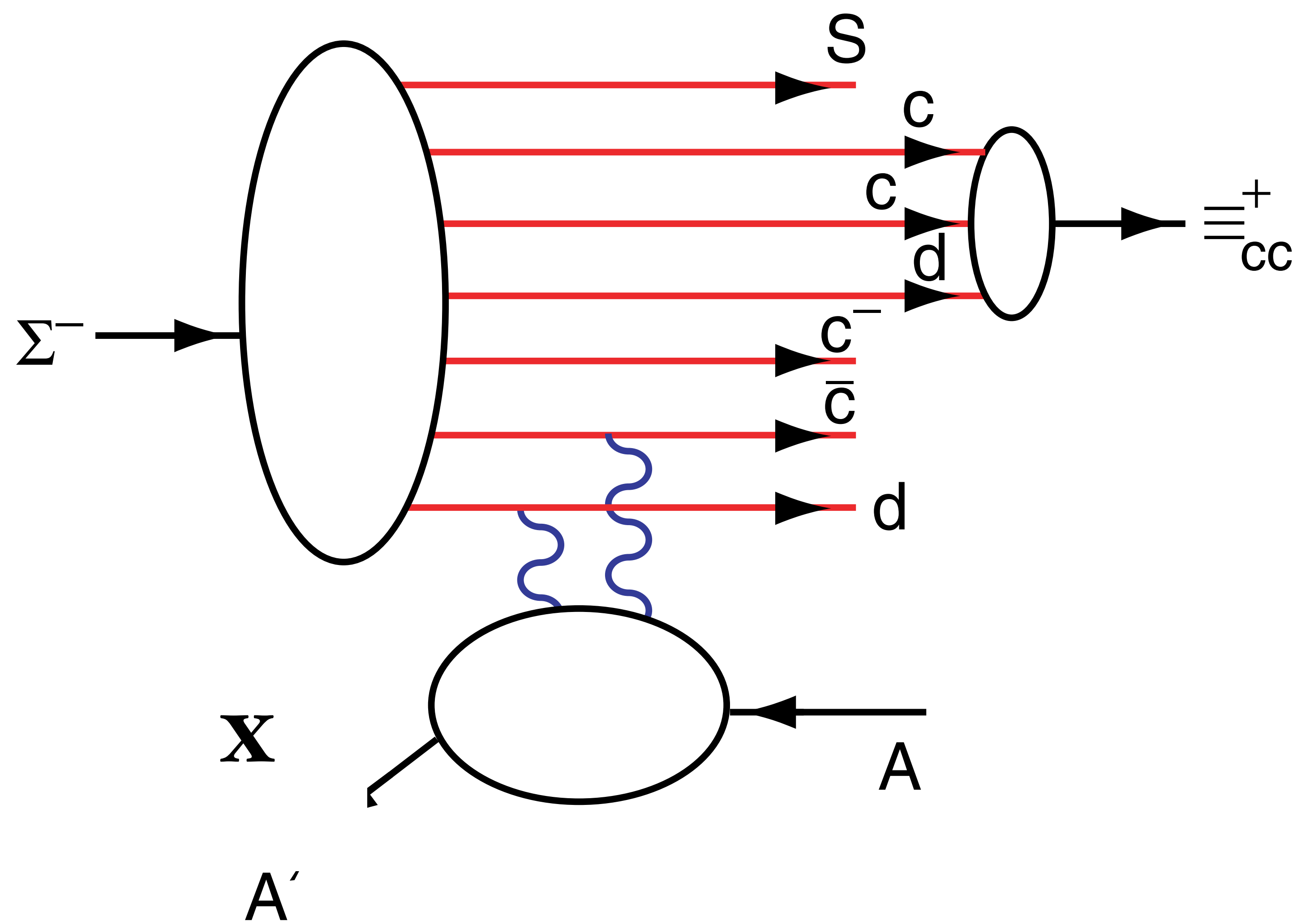



Gaussian fits for $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+$ and $\Xi_{cc}^+ \to pD^+K^-$ (shaded data) on same plot.

# SELEX: Recent Progress in the Analysis of Charm-Strange and Double-Charm Baryons




The $\Lambda_c^+ K^- \pi^+ \pi^+$ invariant mass distribution, for $\Sigma^-$ beam only.




# Production of a Double-Charm Baryon

**SELEX high $x_F$** $\quad < x_F >= 0.33$

$\text{SELEX } (3520 \pm 1\ MeV) \quad J^P = \frac{1}{2}^- \quad |[cd]c>$

Two decay channels: $\Xi_{cc}^+ \to \Lambda_c^+ K^- \pi^+, p D^+ K^-$

$\text{LHCb } (3621 \pm 1\ MeV) \quad J^P = \frac{1}{2}^- \text{ or } \frac{3}{2}^- \quad |(cu)c>$

$\Xi_{cc}^{++} \to \Lambda_c^+ K^- \pi^+ \pi^+$

**Groote, Koshkarev, sjb: SELEX& LHCb could both be correct!**

***Very different production kinematics:***

***LHCb (central region)***

***SELEX (Forward, High $x_F$) where $\Lambda_c$, $\Lambda_b$ were discovered***

NA3: Double J/ψ Hadroproduction measured at High $x_F$

Radiative Decay:

$\text{LHCb}(3621) \to \text{SELEX}(3520) + \gamma$

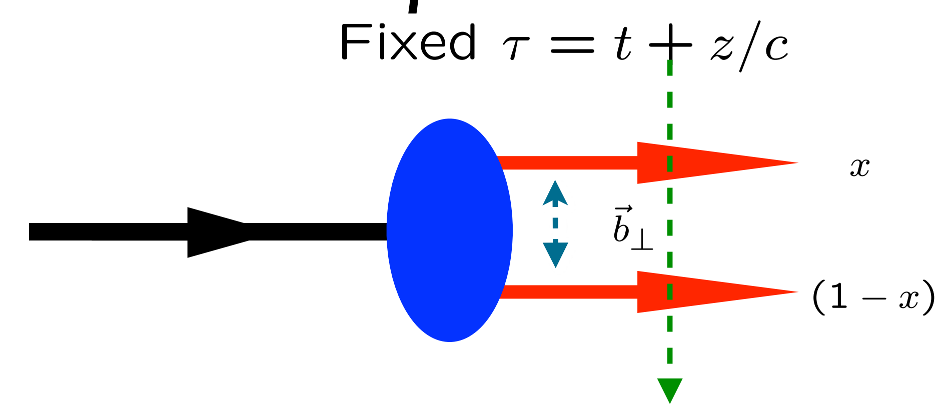
strongly suppressed: $[\frac{100\ MeV}{M_c}]^7$

**Also: Different diquark structure possible for LHCb:** $|(cc)u>$

Karliner and Rosner

# Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame*




- *Quantization at Fixed Light-Front Time* $\tau$
- *Causality: Information within causal horizon*
- *Light-Front Holography: AdS₅ = LF (3+1)*

$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b^2_\perp x(1-x)$$

$$\zeta^2 = x(1-x)b^2_\perp$$

- *Single fundamental hadronic mass scale κ: but retains the Conformal Invariance of the Action (dAFF)!*
- *Unique color-confining LF Potential!* $U(\zeta^2) = \kappa^4 \zeta^2$
- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

# Running Coupling from Modified AdS/QCD

**Deur, de Teramond, sjb**

- Consider five-dim gauge fields propagating in $AdS_5$ space in dilaton background $\varphi(z) = \kappa^2 z^2$

$$e^{\phi(z)} = e^{+\kappa^2 z^2} \qquad\qquad S = -\frac{1}{4}\int d^4x\,dz\,\sqrt{g}\,e^{\varphi(z)}\,\frac{1}{g_5^2}\,G^2$$

- Flow equation

$$\frac{1}{g_5^2(z)} = e^{\varphi(z)}\frac{1}{g_5^2(0)} \quad \text{or} \quad g_5^2(z) = e^{-\kappa^2 z^2} g_5^2(0)$$

where the coupling $g_5(z)$ incorporates the non-conformal dynamics of confinement

- YM coupling $\alpha_s(\zeta) = g_{YM}^2(\zeta)/4\pi$ is the five dim coupling up to a factor: $g_5(z) \to g_{YM}(\zeta)$
- Coupling measured at momentum scale $Q$

$$\alpha_s^{AdS}(Q) \sim \int_0^\infty \zeta d\zeta J_0(\zeta Q)\,\alpha_s^{AdS}(\zeta)$$

- Solution

$$\alpha_s^{AdS}(Q^2) = \alpha_s^{AdS}(0)\,e^{-Q^2/4\kappa^2}.$$

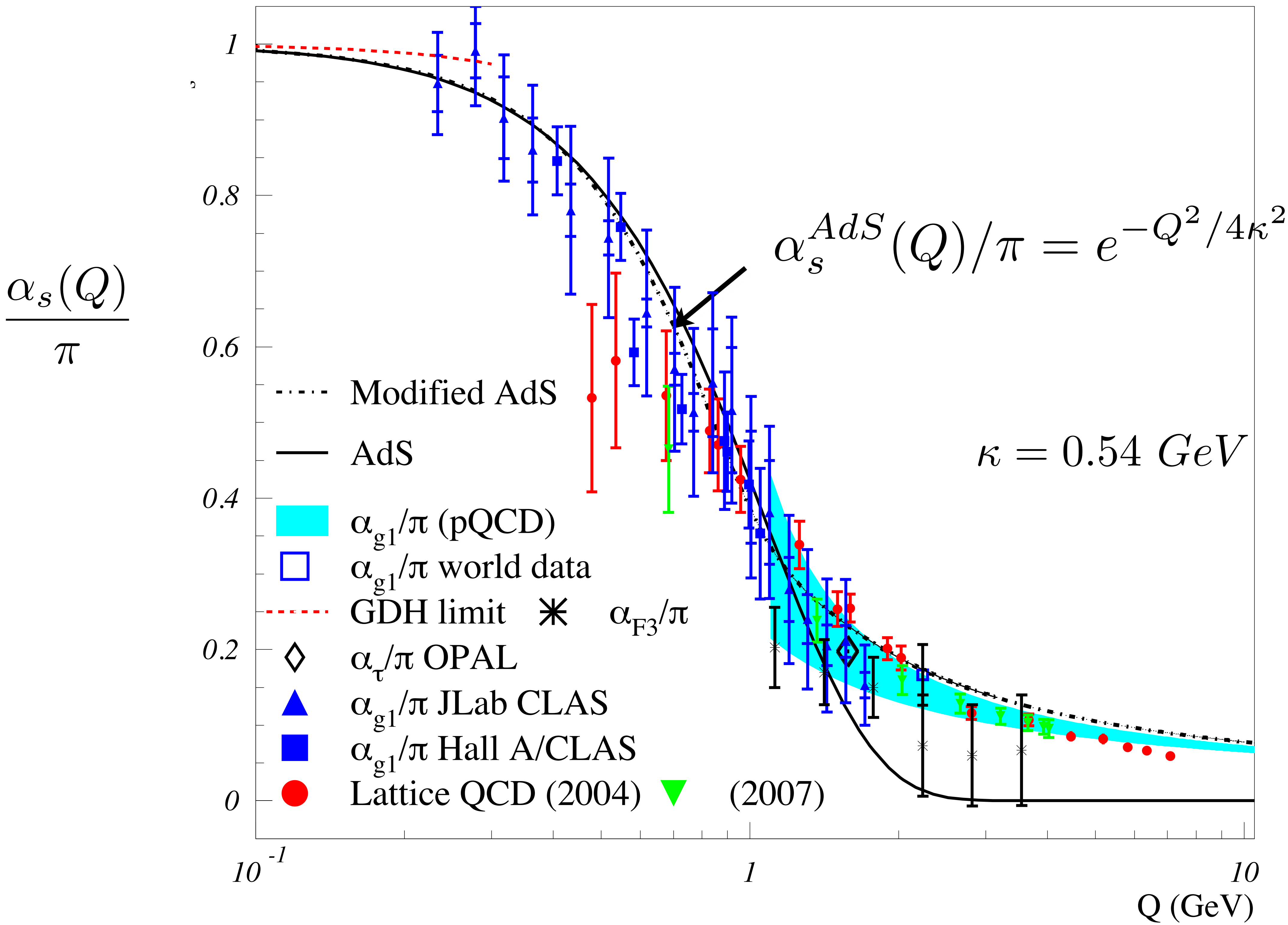
where the coupling $\alpha_s^{AdS}$ incorporates the non-conformal dynamics of confinement

## Bjorken sum rule defines effective charge $\alpha_{g1}(Q^2)$

$$\int_0^1 dx[g_1^{ep}(x,Q^2) - g_1^{en}(x,Q^2)] \equiv \frac{g_a}{6}[1 - \frac{\alpha_{g1}(Q^2)}{\pi}]$$

- ***Can be used as standard QCD coupling***
- ***Well measured***
- ***Asymptotic freedom at large $Q^2$***
- ***Computable at large $Q^2$ in any pQCD scheme***
- ***Universal $\beta_0$, $\beta_1$***

# Analytic, defined at all scales, IR Fixed Point




**AdS/QCD dilaton captures the higher twist corrections to effective charges for Q < 1 GeV**

$$e^{\varphi} = e^{+\kappa^2 z^2}$$

**Deur, de Teramond, sjb**

$m_\rho = \sqrt{2}\kappa$

$m_p = 2\kappa$

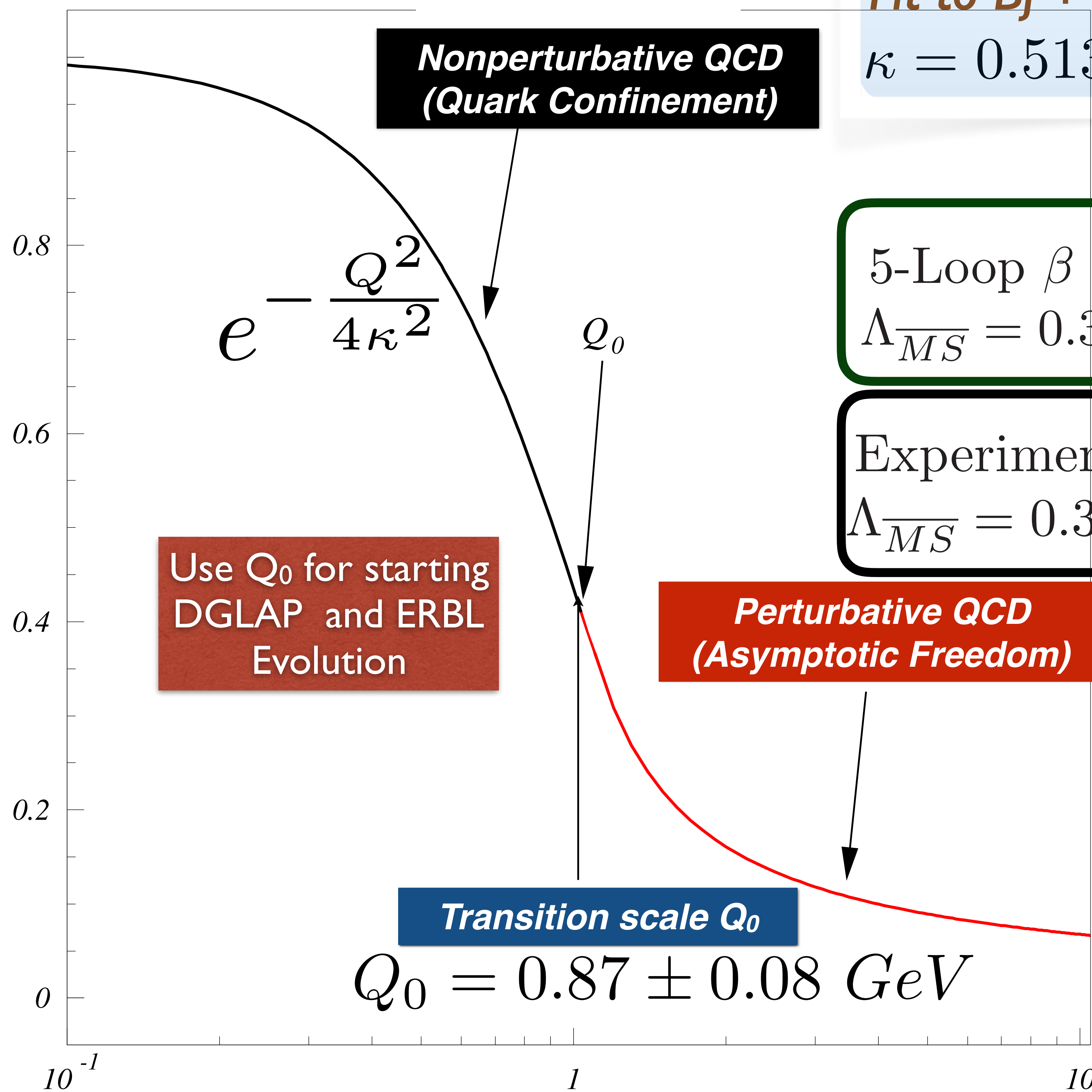
# All-Scale QCD Coupling

**Deur, de Tèramond, sjb**

*Fit to Bj + DHG Sum Rules:*

$\kappa = 0.513 \pm 0.007 \; GeV$

$$\frac{\alpha^s_{g_1}(Q^2)}{\pi}$$




5-Loop $\beta$ Prediction:

$\Lambda_{\overline{MS}} = 0.339 \pm 0.019 \; GeV$

Experiment:

$\Lambda_{\overline{MS}} = 0.332 \pm 0.017 \; GeV$

$\lambda \equiv \kappa^2$

$\overline{MS}$ *scheme*

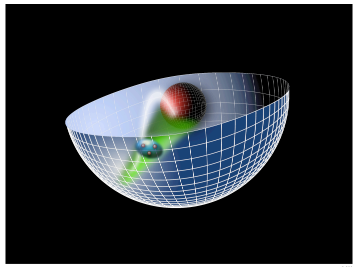
*Reverse Dimensional Transmutation!*

# Underlying Principles

- *Poincarè Invariance: Independent of the observer's Lorentz frame: Quantization at Fixed Light-Front Time τ*

- *Causality: Information within causal horizon: Light-Front*

- *Light-Front Holography: $AdS_5$ = LF (3+1)*



$$z \leftrightarrow \zeta \text{ where } \zeta^2 = b^2_\perp x(1-x)$$

- *Introduce Mass Scale κ while retaining the Conformal Invariance of the Action (dAFF)*

- *Unique Dilaton in $AdS_5$:* $e^{+\kappa^2 z^2}$

- *Unique color-confining LF Potential* $U(\zeta^2) = \kappa^4 \zeta^2$

- *Superconformal Algebra: Mass Degenerate 4-Plet:*

$$\text{Meson } q\bar{q} \leftrightarrow \text{Baryon } q[qq] \leftrightarrow \text{Tetraquark } [qq][\bar{q}\bar{q}]$$

# Features of LF Holographic QCD

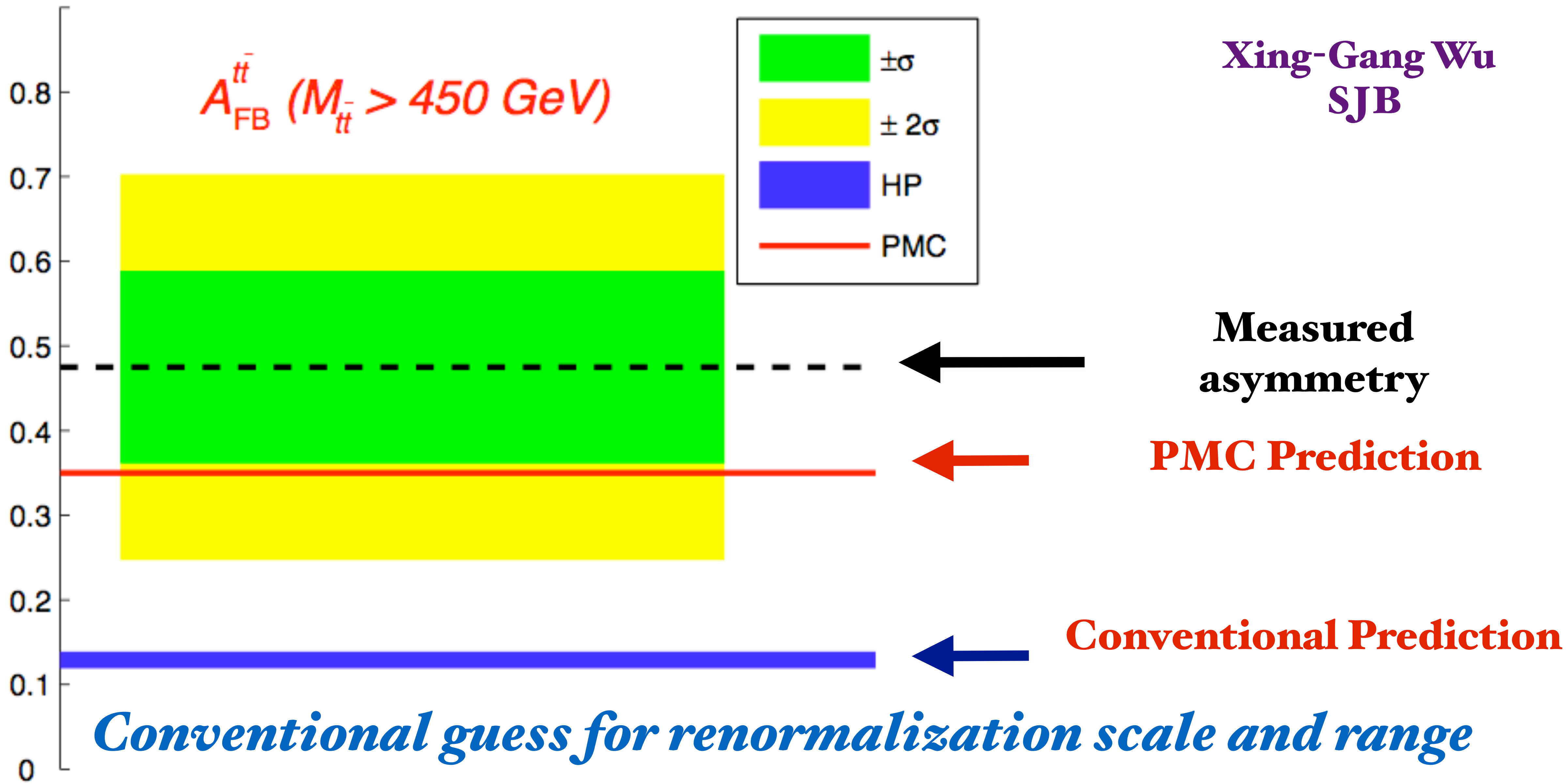
- **Color Confinement, Analytic form of confinement potential**
- **Massless pion bound state in chiral limit**
- **QCD coupling at all scales**
- **Connection of perturbative and nonperturbative mass scales**
- **Poincare' Invariant**
- **Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L**
- **Supersymmetric 4-Plet: Meson-Baryon Tetraquark Symmetry**
- **Light-Front Wavefunctions**
- **Form Factors, Structure Functions, Hadronic Observables**
- **OPE: Constituent Counting Rules**
- **Hadronization at the Amplitude Level**
- **Analytic First Approximation to QCD**
- **Systematically improvable: Basis LF Quantization (BLFQ)**

*Many phenomenological tests*

# Invariance Principles of Quantum Field Theory

- *Polncarè Invariance: Physical predictions must be independent of the observer's Lorentz frame: Front Form*
- *Causality: Information within causal horizon: Front Form*
- *Gauge Invariance: Physical predictions of gauge theories must be independent of the choice of gauge*
- *Scheme-Independence: Physical predictions of renormalizable theories must be independent of the choice of the renormalization scheme — Principle of Maximum Conformality (PMC)*
- *Mass-Scale Invariance: Conformal Invariance of the Action (DAFF)*

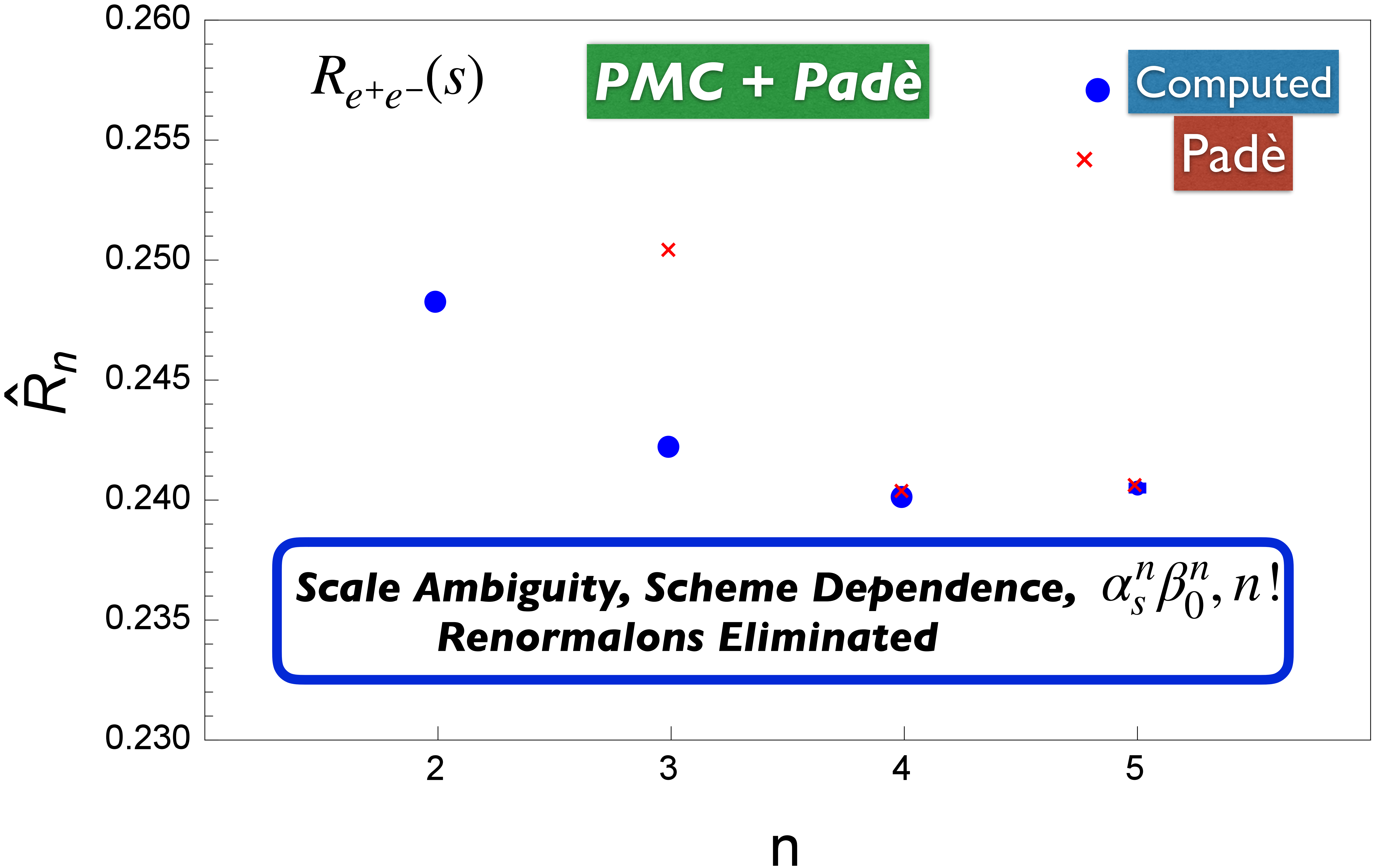
# The Renormalization Scale Ambiguity for Top-Pair Production Eliminated Using the 'Principle of Maximum Conformality' (PMC)




Xing-Gang Wu
SJB

BLM/PMC: Scheme-Independent, same as Gell-Mann-Low in pQED

Top quark forward-backward asymmetry predicted by pQCD NNLO within 1 σ of CDF/D0 measurements using PMC/BLM scale setting




*Extending the Predictive Power of pQCD*

B-L Dun, X-G. Wu, J.M, Shen, sjb

“One of the gravest puzzles of theoretical physics”

**DARK ENERGY AND THE COSMOLOGICAL CONSTANT PARADOX**

A. ZEE

*Department of Physics, University of California, Santa Barbara, CA 93106, USA*
*Kavil Institute for Theoretical Physics, University of California, Santa Barbara, CA 93106, USA*
*zee@kitp.ucsb.edu*

$$(\Omega_\Lambda)_{QCD} \sim 10^{45}$$

$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

***Extraordinary conflict between the conventional definition of the vacuum in quantum field theory and cosmology***

Elements of the solution:

(A) Light-Front Quantization: causal, frame-independent vacuum

(B) New understanding of QCD “Condensates”

(C) Higgs Light-Front Zero Mode

# Two Definitions of Vacuum State

**Instant Form: Lowest Energy Eigenstate of Instant-Form Hamiltonian**

$$H|\psi_0 >= E_0|\psi_0 >, E_0 = min\{E_i\}$$

***Eigenstate defined at one time t over all space; Acausal! Frame-Dependent***

**Front Form: Lowest Invariant Mass Eigenstate of Light-Front Hamiltonian**

$$H_{LF}|\psi_0 >_{LF} = M_0^2|\psi_0 >_{LF}, M_0^2 = 0.$$

***Frame-independent eigenstate at fixed LF time $\tau = t+z/c$ within causal horizon***

*Frame-independent description of the causal physical universe!*

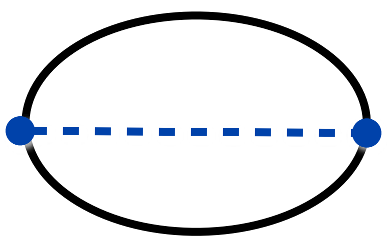
# *Front-Form Vacuum*

All LF propagators have positive $k^+$

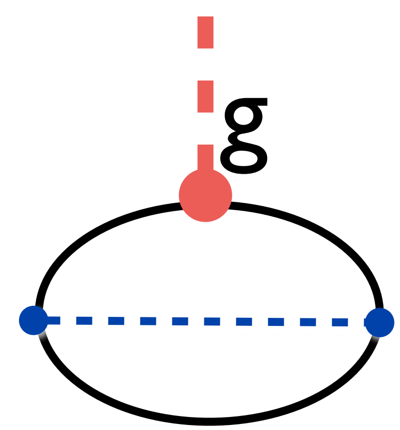
$$k^+ = k^0 + k^3 \geq 0 \text{ since } |\vec{k}| \leq k^0$$

$P^+$ Momentum Conserved

$P^+ = 0$



***zero !!***




***zero !!***

$$< 0|T^{\mu\nu}|0 >= 0$$

Graviton does not couple to LF vacuum!

*Vanishing gravitational coupling even in presence of zero modes*

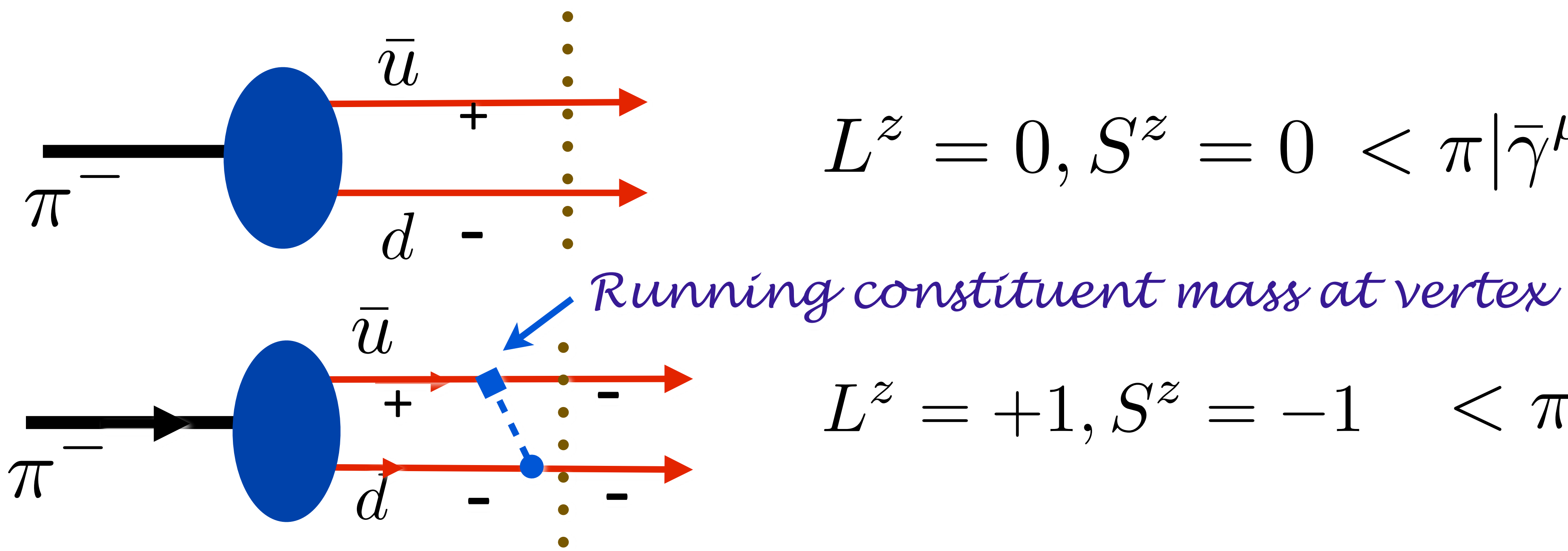
# Light-Front vacuum can simulate empty universe

Shrock, Tandy, Roberts, sjb

- **Independent of observer frame**
- **Causal**
- **Lowest invariant mass state M= 0.**
- **Trivial up to $k^+$=0 zero modes-- already normal-ordering**
- **Higgs theory consistent with trivial LF vacuum (Srivastava, sjb)**
- **QCD and AdS/QCD: "In-hadron"condensates (Maris, Tandy Roberts) -- GMOR satisfied.**
- **QED vacuum; no loops**
- **Zero cosmological constant from QED, QCD, EW**

# Light-Front Pion Valence Wavefunctions

$S^z_{\bar u} + S^z_d = +1/2 - 1/2 = 0$




**Couples to**

$$L^z = 0, S^z = 0 \;\; < \pi|\bar\gamma^\mu q \gamma_5 q|0> \;\sim f_\pi$$

**Couples to**

$$L^z = +1, S^z = -1 \quad < \pi|\bar q \gamma_5 q|0> \;\sim \rho_\pi$$

$S^z_{\bar u} + S^z_d = -1/2 - 1/2 = -1$

***Angular Momentum Conservation***

$$J^z = \sum_i^n S_i^z + \sum_i^{n-1} L_i^z$$

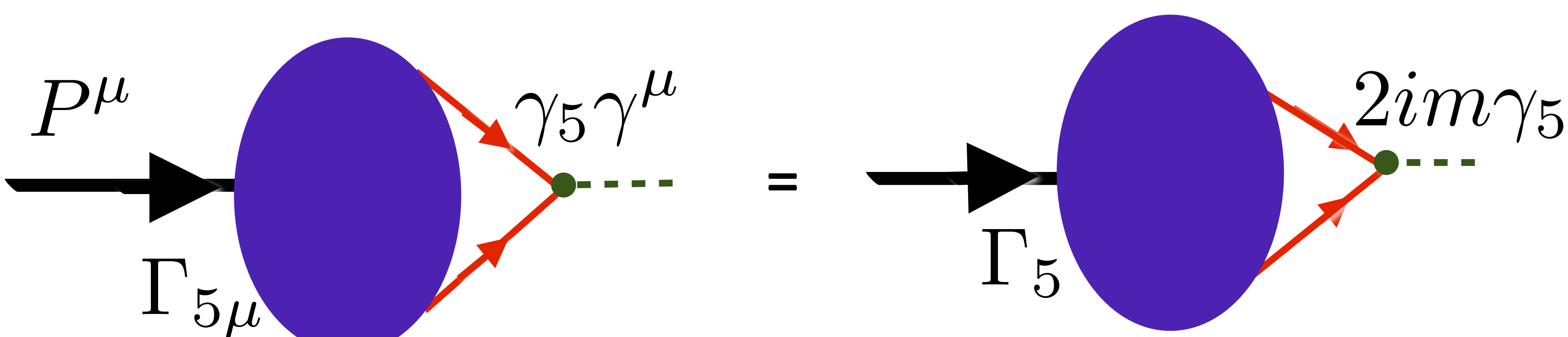
Maris, Roberts, Shrock, Tandy, sjb

# Ward-Takahashi Identity for axial current

**GMOR satisfied, no VEV**

$$P^{\mu}\Gamma_{5\mu}(k,P) + 2im\Gamma_5(k,P) = S^{-1}(k+P/2)i\gamma_5 + i\gamma_5 S^{-1}(k-P/2)$$

$$S^{-1}(\ell) = i\gamma\cdot\ell A(\ell^2) + B(\ell^2) \qquad m(\ell^2) = \frac{B(\ell^2)}{A(\ell^2)}$$




plus non-pole

Identify pion pole at $P^2 = m_\pi^2$

$$P^{\mu} < 0|\bar{q}\gamma_5\gamma^{\mu}q|\pi >= 2m < 0|\bar{q}i\gamma_5 q|\pi >$$

$$f_\pi m_\pi^2 = -(m_u+m_d)\rho_\pi$$

# Revised Gell Mann-Oakes-Renner Formula in QCD

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi^2} < 0|\bar{q}q|0 >$$

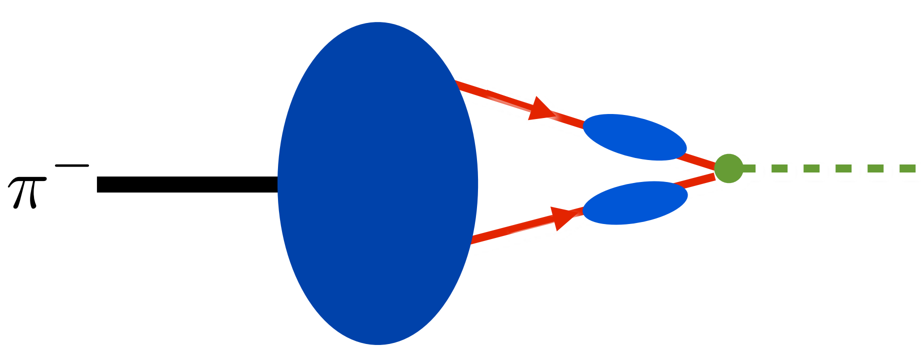
**current algebra: effective pion field**

$$m_\pi^2 = -\frac{(m_u + m_d)}{f_\pi} < 0|i\bar{q}\gamma_5 q|\pi >$$

**QCD: composite pion Bethe-Salpeter Eq.**

No VEV!

*vacuum condensate actually is an "in-hadron condensate"*



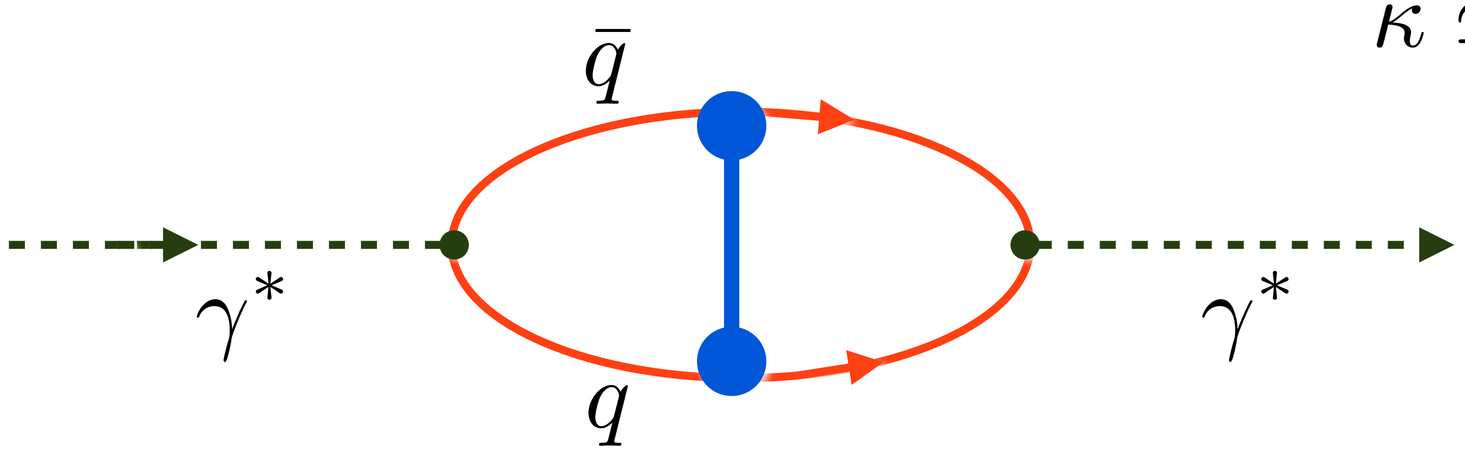

$$< 0|\bar{q}\gamma_5 q|\pi >$$

Maris, Roberts, Tandy

*Effective Confinement potential from soft-wall AdS/QCD gives Regge Spectroscopy plus higher-twist correction to current propagator*

$$M^2 = 4\kappa^2(n + L + S/2)$$ light-quark meson spectra

$\kappa \simeq 0.5$ GeV




$$R_{e^+e^-}(s) = N_c \sum_q e_q^2 (1 + \mathcal{O}\frac{\kappa^4}{s^2} + \cdots)$$

*mimics dimension-4 gluon condensate* $< 0|\frac{\alpha_s}{\pi} G^{\mu\nu}(0) G_{\mu\nu}(0)|0 >$ *in*

$e^+e^- \to X$, $\tau$ decay, $Q\bar{Q}$ phenomenology

PHYSICAL REVIEW C **82**, 022201(R) (2010)

# New perspectives on the quark condensate

Stanley J. Brodsky,[1,2] Craig D. Roberts,[3,4] Robert Shrock,[5] and Peter C. Tandy[6]
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[2]*Centre for Particle Physics Phenomenology: CP$^3$-Origins, University of Southern Denmark, Odense 5230 M, Denmark*
[3]*Physics Division, Argonne National Laboratory, Argonne, Illinois 60439, USA*
[4]*Department of Physics, Peking University, Beijing 100871, China*
[5]*C.N. Yang Institute for Theoretical Physics, Stony Brook University, Stony Brook, New York 11794, USA*
[6]*Center for Nuclear Research, Department of Physics, Kent State University, Kent, Ohio 44242, USA*
(Received 25 May 2010; published 18 August 2010)

We show that the chiral-limit vacuum quark condensate is qualitatively equivalent to the pseudoscalar meson leptonic decay constant in the sense that they are both obtained as the chiral-limit value of well-defined gauge-invariant hadron-to-vacuum transition amplitudes that possess a spectral representation in terms of the current-quark mass. Thus, whereas it might sometimes be convenient to imagine otherwise, neither is essentially a constant mass-scale that fills all spacetime. This means, in particular, that the quark condensate can be understood as a property of hadrons themselves, which is expressed, for example, in their Bethe-Salpeter or light-front wave functions.

# *Quark and Gluon condensates reside within hadrons, not vacuum*

**Casher and Susskind** **Maris, Roberts, Tandy** **Shrock and sjb**

- **Bound-State Dyson Schwinger Equations**
- **AdS/QCD**
- **Implications for cosmological constant -- Eliminates 45 orders of magnitude conflict**

“One of the gravest puzzles of theoretical physics”

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$$(\Omega_\Lambda)_{EW} \sim 10^{56}$$

$$\Omega_\Lambda = 0.76(expt)$$

$$(\Omega_\Lambda)_{QCD} = 0 \qquad (\Omega_\Lambda)_{EW} = 0$$

Central Question: What is the source of Dark Energy?

$$\Omega_\Lambda = 0.76(expt)$$

Higgs Zero-Mode Curvature?

# Advantages of the Dirac's Front Form for Hadron Physics
## Poincare' Invariant

## *Physics Independent of Observer's Motion*

- Measurements are made at fixed τ
- Causality is automatic
- Structure Functions are squares of LFWFs
- Form Factors are overlap of LFWFs
- LFWFs are frame-independent: no boosts, no pancakes!

*Penrose, Terrell, Weisskopf*

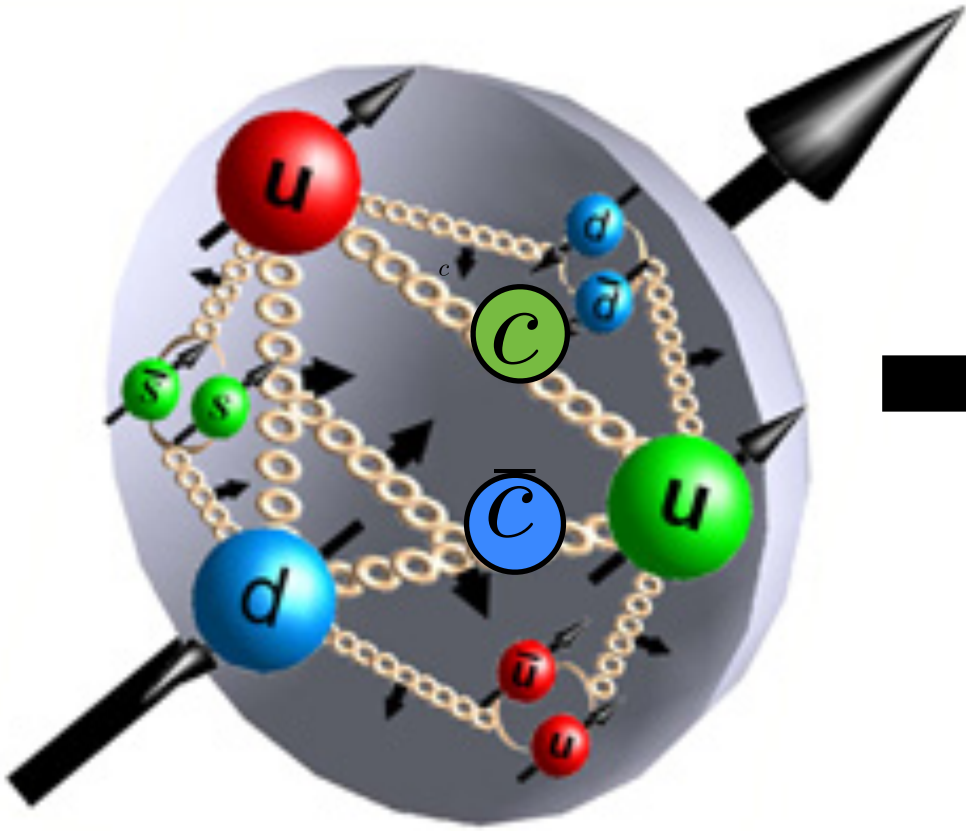
- Same structure function measured at an e p collider and the proton rest frame
- No dependence of hadron structure on observer's frame
- Jz Conservation, bounds on ΔLz *Chiu, sjb*
- LF Holography: Dual to AdS space
- LF Vacuum trivial -- no vacuum condensates!

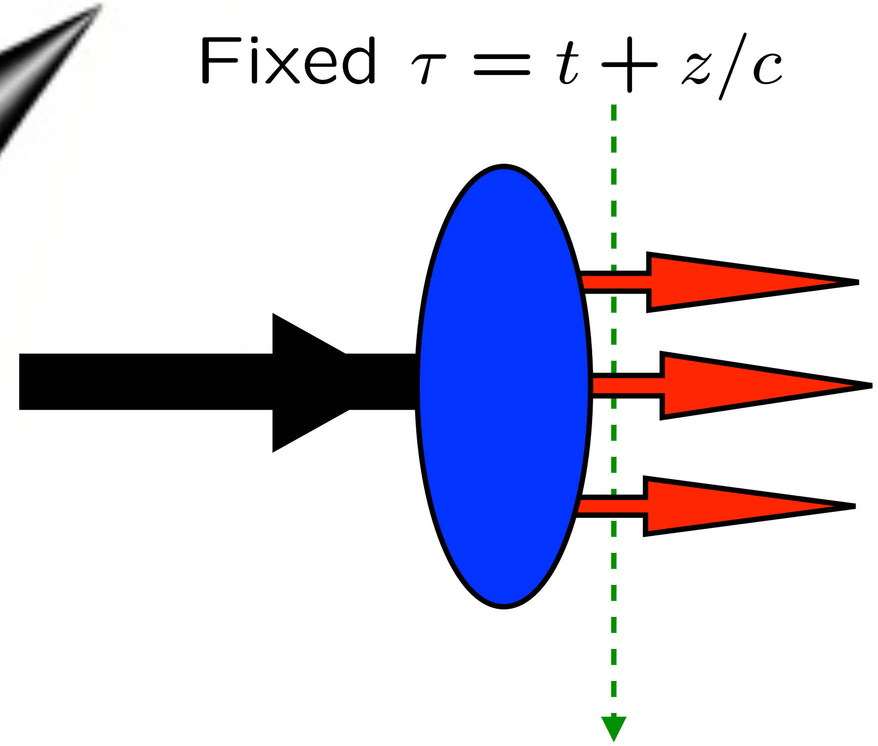
*Roberts, Shrock, Tandy, sjb*

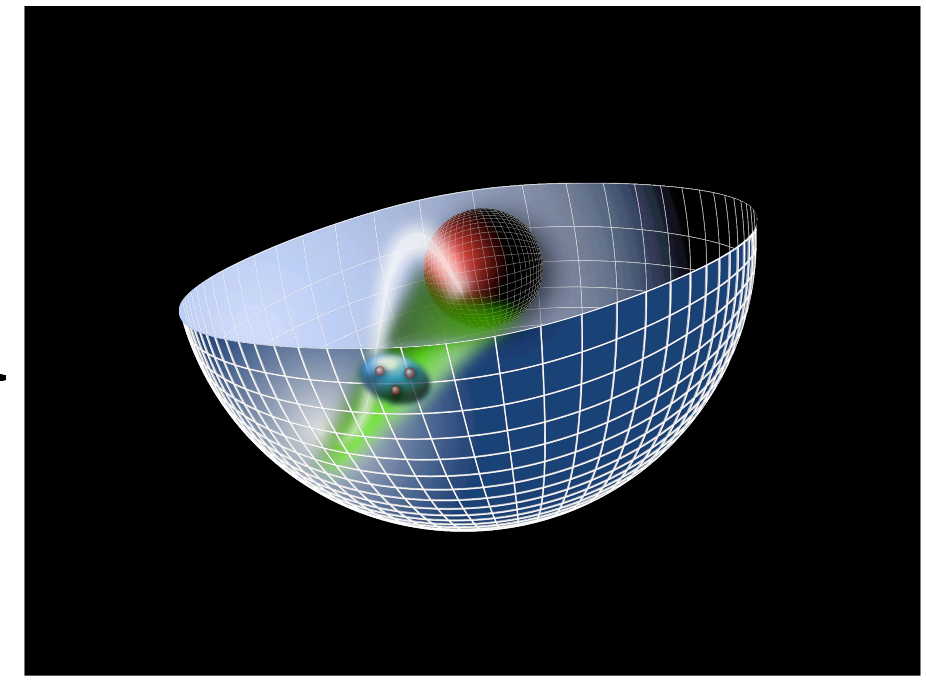
# Light-Front Holography: First Approximation to QCD

- Color Confinement, Analytic form of confinement potential
- Retains underlying conformal properties of QCD despite mass scale (DeAlfaro-Fubini-Furlan Principle)
- Massless quark-antiquark pion bound state in chiral limit, GMOR
- QCD coupling at all scales
- Connection of perturbative and nonperturbative mass scales
- Poincarè Invariant
- Hadron Spectroscopy-Regge Trajectories with universal slopes in n, L
- Supersymmetric 4-Plet: Meson-Baryon -Tetraquark Symmetry
- Light-Front Wavefunctions
- Form Factors, Structure Functions, Hadronic Observables
- OPE: Constituent Counting Rules
- Hadronization at the Amplitude Level: Many Phenomenological Tests
- Systematically improvable: Basis LF Quantization (BLFQ)

# *Emergent Hadron Mass from Light-Front Holography and Superconformal Quantum Mechanics*

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***with Guy de Tèramond, Hans Günter Dosch, C. Lorce, K. Chiu, R. S. Sufian, A.Deur, C. Roberts***

## *Emergent Mass and its Consequences in the Standard Model*