3D quantum gravity on the edge

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I
Context and
Classical Theory

Question

Understand holography from the perspective of

non-perturbative models of Quantum Gravity

in models that

- are mathematically under control
- provide a clear picture of quantum geometry
- have exact realization of a quantum version of *diffeo* symmetry

In particular, clarify origin and dynamics of quantum boundary dofs

Set-up

To begin with, we consider

Euclidean 3d Quantum Gravity with $\Lambda = 0$

With focus on

- extended boundaries at a *finite distance*
- with **Gibbons-Hawking-York** (GHY) type *boundary conditions* (i.e. fixed induced metric)

Results

3d gravity is topological: **boundary theory from 'would-be-gauge' dofs**

(1)

nonperturbative boundary theories in terms of spin-chains;
in this context we 'rediscover' dualities in statistical mechanics
and integrable systems from a natural gravitational perspective

For appropriate **semiclassical boundary conditions**,

(2)

we recover results of perturbative gravity (*1-loop partition function*)
and *reconstruct bulk geometry* from boundary sigma model

3D Gravity [Λ = 0]

Action principle /1

Einstein-Hilbert

$$S[g_{\mu\nu}] = \frac{1}{2\ell_{\rm Pl}} \int \sqrt{g} R + \frac{1}{\ell_{\rm Pl}} \oint \sqrt{h} K$$

Local symmetries (diffeos)

$$\xi \in \mathrm{Vect}(M): \qquad \delta_\xi g_{\mu\nu} = \pounds_\xi g_{\mu\nu}$$

E.o.m. $\delta S = 0 \;\Leftrightarrow\; R_{\mu\nu} = 0 \;\Leftrightarrow\; R^\rho{}_{\mu\sigma\nu} = 0$ **No local dofs**

$$g_{\mu\nu} = \eta_{ab} e^a_\mu e^b_\nu$$

$$\omega^a_\mu = \left(\partial_\mu e^b_\nu - \Gamma^\rho_{\mu\nu} e^b_\rho\right)(e^{-1})^\nu_c \epsilon_b{}^{ca}$$

3D Gravity [Λ = 0]

Action principle /2

Einstein-Cartan (1st order)

conjugate variables

$$S[e^a_\mu, \omega^a_\mu] = \frac{1}{2\ell_{\rm Pl}} \int \mathrm{Tr}\Big(e \wedge F[\omega]\Big)$$

Local symmetries (diffeos + **Lorentz** $\mathrm{SU}(2)$ + **shift** $\mathrm{su}(2)_+$)

$$\begin{cases} \xi \in \mathrm{Vect}(M): & \delta_\xi e^a_\mu = \pounds_\xi e^a_\mu \quad \text{and} \quad \delta_\xi \omega^a_\mu = \pounds_\xi \omega^a_\mu \\ X \in \mathrm{su}(2): & \delta_X e = [e, X] \quad \text{and} \quad \delta_X \omega = \mathrm{D}_\omega X \\ \phi \in \mathbb{R}^3: & \delta_\phi e = \mathrm{D}_\omega \phi \quad \text{and} \quad \delta_\phi \omega = 0 \end{cases}$$

$\boxed{\text{No local dofs}}$

E.o.m. $\quad F[\omega] = 0 \quad \& \quad \mathrm{D}_\omega e = 0 \;\Leftrightarrow\; \omega = g^{-1}\mathrm{d}g \quad \& \quad e = g^{-1}\mathrm{d}\phi\, g$

$\mathrm{SU}(2)$ *Lorentz symmetry*

triad shift symmetry

$$X \in \mathrm{su}(2): \qquad \delta_X e = [e, X] \quad \text{and} \quad \delta_X \omega = \mathrm{D}_\omega X$$

3D Gravity [Λ = 0]

$$\phi \in \mathbb{R}^3: \qquad \delta_\phi e = \mathrm{D}_\omega \phi \quad \text{and} \quad \delta_\phi \omega = 0$$

Action principle /2

Einstein-Cartan (1st order) *conjugate variables*

$$S[e^a_\mu, \omega^a_\mu] = \frac{1}{2\ell_{\mathrm{Pl}}} \int \mathrm{Tr}\Big(e \wedge F[\omega]\Big)$$

Internal symmetries (**Lorentz** $\mathrm{SU}(2)$ + **shift** $\mathrm{su}(2)_+$)

organize into the *Poisson-Lie* group structure

$$D \cong \mathrm{SU}(2) \ltimes \mathrm{su}(2) \cong \mathrm{T}^*\mathrm{SU}(2)$$

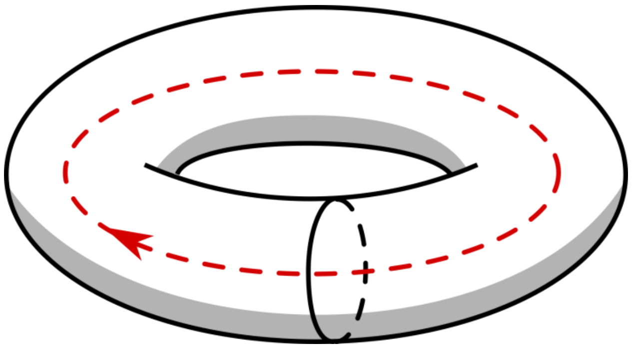
conjugate symmetries

[Admits more subtle generalizations to Λ ≠ 0]

3D Gravity: bulk topological dofs

Topological dofs

However, non-trivial topology unfreezes global dofs



E.g. holonomies around non-trivial cycles in the bulk

3D Gravity: boundary dofs

Boundary dofs

Bdry cond. generically break bulk gauge symmetries and diffeo invariance

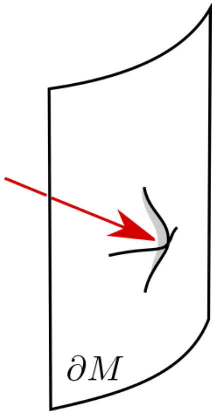
new dofs to restore these symmetries

these are bdry '*would-be-gauge*' dofs (prototypical ex is WZW)

Here, shift symmetry explicitly broken,

$\rightsquigarrow$ GHY boundary conditions *break diffeomorphism invariance*

$\rightsquigarrow$ E.g. Carlip: « dual Liouville = 'would-be-gauge' dof from diffeo normal to boundary »



[Another perspective: bdry dofs are needed for covariantly gluing back regions along hypersurfaces]

[see e.g. Brawn & Henneaux 1986, Carlip 1995 + 2005, Balachandran Et Al 1996, Freidel & Donnelly 2016, Gomes & AR 2017]

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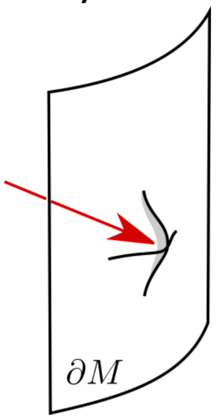


Here, depending on presentation,

either shift or Lorentz symmetry is explicitly broken

$\rightsquigarrow$ dual descriptions

$\rightsquigarrow$ E.g. shift reproduces Carlip's

« dual Liouville = 'would-be-gauge' dof from diffeo normal to bdry »

[Another perspective: bdry dofs are needed for covariantly gluing back regions along hypersurfaces]

[see e.g. Brawn & Henneaux 1986, Carlip 1995 + 2005, Balachandran Et Al 1996, Freidel & Donnelly 2016, Gomes & AR 2017]

II
Non-perturbative
3D quantum gravity

Roadmap

1. Discretization and smeared variables

2. Action of (discrete) symmetries

3. Amplitude kernel (for 3-ball and solid-torus topologies)

4. Quantum amplitude for arbitrary quantum bdry conditions (“bdry state”)

5. Bdry state encoding quantum GHY bdry conditions (fixed bdry metric)

→ Part III: Holography -- map 3d QG amplitude on a dual 2d model

1. Discretization and smeared variables

3d QG can be exactly quantized in non-perturbative covariant fashion:

Ponzano-Regge-Turaev-Viro model [Euclidean, $\Lambda = 0$]

[equivalent to CS combinatorial quantiz.; cf. Kitaev models for topological phases of 2+1d matter]

Model based on a *discretization*,

thanks to topological invariance the discretization is *inconsequential*

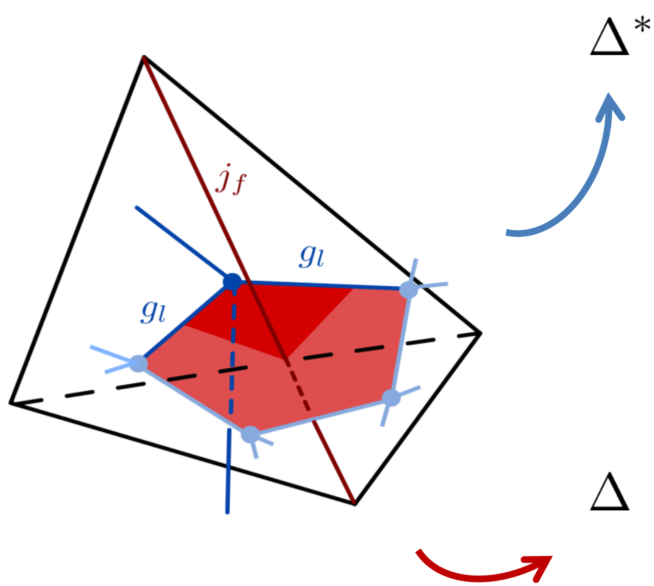
– *in the bulk*

PR-TV MODEL – a very partial list of references:
Ponzano & Regge 1968, Turaev & Viro 1992, Perez & Noui 2003, Freidel & Louapre 2004,
Barrett & Naish-Guzmann 2009, Meusburger & Noui 2010, Bonzom & Smerlak 2011-12

2. Symmetries

Ponzano-Regge-Turaev-Viro model [Euclidean, Λ = 0]

Discretization



Connection variables

$$\omega \rightsquigarrow g_{\ell^*} = P\exp\int_{\ell^*}\omega$$

Lorentz symmetry

$$g_\ell \mapsto h_t g_\ell h_s^{-1}$$

Metric variables

$$||e_\mu \mathrm{d}x^\mu|| \rightsquigarrow L_\ell = \ell_{\mathrm{Pl}}\sqrt{j_\ell(j_\ell+1)}$$

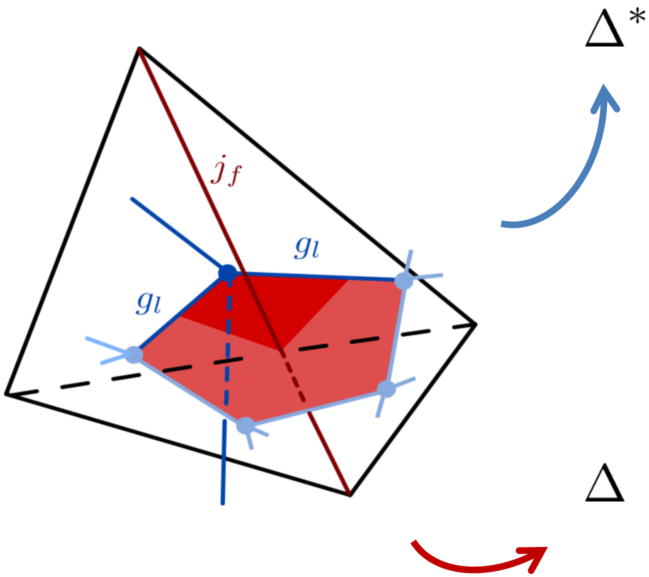
Shift symmetry

$$j_\ell \mapsto j_\ell + \tfrac{1}{2}\epsilon_{v,\ell}$$

2. Symmetries

Ponzano-Regge-Turaev-Viro model [Euclidean, Λ = 0]

Discretization



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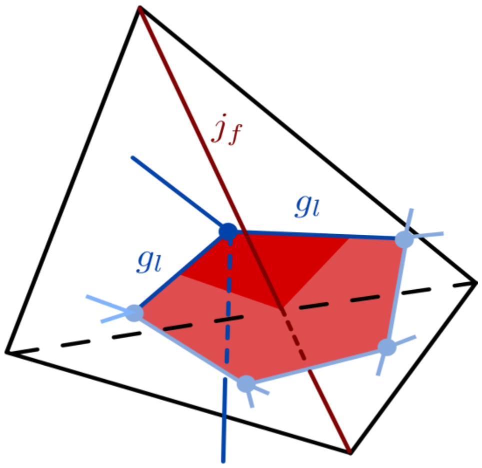
$$||e_\mu \mathrm{d}x^\mu|| \rightsquigarrow L_\ell = \ell_{\rm Pl}\sqrt{j_\ell(j_\ell+1)}$$

'Active diffeo' = vertex displacement

$$j_\ell \mapsto j_\ell + \tfrac{1}{2}\epsilon_{v,\ell}$$

Length: discrete spectrum

3. Amplitude kernel



Closed manifold

Einstein-Cartan QG gravity

$$Z_\omega = \int \mathcal{D}[e]\mathcal{D}[\omega] e^{\frac{i}{2\ell_{\mathrm{Pl}}}\int \mathrm{Tr}(e\wedge F[\omega])} \rightsquigarrow Z_\omega = \int \mathcal{D}[\omega]\delta(F[\omega])$$

Introduce a discretization of the manifold completely capturing its topology

Δ^*

$$Z_\omega = \Big[\prod_l \int \mathrm{d}g_l\Big] \prod_f \delta\Big(\overleftarrow{\prod_{l\in\partial f}} g_l \Big)$$

A theory of Flat connections

To switch to “metric” variables, use Peter-Weyl thm + SU(2) recoupling theory

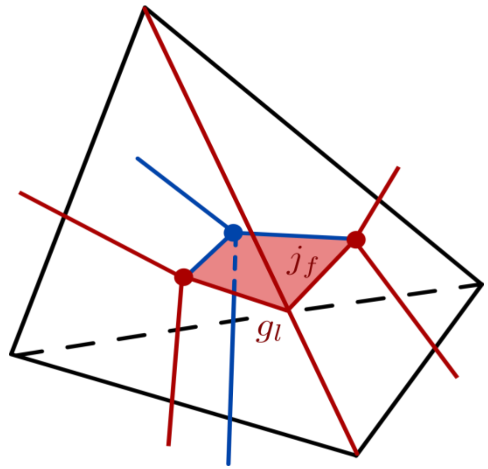
Δ

$$Z_e = \sum_{\{j_f\}} \prod_f (-1)^{2j}(2j+1) \prod_l (-1)^{j_1+j_2+j_3} \prod_v \{6j\}$$

The *j*’s are eigenvalues of length operator

For large spin, it reproduces discrete Einstein-Hilbert

4. Quantum amplitude



Manifold with boundaries

Start by fixing values of a variable on the boundary, e.g.

$$Z_\omega(g_{l_\partial}) = \Big[\prod_{l \neq l_\partial} \int \mathrm{d}g_l\Big] \prod_f \delta\Big(\overleftarrow{\prod_{l \in \partial f}} g_l\Big)$$

(Choice of **connection-variable polarization**)

$\rightsquigarrow$ basis for amplitudes of arbitrary superpositions of $\{g_{l_\partial}\}$ weighted by Ψ :

$$\langle Z|\Psi\rangle = \Big[\prod_{l_\partial} \int \mathrm{d}g_{l_\partial}\Big] \overline{Z_\omega(g_{l_\partial})}\Psi(g_{l_\partial})$$

Note: Ψ is called a *boundary state*,

it can impose *any* (quantum) boundary conditions, even metric ones

4. Quantum amplitude

Three ball

No non-trivial holonomies,

flatness + gauge invariance $\Rightarrow$ all $g_l = \mathbb{1}$

$$\langle Z|\Psi\rangle = \Psi(g_l = \mathbb{1})$$

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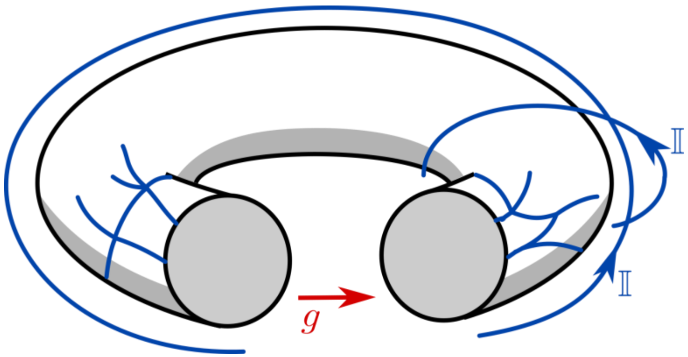

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Solid torus

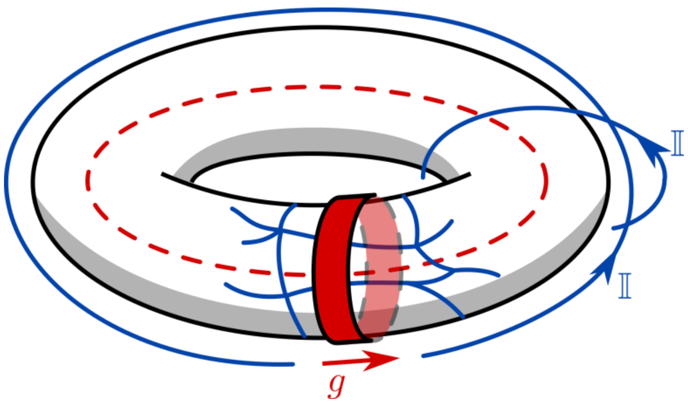
One residual holonomy,

flatness + gauge invariance $\Rightarrow$ all $g_l = \mathbb{1}$, except across a “dual” ring

$$\langle Z|\Psi\rangle = \int \mathrm{d}g \; \Psi(g_l = \mathbb{1}, g_{l\in\mathrm{ring}} = g)$$

finite expressions

4. Quantum amplitude



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finite expressions

5. Boundary state for Gibbons-Hawking-York boundary conditions

GHY – i.e. metric – boundary conditions

(Gauge-)invariant tensors
at vertices of the boundary graph:
intertwiners

We need to keep the spins j_∂ fix

Hence $$\Psi_{j,\iota}(g_l) = \left[\prod_l D^j(g_l)^m{}_{m'}\right] \bullet \left[\prod_v \iota^{m'_1\cdots}{}_{m_r\cdots}\right]$$

Wigner matrices
irrep of spin *j*
magnetic numbers *m, m'*
[non-Abelian analogues of e^{ipx}]

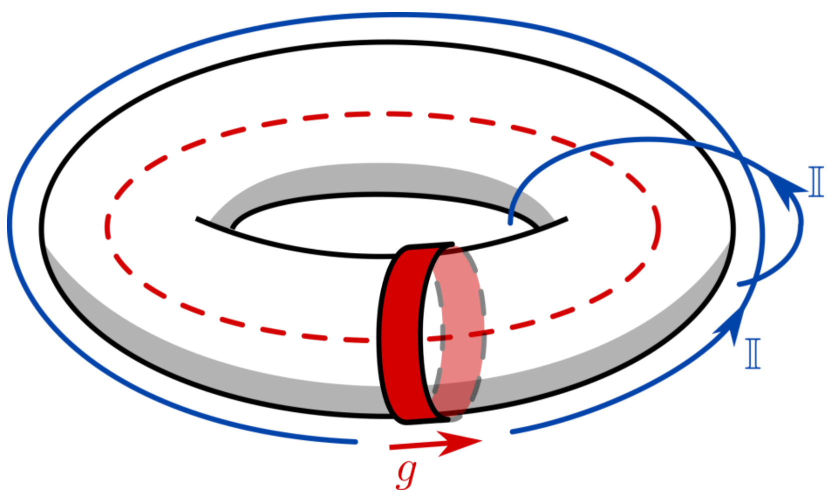
Intertwiners are unique for 3-valent graphs (triangulations): Clebsch-Gordan

5. Boundary state for Gibbons-Hawking-York boundary conditions

Amplitude (for the torus)

$$\langle Z|\Psi\rangle = \int \mathrm{d}g \left[\prod_{l\in\mathrm{ring}} D^j(g)^m{}_{m'} \prod_{l\notin\mathrm{ring}} \delta^m{}_{m'} \right] \bullet \left[\prod_v \iota^{m'_1\cdots}{}_{m_r\cdots} \right]$$

Residual effect of
non-trivial topological cycles



III

Holography

--

map 3d QG amplitude to dual 2d moels

Boundary state for GHY bdry conditions

Amplitude

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Residual effect of
a non-trivial topological cycle
(solid torus)

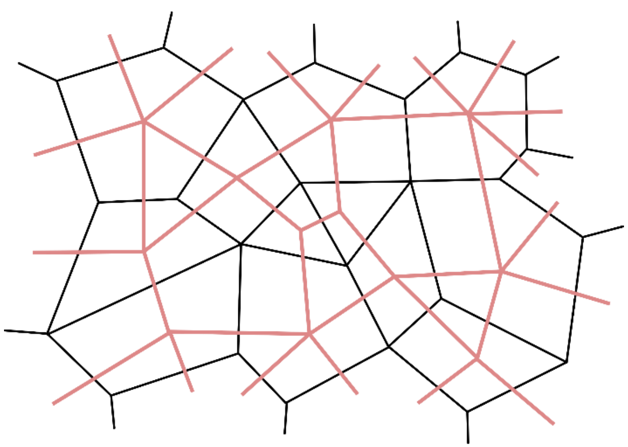
Boundary state for GHY bdry conditions

Amplitude

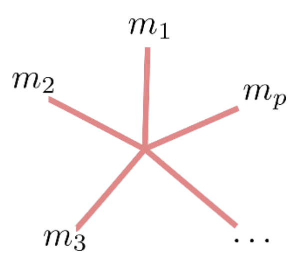
$$\langle Z|\Psi\rangle = \cancel{\int \mathrm{d}g \left[\prod_{l\in\mathrm{ring}} D^j(g)^m{}_{m'}\right.} \prod_{l\notin\mathrm{ring}} \delta^m{}_{m'}\Bigg] \bullet \left[\prod_v \iota^{m'_1\dots}{}_{m_r\dots}\right]$$

Forget for the moment

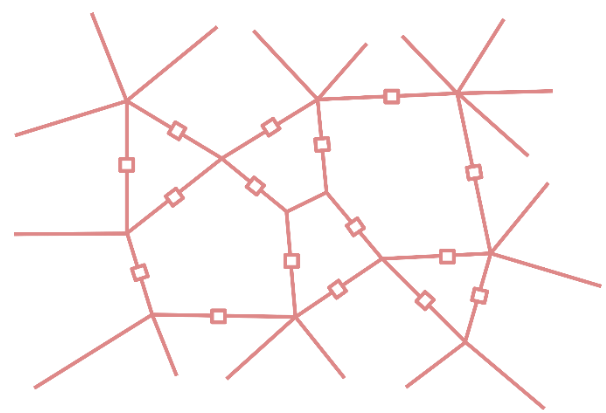
Magnetic indices
are *edge variables*



discretized boundary
and its dual



$\iota^{m_1\dots}{}_{m_r\dots}$



box = sum over
magnetic indices

Boundary state for GHY bdry conditions
- statistical model representation

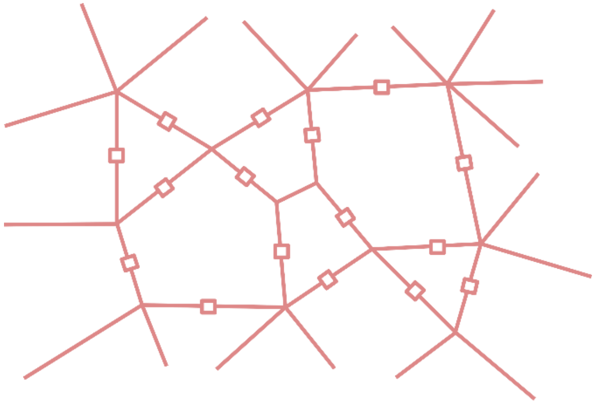
Amplitude

$$\langle Z|\Psi\rangle = \sum_{\{m_l\}} \iota_v(m_{l\supset v}\dots)$$

This is a peculiar **statistical model** where

$\{m_i,\dots\} \rightsquigarrow$ *magnetic indices = dofs at edges*

$\iota^{m_1\cdots}{}_{m_r\dots} \rightsquigarrow$ *(complex) Boltzmann weights of a vertex model* $[R^{m_1\cdots}_{m_r\dots}]$



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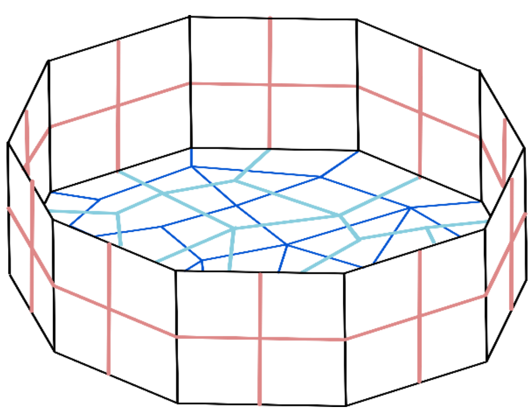

Transfer matrix picture

$$T_\iota : (V_j)^{\otimes N_x} \to (V_j)^{\otimes N_x}$$

$$T_\iota = \mathrm{Tr}_{\mathrm{horiz}}[\iota^{\otimes N_x}]$$

[spin-chain]

Boundary state for GHY bdry conditions
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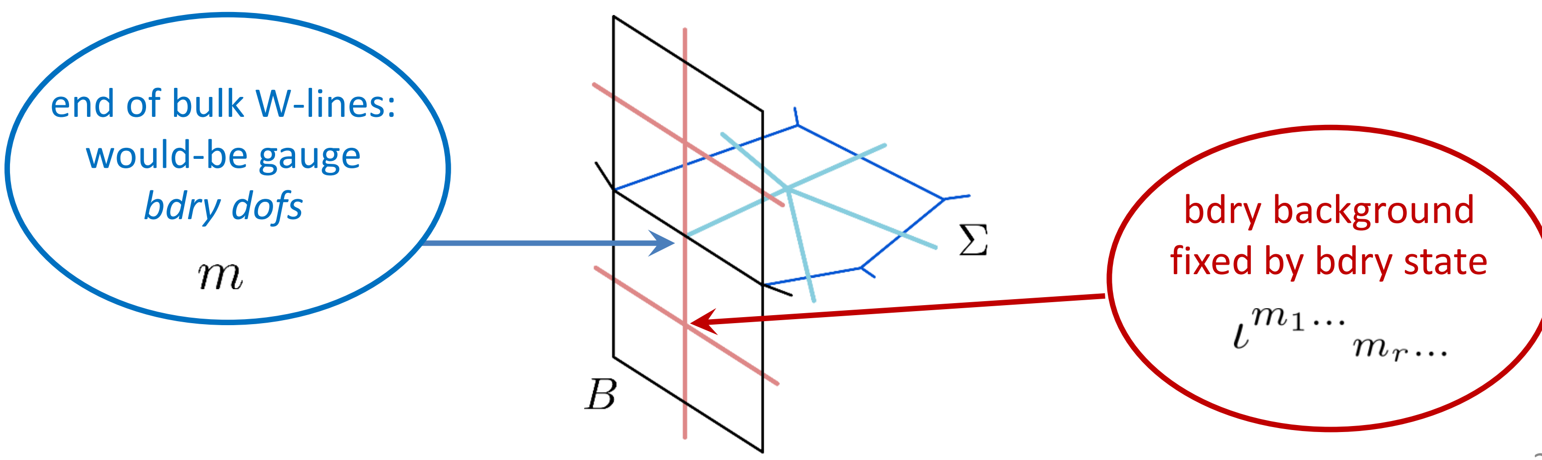


Boundary state for GHY bdry conditions - discrete sigma model representation

From magnetic indices to group elements

Using a different basis of intertwiners (coherent LS intertwiners)…

…it is possible to trade sum over magnetic indices

for integrals over *SU(2) variables living at vertices*

$$S[G_v] = \sum_l 2j_l \ln\langle \xi^l_{v+l} | G^{-1}_{v+l} G_v | \xi^l_v \rangle$$

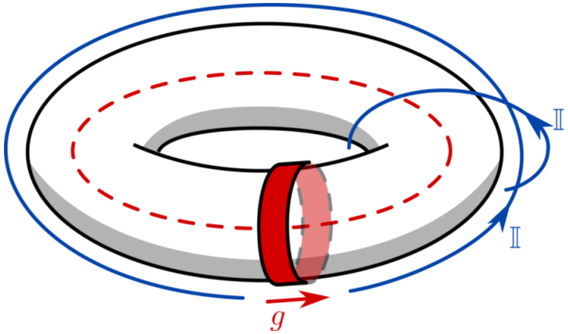
$\rightsquigarrow$ *Lorentz 'would-be-gauge' dofs*

over a background provided by the bdry state

(local Lorentz frame at boundary become dynamical)

This gives a formulation in terms of a sigma-model similar in spirit to WZW

Boundary state for GHY bdry conditions



Solid torus topology

$$\langle Z|\Psi\rangle = \int \mathrm{d}g \left[\prod_{l\in\text{ring}} D^j(g)^m{}_{m'} \prod_{l\notin\text{ring}} \delta^m{}_{m'} \right] \bullet \left[\prod_v \iota^{m'_1\cdots}{}_{m_r\cdots} \right]$$

In the spin chain it corresponds to the insertion of a

projector on the *total* spin zero sector

Transfer matrix representation: $\langle Z|\Psi\rangle = \mathrm{Tr}[T_\iota^{\otimes N_t} P_{S=0} U_{N_\gamma}]$

RMK

if all bdry spins are j = ½, then the bdry model is the XXX spin-chain (integrable)

Boundary state for GHY bdry conditions - duality with RSOS model

Question*:*

what does the model look like in the spin/metric representation ?

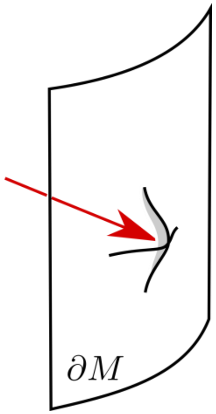
Idea, instead of solving for face delta-functions,

‘Fourier’ transform them and use SU(2) recoupling

Result: **IRF model of the RSOS** type modeling surface growth

dofs are given by *spin of edges normal to boundary*

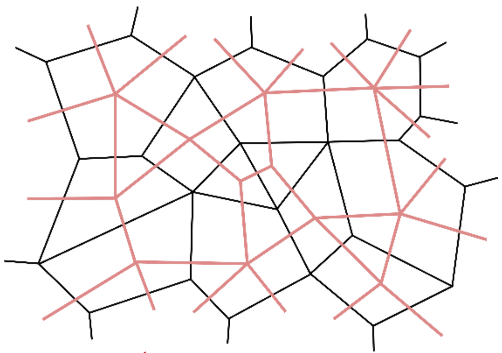
→ quantum version of Carlip’s ‘would-be-normal-diffeos’



For 3-valent boundary graphs: PR model on minimal bulk triangulation (one internal vertex)

[Pasquier 1990, Witten 1990]

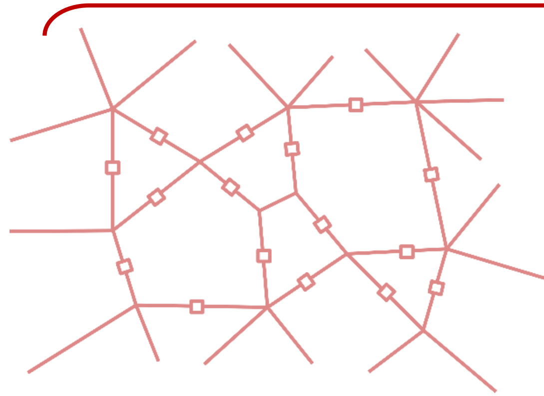
Duality structure



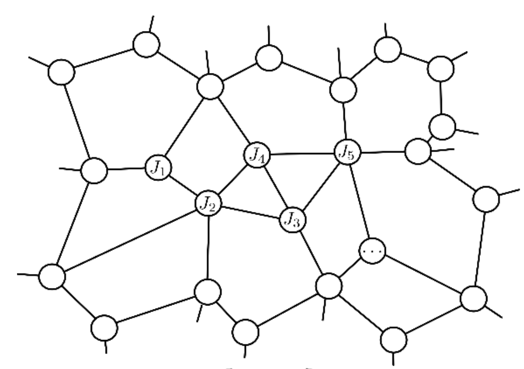
Poisson-Lie
symmetry

$$(\Delta^*, \Delta) \leftrightarrow (\omega, e) \leftrightarrow \mathrm{SU}(2) \ltimes \mathrm{su}(2) \leftrightarrow \text{2d (vertex, face) stat model}$$

canonical
phase space



$\{m_l\} \leftrightarrow \{G_v\}$



$\{J_\perp\}$

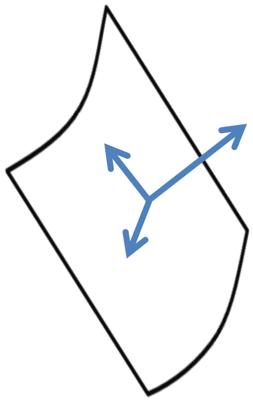
Geometrical meaning of the boundary variables

Embedding on the boundary into the bulk

fixed by…

$$G_v \leftrightarrow \mathrm{SU}(2) \leftrightarrow \text{2d (discrete) sigma model}$$

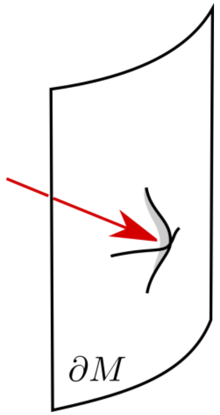
…local orientation on the boundary into the bulk



or

$$J_\perp \leftrightarrow \mathrm{su}(2) \cong \mathbb{R}^3 \leftrightarrow \text{2d RSOS model}$$

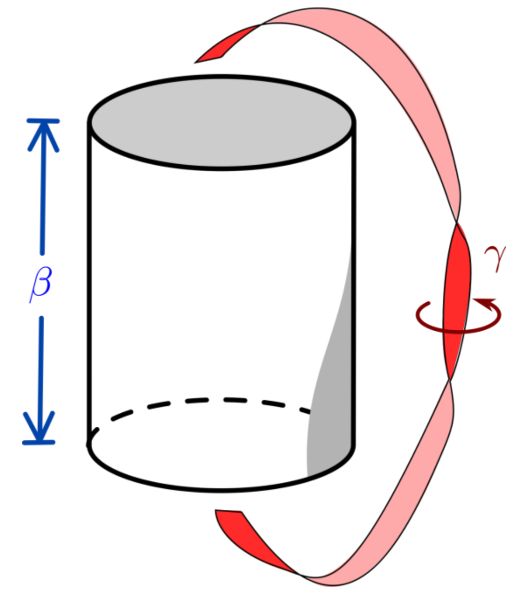
…distance from the “center” of the spacetime



These are the variables “summed” over in the amplitude:
extrinsic shape of surface is conjugated to intrinsic metric (GHY)

IV
From semiclassical coherent states to BMS_3 characters

Partition function of 3d gravity on twisted thermal Minkowski



Thermal

Solid torus topology,

Euclidean periodic time $\Delta t = \beta$

Twisted

Before identification,

turn by an angle γ

Partition function

Corresponds to the $\Lambda \to 0$ limit of thermal AdS_3 partition function

& BMS_3 characters correspond to $\Lambda \to 0$ limit of Virasoro chararacters

[Maloney & Witten 2007, Giombi, Maloney & Yin 2008, Barnich Et Al 2014, Oblak 2015]

Partition function of 3d gravity on twisted thermal Minkowski

Result

Formally:

$$Z \sim e^{\frac{(2)\pi i \beta}{\ell_{\mathrm{Pl}}}} \prod_{p\geq 2} \frac{1}{2-2\cos(\gamma p)}$$

Classical action (GHY term only)

bulk/gravity

1-loop corrections

0-modes of supertranslation (“boundary vacuum energy”)

boundary/ BMS_3 character

Effect of (super)rotations on modes of momentum *p*

[Maloney & Witten 2007, Giombi, Maloney & Yin 2008, Barnich Et Al 2014, Oblak 2015]

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Partition function of 3d gravity on twisted thermal Minkowski

Linearized quantum Regge calculus

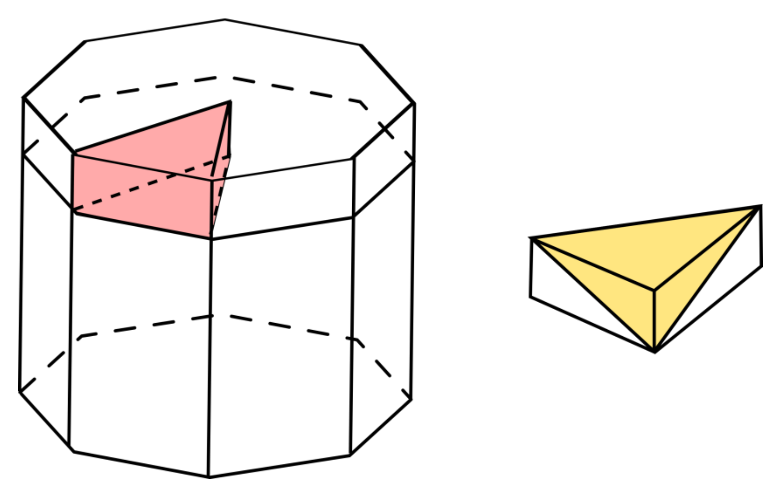
Regge calculus (RC) = simplicial version of metric gravity

bulk edges are discrete metric variables

Quantum linearized RC is *triangulation invariant* $\leftrightarrow$ diffeo invariant

"1-loop" partition function on a *finite* discretized twisted solid torus:

$$Z \sim e^{\frac{2\pi i \beta}{\ell_{\mathrm{Pl}}}} \prod_{p=2}^{(N_x-1)/2} \frac{1}{2 - 2\cos(\gamma p)}$$



[Bonzom & Dittrich 2015]

WKB evaluation of the amplitude

Result

$$\langle Z|\Psi\rangle \sim \sum_n e^{\,S(n)_\Psi|_{\text{crit.pt.}}}\mathcal{A}(n)\mathcal{D}(n,\gamma)$$

WKB evaluation of the amplitude

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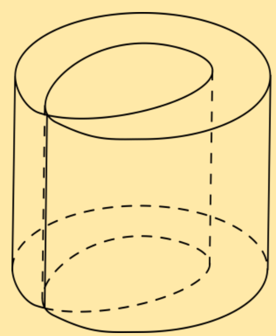
“Windings”



$n = 2$

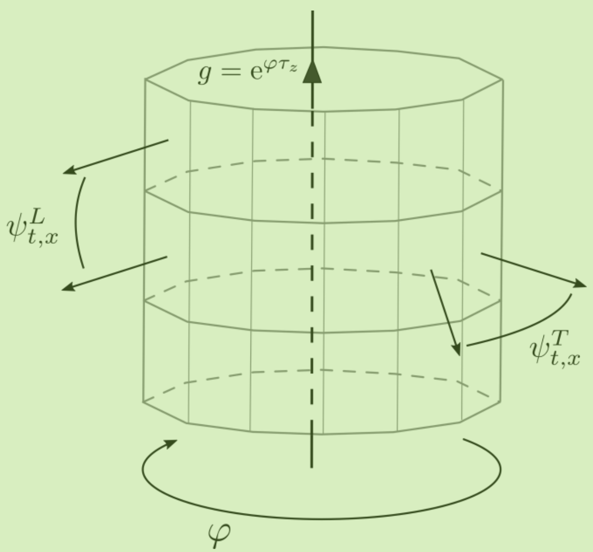
WKB evaluation of the amplitude

Result

$$\langle Z|\Psi\rangle \sim \sum_n e^{S(n)_\Psi|_{\text{crit.pt.}}} \mathcal{A}(n)\mathcal{D}(n,\gamma)$$

Classical action

$$e^{i\sum j\psi} = e^{2\pi n i N_t j_t} = e^{\frac{2\pi n i\beta}{\ell_{\rm Pl}}}$$



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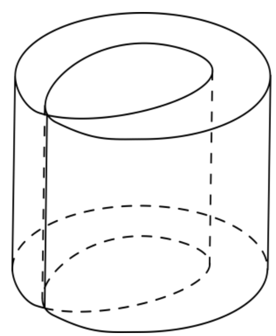

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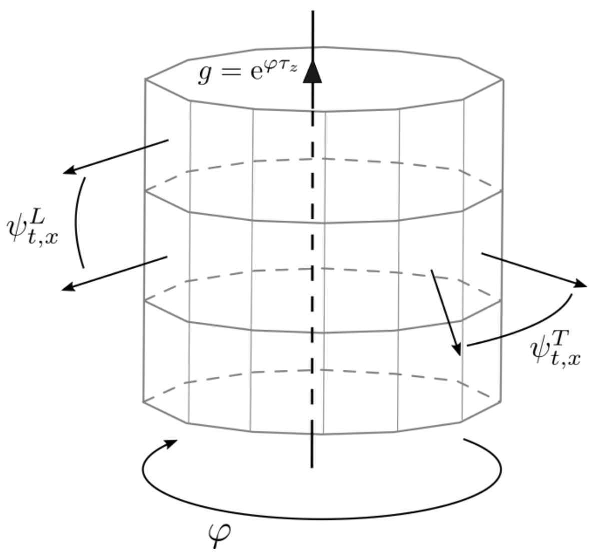

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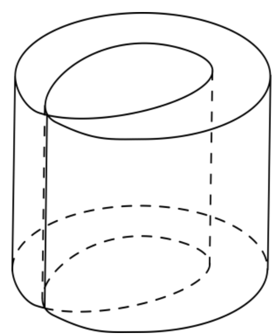

$n = 2$

Twist dependence

$$\mathcal{D}(n,\gamma) = \prod_{\substack{p>0\\p\neq n}}^{(N_x-1)/2} \frac{1}{2-2\cos(p\gamma)}$$

***p* = 0 correspond to a global sym**
***p* = *n* 'killed' by measure over φ**

WKB evaluation of the amplitude

Result

$$\langle Z|\Psi\rangle \sim \sum_n e^{S(n)_\Psi|_{\text{crit.pt.}}} \mathcal{A}(n)\mathcal{D}(n,\gamma)$$

Classical action

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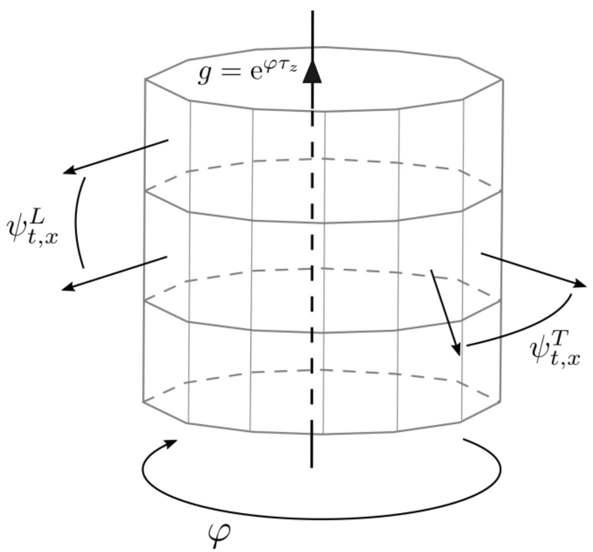

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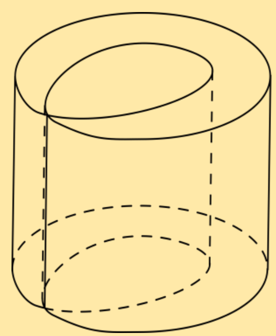


$n = 2$

Overwhelmingly dominated by *n*=1 and *n*=(*N*$_x$-1)/2

Twist dependence

$$\mathcal{D}(n,\gamma) = \prod_{\substack{p>0 \\ p\neq n}}^{(N_x-1)/2} \frac{1}{2-2\cos(p\gamma)}$$

p = 0 correspond to a global sym

p = *n* 'killed' by measure over φ

killed by *minimal* knowledge of extrinsic curvature

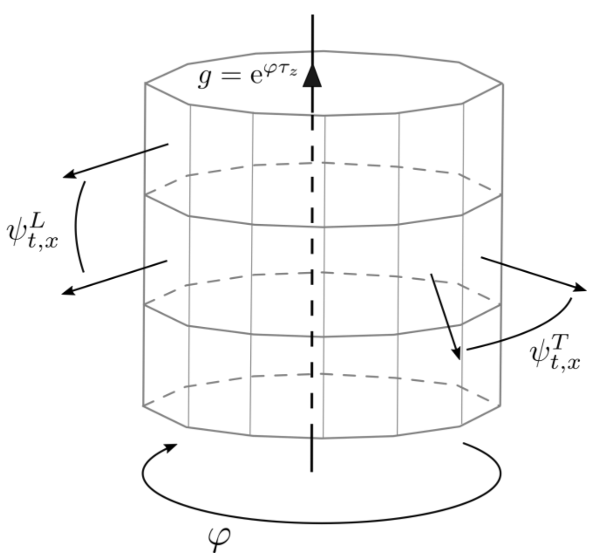
WKB evaluation of the amplitude

Result

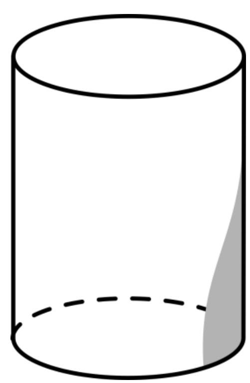
$$\langle Z|\Psi_{\text{coh}}\rangle \sim e^{\frac{2\pi i\beta}{\ell_{\text{Pl}}}} \prod_{p\geq 2}^{(N_x-1)/2} \frac{1}{2-2\cos(p\gamma)}$$

Classical action

$$(-1)^{2N_t j_t}$$



"Windings"



Overwhelmingly dominated by
***n* = 1**

Twist dependence

$$\mathcal{D}(n=1,\gamma) = \prod_{\substack{p>0\\ p\neq 1}}^{(N_x-1)/2} \frac{1}{2-2\cos(p\gamma)}$$

p = 0 correspond to a global sym

***p* = 2 'killed' by measure over φ**

Result

Result

At large spins, states encoding a classical flat intrinsic boundary geometry,

- $\rightsquigarrow$ induce a bdry theory that reproduces extrinsic torus geometry
- $\rightsquigarrow$ have an amplitude that reproduces the expected continuum result for irrational twists, corresponding to truncated BMS_3 character
- $\rightsquigarrow$ always finite result, even at rational "poles"
- $\rightsquigarrow$ produce corrections for small and quantum boundaries

First time such an explicit check over extended boundary states is performed

Conclusions
&
Outlook

Conclusions

On the boundary of 3d QG

Boundary states of non-perturbative 3d QG are very rich objects

Impose quantum bdry conditions – here we explored only one class of them

Their dynamical evaluation (amplitude), interpreted as boundary field theories

Boundary dofs

magnetic indices $\leftrightarrow$ reference frame orientation $[G_v]$

$\rightsquigarrow$ dofs come from SU(2)-gauge sym at bdry [‘would-be-gauge’ dofs]

or, dually:

radial spins $\rightsquigarrow$ quantum version of ‘would-be-normal-diffeos’

Outlook

Phase transitions

What is the geometrical significance

of a phase transition of the boundary theroy?

Relations to boundary continuum limit / discretization invariance?

Symmetries of the boundary theories

Can we have a more concrete grasp on the emergence of BMS_3 – or Virasoro?

In particular, how can it be understood directly as a

symmetry of the boundary theory?

Outlook

Other observables?

E.g. in spin chain: Correlations from coupling to bulk Wilson lines?

(A)dS?

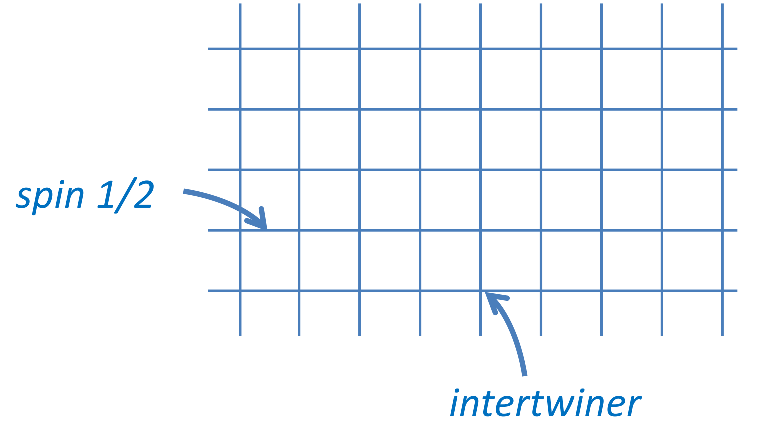
We know that curved geometries correspond to *q*-deformed spin-networks

Extend our result to those cases? [E.g. full 6v models, with phases transitions?]

Holographic RG? Liouville bdry theory?

Thank you

Spins ½ and the XXX chain

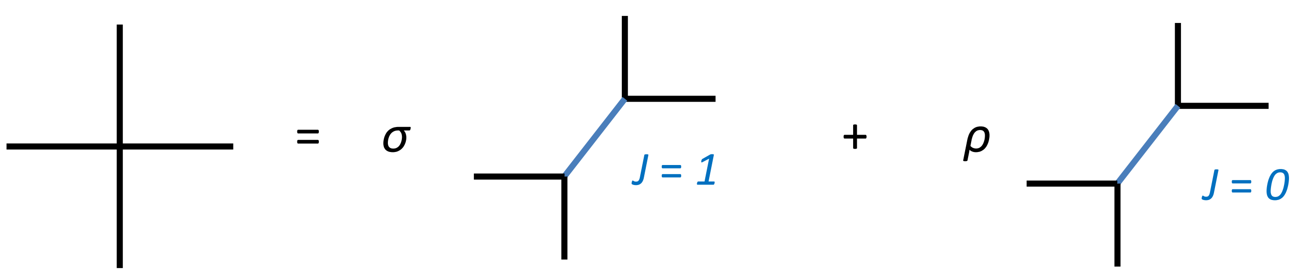


GHY b.c. on a square lattice

Let’s start from topologically trivial case

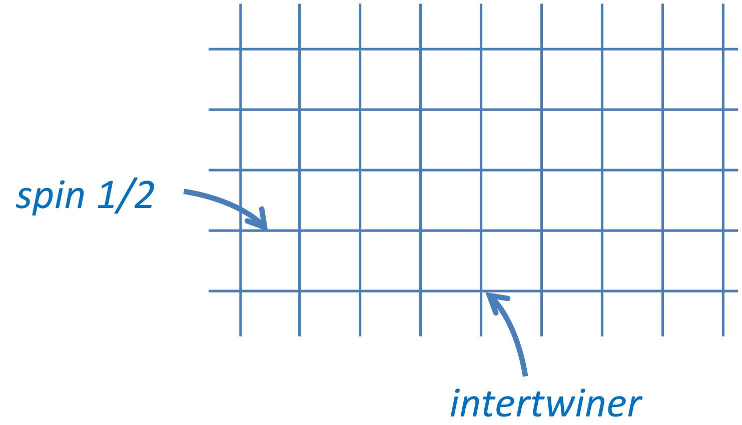
and 4-valent graph, with all spins *j = 1/2*

At each vertex, the intertwiner space is two dimensional



[Kaufmann 1983, Pasquier 1990, Witten 1990, Turaev 1992, AR Et Al 2017]

Spins ½ and the XXX chain



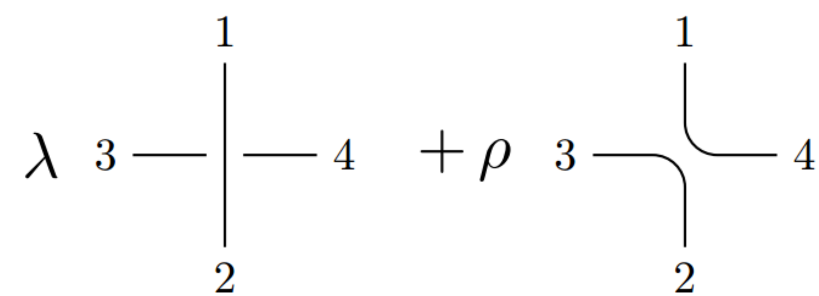
GHY b.c. on a square lattice

Let’s start from topologically trivial case

and 4-valent graph, with all spins *j = 1/2*

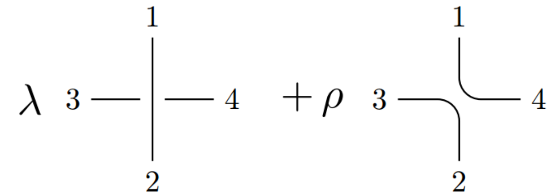
A more enlightening basis is, however, the following

$$|\iota[\lambda, \rho]\rangle :=$$

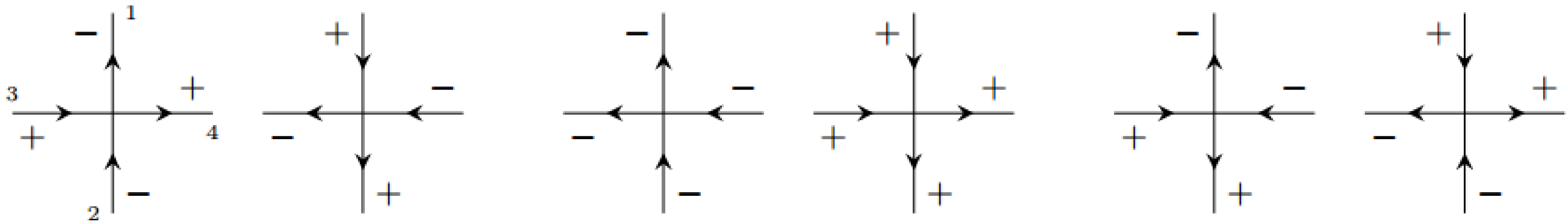


[Kaufmann 1983, Pasquier 1990, Witten 1990, Turaev 1992, AR Et Al 2017]

Spins ½ and the XXX chain



6-vertex model (ice-type model)



$\omega(\mathrm{I}) = a$ $\omega(\mathrm{II}) = a$ $\omega(\mathrm{III}) = b$ $\omega(\mathrm{IV}) = b$ $\omega(\mathrm{V}) = c$ $\omega(\mathrm{VI}) = c$

$$Z_{6v} = \sum_{\text{arrows}} a^{\#I+\#II} b^{\#III+\#IV} c^{\#V+\#VI}$$

Matches the gravitational amplitude if

$$a = \lambda \qquad b = \lambda + \rho \qquad c = \rho$$

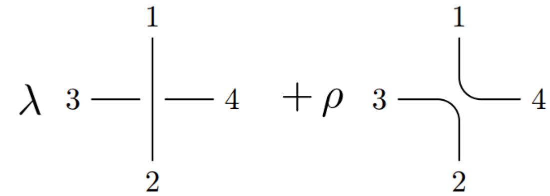
or

$$a = \lambda + \eta \qquad b = \lambda \qquad c = \eta$$

6v lattice 90° rot
≡
u vs *t* channel basis

[Kaufmann 1983, Pasquier 1990, Witten 1990, Turaev 1992, AR Et Al 2017]

Spins ½ and the XXX chain



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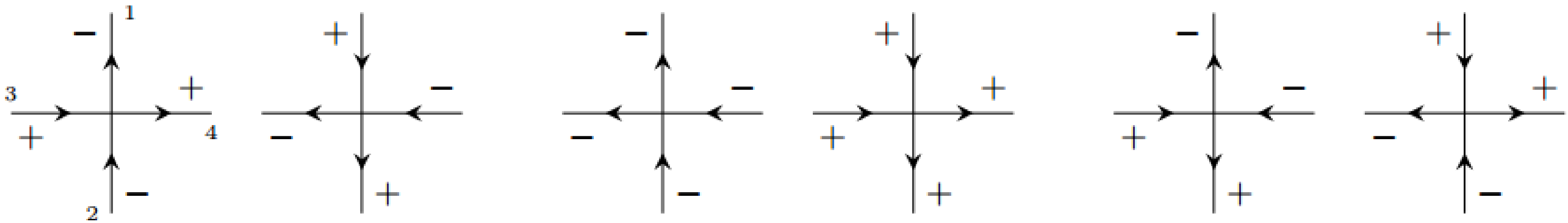


$\omega(\mathrm{I}) = a$ $\omega(\mathrm{II}) = a$ $\omega(\mathrm{III}) = b$ $\omega(\mathrm{IV}) = b$ $\omega(\mathrm{V}) = c$ $\omega(\mathrm{VI}) = c$

$$Z_{6v} = \sum_{\text{arrows}} a^{\#I+\#II} b^{\#III+\#IV} c^{\#V+\#VI}$$

Matches the gravitational amplitude if

$$a + c - b = 0$$

or

$$b + c - a = 0$$

6v lattice 90° rot
≡
u vs *t* channel basis

[Kaufmann 1983, Pasquier 1990, Witten 1990, Turaev 1992, AR Et Al 2017]

Spins ½ and the XXX chain

Spin chains

The 6v model is an integrable system related to

the XXZ Heisenberg spin chain

Spin 1/2
boundary states

$$H_{\mathrm{XXZ}} = -J \sum_i \left(\sigma_i^x \sigma_{i+1}^x + \sigma_i^y \sigma_{i+1}^y + \Delta \sigma_i^z \sigma_{i+1}^z \right)$$

with trivial anisotropy Δ

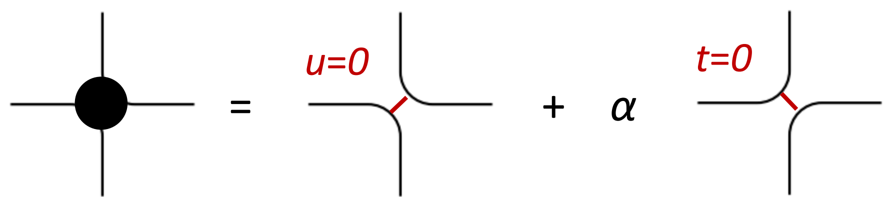
$$\Delta = \frac{a^2 + b^2 - c^2}{2ab} \equiv 1$$

SU(2) theory
[Λ = 0]

Thus, *exact* correspondence between GHY boundary states at spin 1/2 and an integrable statistical model, the XXX spin chain

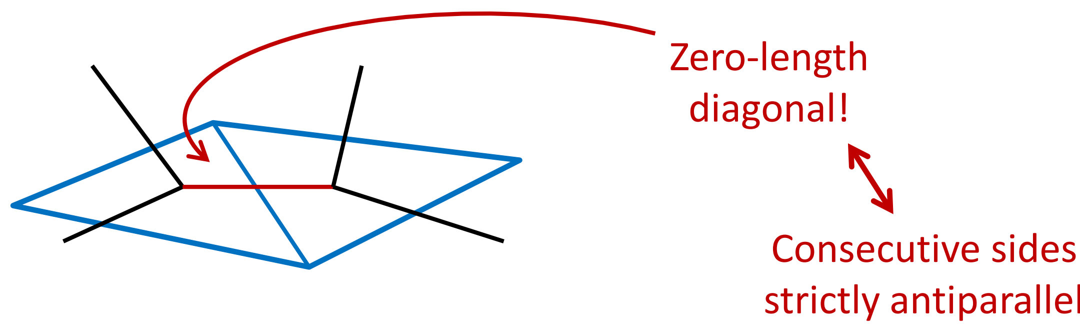
From Witten's weaves to fuzzy parallelograms

Spin 0 in *u* and *t* recoupling channel



These intertwiners produce other integrable models (IRF, RSOS type)

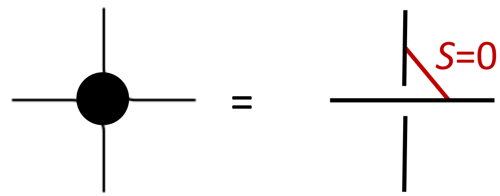
Unfortunately, they have a very degenerate geometrical interpretation



[Kaufmann 1983, Pasquier 1990, Witten 1990, Turaev 1992, AR Et Al 2017]

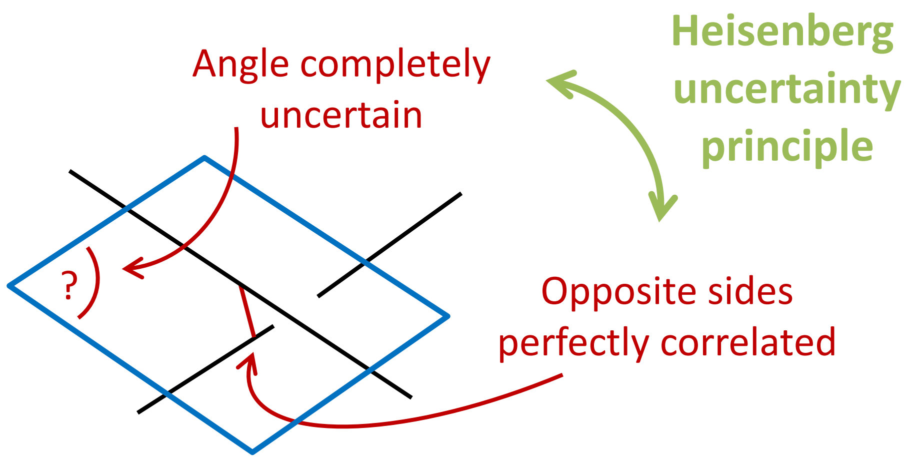
From Witten's weaves to fuzzy parallelograms

Spin 0 in *s* recoupling channel



[Witten 1990]

More sensible, but still no definite shape!



Questions

Unfreeze the spins

What if we relax strict GHY conditions and superpose different spins?

Non-trivial topology

How is a non-trivial topology encoded in these statistical models?

Fuzzy parallelograms

Can one build coherent (i.e. “not-too-fuzzy”) parallelograms?

Ising model
from spin-network superpositions

Spin-network generating functions

It turns out that considering superpositions of *all* spins and intertwiners,
one can build states dual to the Ising model

These states are actually generating functions for all spin networks states

The duality is through a SUSY,
it transforms group averaging at vertices
into Grassmannian spinors whose pairings give Ising's loop expansion

The intertwiners's inner structure gives
Ising's coupling constants $\rightsquigarrow$ *criticality = specific intrinsic geometries*

[Dittrich & Hnybida 2013 , Bonzom, Costantino & Livine 2015]

Dependence on γ

$$Z \sim e^{\frac{2\pi i \beta}{\ell_{\rm Pl}}} \prod_{p\geq 2}^{(N_x-1)/2} \frac{1}{2-2\cos(\gamma p)}$$

p ≥ 2

1-loop 3d EH: ghosts "eat" $p = 0, \pm 1$

Regge calculus: modes with $p = 0, \pm 1$ correspond to discrete diffeos

BMS_3 character : $p = 0, \pm 1$ special because of Schwarzian derivative

Rational vs. irrational γ

In the continuum, poles at rational twists, i.e. $\gamma \in 2\pi\mathbb{Q}$

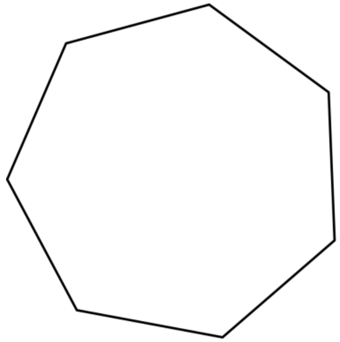
In the discrete, $\gamma = 2\pi\frac{N_\gamma}{N_x}$ and "irrational" means $\mathrm{GCD}(N_\gamma, N_x) = 1$

[Maloney & Witten 2007, Giombi, Maloney & Yin 2008,
Barnich Et Al 2014, Oblak 2015, Bonzom & Dittrich 2015]

Rational vs. irrational twist

Result

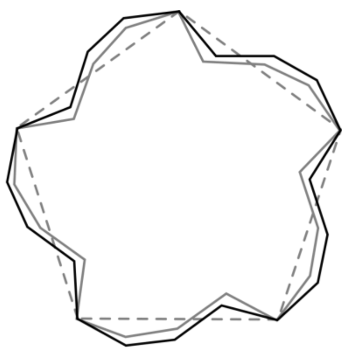
$$\mathcal{D}(\gamma, n) = \prod_{\substack{p>0 \\ p\neq n}}^{(N_x-1)/2} \frac{1}{2-2\cos(p\gamma)}$$



$N_x = 7,\ K = 1$

is divergent for ‘rational’ twist, i.e. iff

$$\gamma = 2\pi \frac{N_\gamma}{N_x} \qquad \text{and} \qquad \mathrm{GCD}(N_\gamma, N_x) > 1$$



$N_x = 20,\ K = 4$

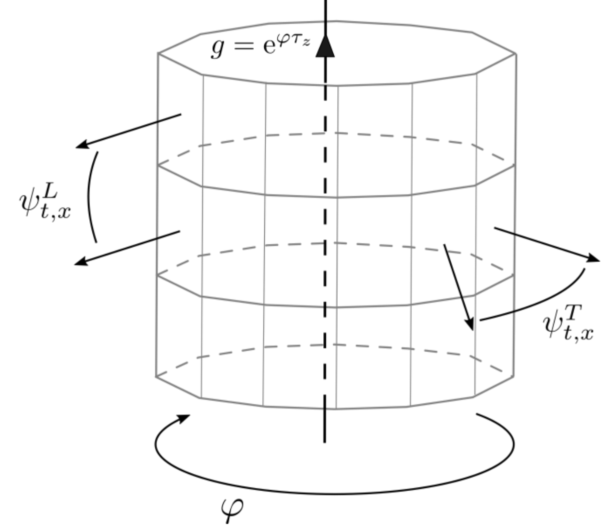
This is an artifact of the approximation, though:

if GCD > 1 then the stationary points are not isolated,

and the Hessian is degenerate

Divergences = breakdown of simple stationary phase method

Non-perturbative 3d QG



Design boundary state

Coherent state techniques

to build states encoding *flat* rectangular plaquettes

$$|\iota\rangle = \int \mathrm{d}G \bigotimes_{l=1}^{4} D^{j_l}(G)|j_l, \xi_l\rangle$$

where: spin j_l = length of the *l-th* side

spinor = its direction

bdry dofs = magnetic indices $\leftrightarrow$ group variable [rotation invariance]

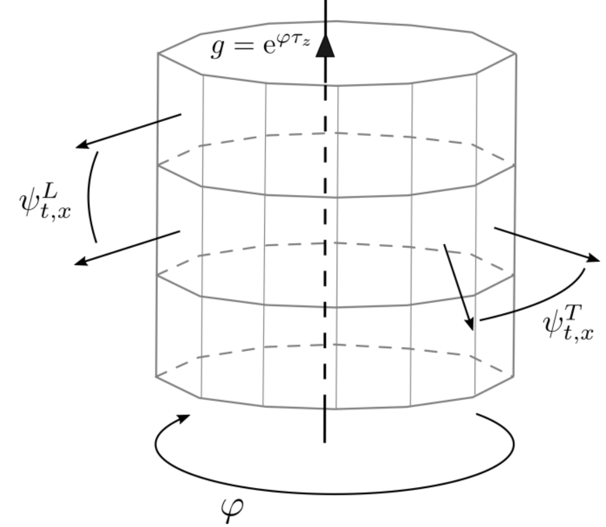
Evaluate its amplitude

$$\langle Z|\Psi_{\mathrm{coh}}\rangle = \int \mathrm{d}\varphi \sin^2\tfrac{\varphi}{2}\Big[\prod_v \int \mathrm{d}G_v\Big] e^{\sum_v 2j_t \ln\langle\uparrow|G^{-1}_{v+x}G_v|\uparrow\rangle + 2j_x \ln\langle +|G^{-1}_{v+t} e^{\frac{\varphi}{N_t}\tau_z} G_v|+\rangle}$$

finite expression

[Livine & Speziale 2007, Barrett Et Al 2008, Freidel & Krasnov 2008, Dowdall , Hellmann & Gomes 2010, AR Et Al. 2015, AR Et. Al. 2017]

Non-perturbative 3d QG



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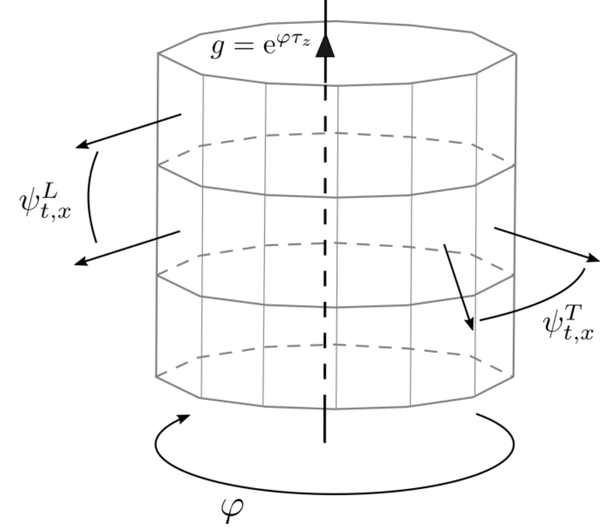
Spins:
size of the plaquette

Non-perturbative 3d QG



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Coherent state techniques

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where: spin j_l = length of the *l-th* side

spinor = its direction

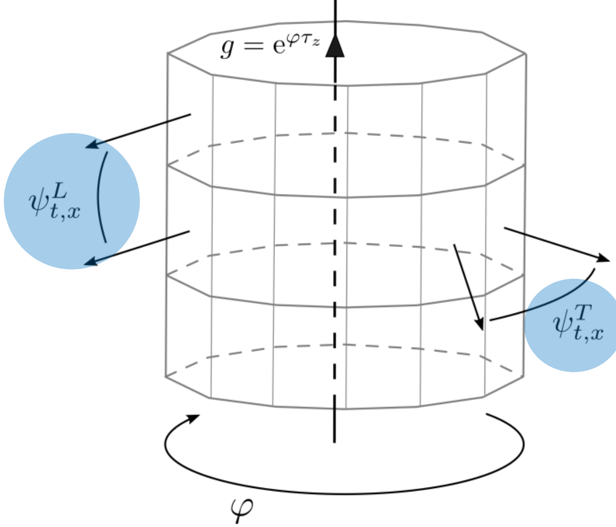
bdry dofs = magnetic indices $\leftrightarrow$ group variable [rotation invariance]

Spinors:
rectangle in the x-z plane

Evaluate its amplitude

$$\langle Z|\Psi_{\text{coh}}\rangle = \int \mathrm{d}\varphi \sin^2 \tfrac{\varphi}{2} \Big[\prod_v \int \mathrm{d}G_v\Big] e^{\sum_v 2j_t \ln\langle\uparrow|G^{-1}_{v+x} G_v|\uparrow\rangle + 2j_x \ln\langle +|G^{-1}_{v+t} e^{\frac{\varphi}{N_t}\tau_z} G_v|+\rangle}$$

Non-perturbative 3d QG



Design boundary state

Coherent state techniques

to build states encoding *flat* rectangular plaquettes

$$|\iota\rangle = \int \mathrm{d}G \bigotimes_{l=1}^{4} D^{j_l}(G)|j_l, \xi_l\rangle$$

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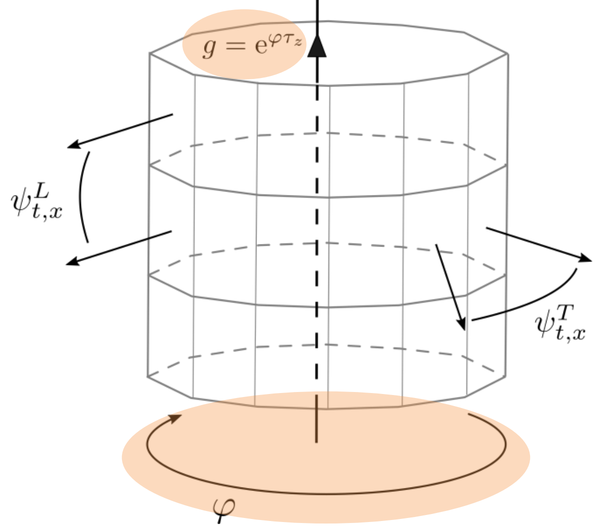
bdry dofs = magnetic indices ↔ group variable [rotation invariance]

Evaluate its amplitude

$$\langle Z|\Psi_{\mathrm{coh}}\rangle = \int \mathrm{d}\varphi \sin^2\frac{\varphi}{2}\left[\prod_v \int \mathrm{d}G_v\right] e^{\sum_v 2j_t \ln\langle\uparrow|G^{-1}_{v+x}G_v|\uparrow\rangle + 2j_x \ln\langle +|G^{-1}_{v+t}e^{\frac{\varphi}{N_t}\tau_z}G_v|+\rangle}$$

$$\boxed{\text{3d orientation of plaquettes (extrinsic curvature)}}$$

Non-perturbative 3d QG



Design boundary state

Coherent state techniques

to build states encoding *flat* rectangular plaquettes

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where: spin j_l = length of the *l-th* side

spinor = its direction

bdry dofs = magnetic indices $\leftrightarrow$ group variable [rotation invariance]

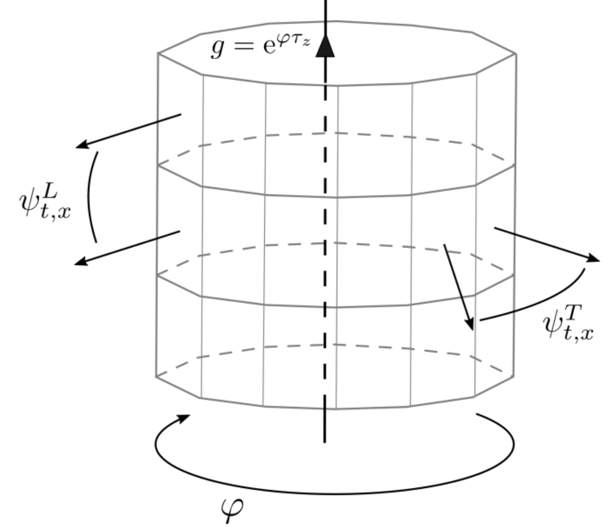
Amplitude

Bulk monodromy, non-local insertion

$$\langle Z|\Psi_{\mathrm{coh}}\rangle = \int \mathrm{d}\varphi \sin^2\frac{\varphi}{2}\left[\prod_v \int \mathrm{d}G_v\right] e^{\sum_v 2j_t \ln\langle\uparrow|G^{-1}_{v+x}G_v|\uparrow\rangle + 2j_x \ln\langle +|G^{-1}_{v+t} e^{\frac{\varphi}{N_t}\tau_z} G_v|+\rangle}$$

Bulk monodromy measure

Non-perturbative 3d QG



Design boundary state

Coherent state techniques

to build states encoding *flat* rectangular plaquettes

$$|\iota\rangle = \int \mathrm{d}G \bigotimes_{l=1}^{4} D^{j_l}(G)|j_l,\xi_l\rangle$$

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Evaluate its amplitude

$$\langle Z|\Psi_{\mathrm{coh}}\rangle = \int \mathrm{d}\varphi \sin^2\tfrac{\varphi}{2}\Big[\prod_v \int \mathrm{d}G_v\Big] e^{\sum_v 2j_t \ln\langle\uparrow|G^{-1}_{v+x}G_v|\uparrow\rangle + 2j_x \ln\langle +|G^{-1}_{v+t}e^{\frac{\varphi}{N_t}\tau_z}G_v|+\rangle}$$

$$S \sim \oint \langle\sqrt{q}^{\wedge}|\mathrm{d} + G^{-1}\omega^o G + G^{-1}\mathrm{d}G|\sqrt{q}^{\wedge}\rangle$$

[Wieland 2017]

WKB evaluation of the amplitude

Semiclassical limit

$$\langle Z|\Psi_{\mathrm{coh}}\rangle = \int \mathrm{d}\varphi \sin^2 \tfrac{\varphi}{2} \left[\prod_v \int \mathrm{d}G_v\right] e^{\sum_v 2j_t \ln\langle\uparrow|G_{v+x}^{-1} G_v|\uparrow\rangle + 2j_x \ln\langle +|G_{v+t}^{-1} e^{\frac{\varphi}{N_t}\tau_z} G_v|+\rangle}$$

The semiclassical limit corresponds to uniform “large *j*” limit = $\hbar \to 0$

$\rightsquigarrow$ apply stationary phase methods

Geometry from e.o.m.

Fix G_v to correspond to local embedding of the torus boundary in R^3

Fix φ to be equal to the twist angle *γ*

The e.o.m. for φ fixes which cycle of the torus can carry extrinsic curvature

[Livine & Speziale 2007, Barrett Et Al 2008, Freidel & Krasnov 2008, Dowdall , Hellmann & Gomes 2010, AR Et Al. 2015, AR Et. Al. 2017]