Lattice quantum gravity and its continuum limit in the group field theory formalism

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Simplicial path integral for quantum gravity

path integral for simplicial geometries associated to triangulation

$$\mathcal{A}_\Delta = \int \mathcal{D}g_\Delta \, e^{i\, S_\Delta(g_\Delta)}$$

dynamical variables should encode the geometry of lattices

action should correspond to the discretisation of some continuum gravity action

measure should encode some essential symmetries

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can be defined using group-theoretic ingredients **Part 1**

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then, define strategy/procedure for continuum limit **Part 2**

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can be defined using group-theoretic ingredients **Part 1**

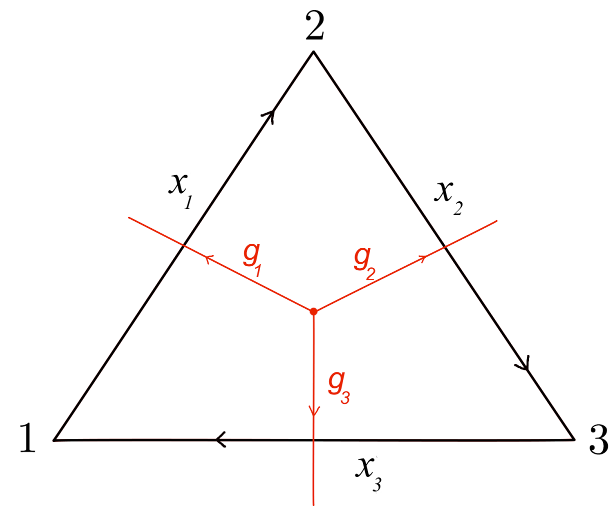
can be embedded in QFT formalism: Group Field Theory **Part 2**

define strategy/procedure for continuum limit via GFT renormalization **Part 3**

Discrete quantum gravity path integrals

in group-theoretic variables

Quantum triangle in 3d - classical phase space

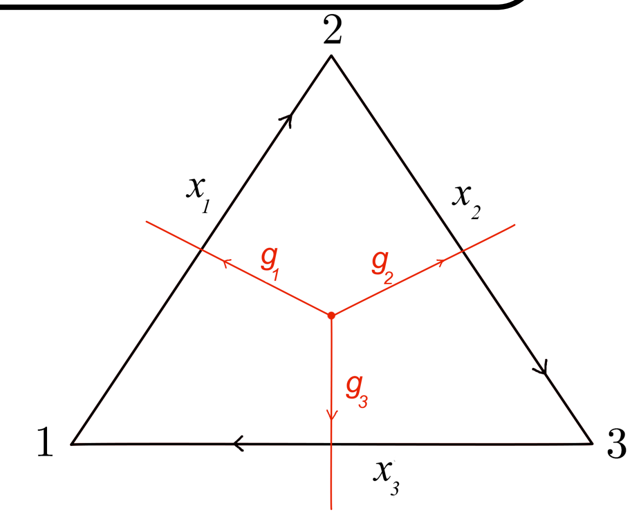


see talks by A. Riello and B. Dittrich

Quantum triangle in 3d - classical phase space

intrinsic geometry of classical triangle in $\mathbb{R}^3$

3 edge vectors that close $x_1, x_2, x_3 \in \mathbb{R}^3 \quad s.t. \quad \sum_i x_i = 0$



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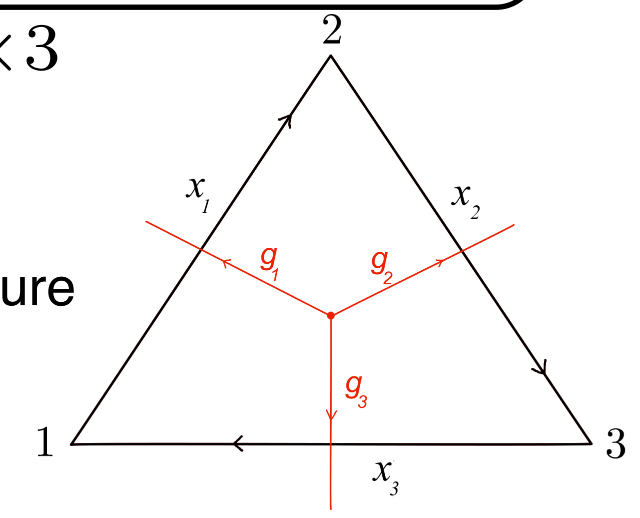
group elements $\{g_i\} =$ discrete connection, encoding extrinsic geometry/curvature



Phase space for triangle in discrete 3d gravity

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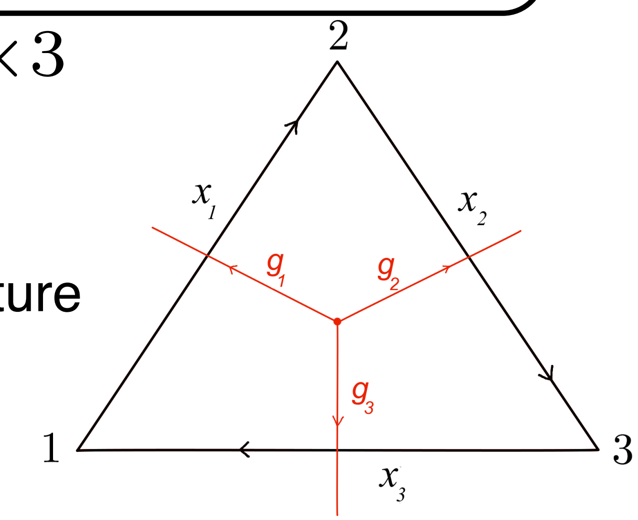

Phase space for triangle in discrete 3d gravity

discretised 3d gravity variables from continuum theory: $S(e,\omega) = \int Tr(e \wedge F(\omega))$

triad Lie-algebra valued 1-form ——> Lie algebra element

$$\int_i e = b_i \in \mathfrak{su}(2)$$

connection Lie algebra-valued 1-form ——> group-valued parallel transport ——> group element

$$\mathcal{P}e^{\int_{i*}\omega} = g_i \in SU(2)$$

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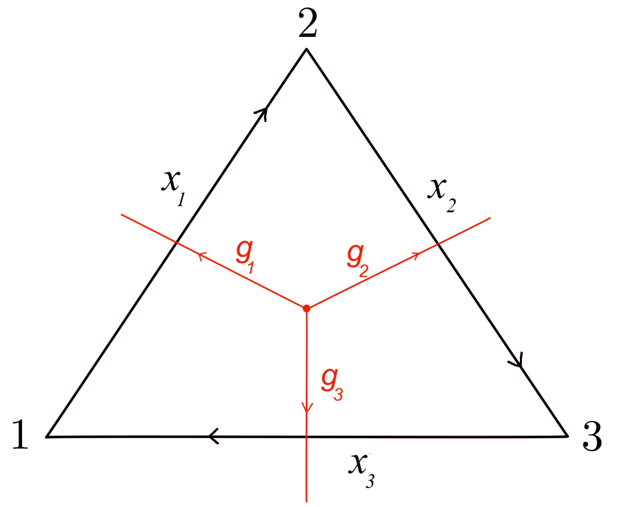
Quantum triangle in 3d - Hilbert space, representations

quantum triangle: $$\mathcal{H}_{triangle} = Inv\left(\bigotimes_i \mathcal{H}_i^{SU(2)}\right)$$



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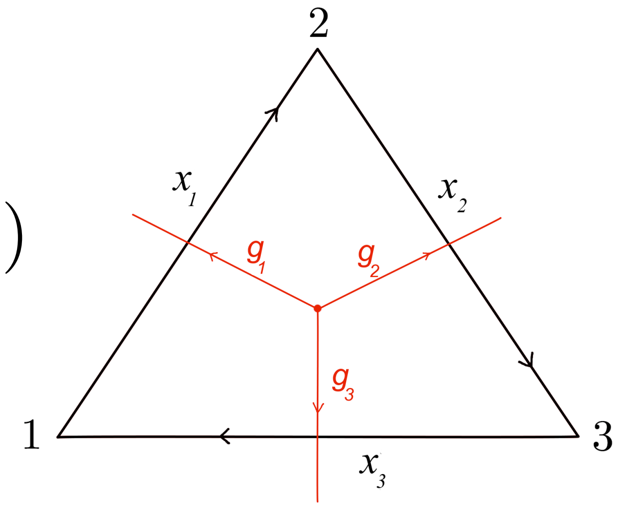

- non-commutative Lie algebra (edge vector) representation: $\mathcal{H}_i^{SU(2)} = L^2_\star(\mathbb{R}^3)$

for given quantisation map $f_\star \star g_\star = \mathcal{Q}^{-1}(\mathcal{Q}(f_\star)\mathcal{Q}(g_\star))$

complete basis of non-commutative plane waves $\left(e_{g_1} \star e_{g_2}\right)(x) \equiv e_{g_1 g_2}(x)$

L. Freidel, E. Livine, '05; L. Freidel, S. Majid, '06; A. Baratin, D. Oriti, '10; C. Guedes, DO, M. Raasakka, '13

$$\mathcal{H}_{triangle} = Inv\left(\otimes_i \mathcal{H}_i^{SU(2)}\right) \ni \psi(x_1, x_2, x_3) \star \delta\left(x_1 + x_2 + x_3\right)$$

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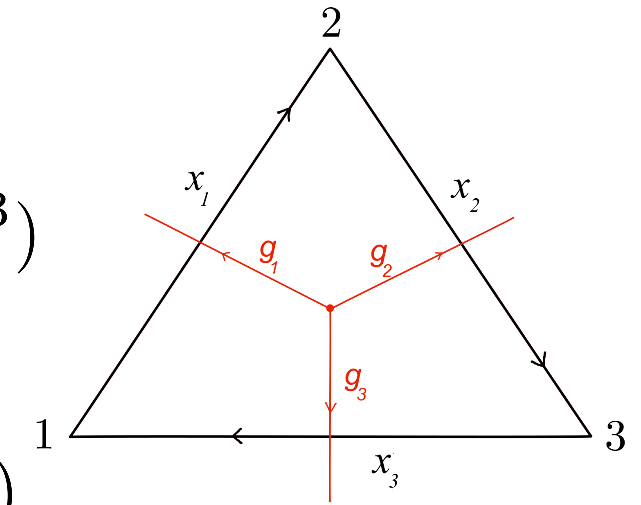

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- group (connection) representation: $\mathcal{H}_i^{SU(2)} = L^2(SU(2))$

$$\psi(x_1, x_2, x_3) \star \delta(x_1 + x_2 + x_3) = \int [dg]^3 \int dh\, \psi(g_1 h, g_2 h, g_3 h)\, e_{g_1}(x_1) e_{g_2}(x_2) e_{g_3}(x_3)$$

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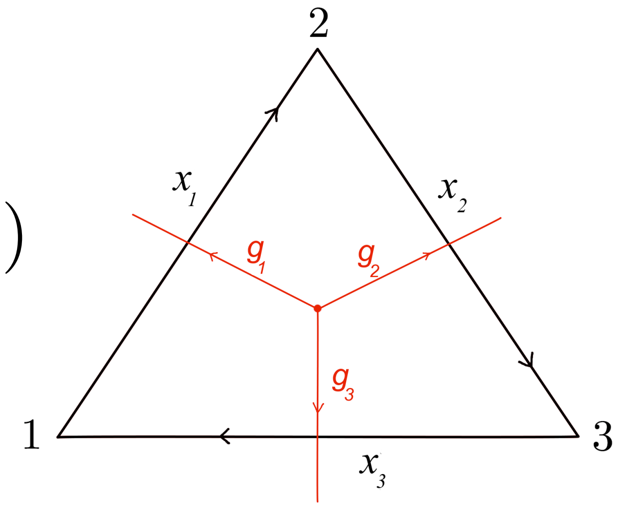

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- spin representation (via Peter-Weyl decomposition): $\mathcal{H}_i^{SU(2)} = \bigoplus_{j_i \in \mathbb{N}/2} \mathcal{H}^{j_i}$

$$\psi(g_1, g_2, g_3) = \sum \psi^{j_1 j_2 j_3}_{m_1 m_2 m_3}\, C^{j_1 j_2 j_3}_{n_1 n_2 n_3}\, D^{j_1}_{m_1 n_1}(g_1) D^{j_2}_{m_2 n_2}(g_2) D^{j_3}_{m_3 n_3}(g_3)$$

3j-symbol - intertwiner

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use simplicial complex Δ and its dual complex Γ, with assigned same group-theoretic variables

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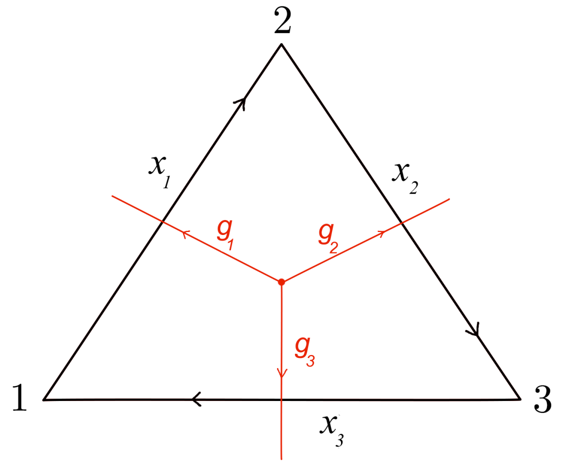
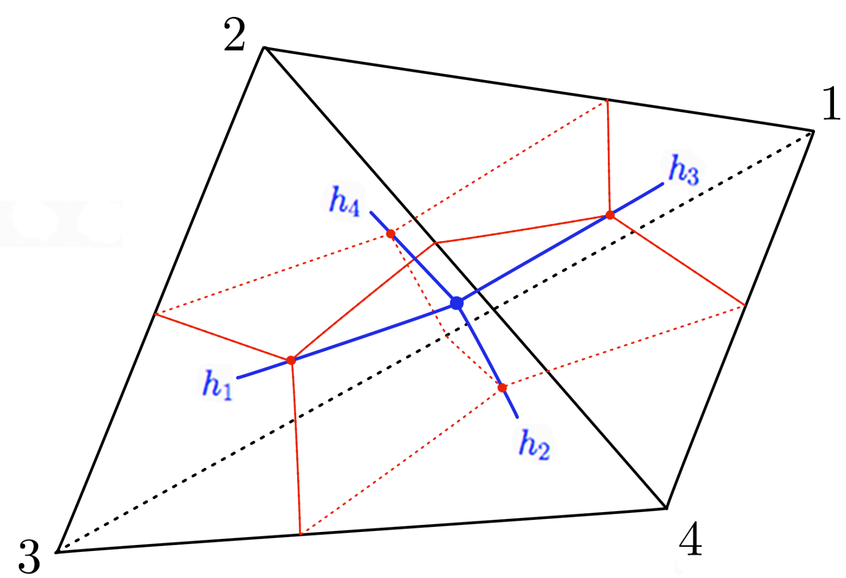
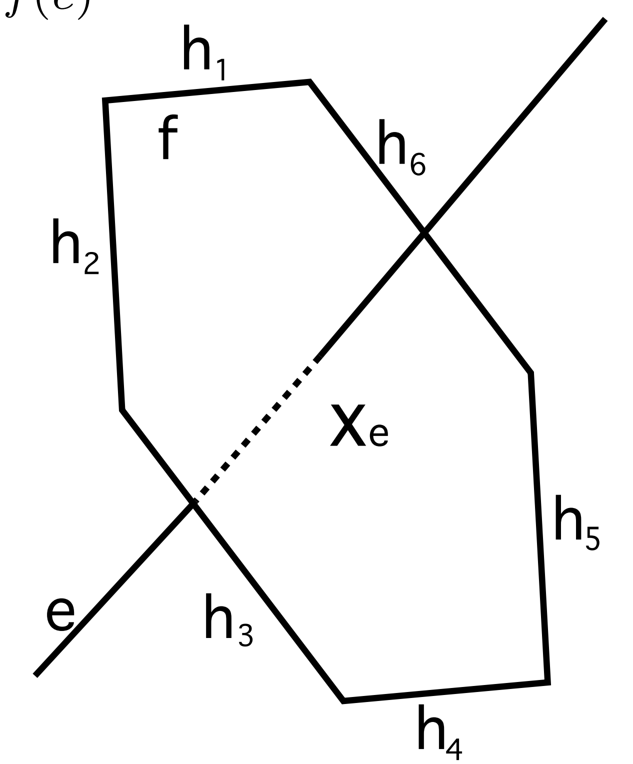
$$S(x_e, h_l) = \sum_e tr\,[x_e\, H_e(h_l)] \qquad H_e = \prod_{l \in \partial f(e)} h_l \in SO(3)$$







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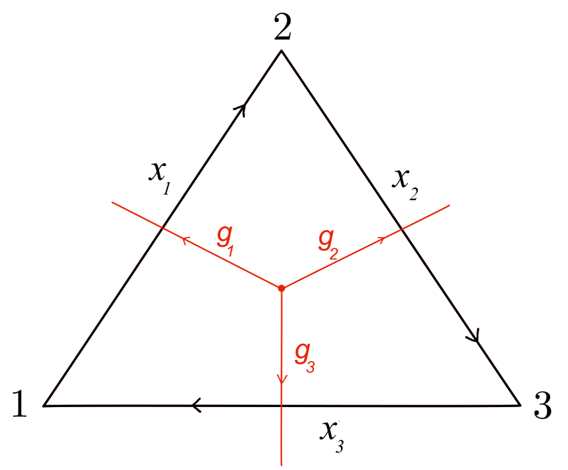

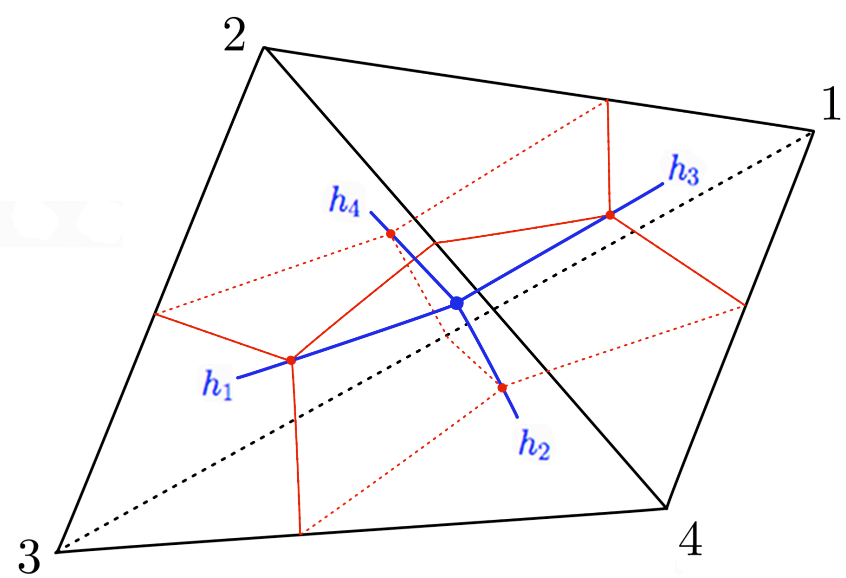

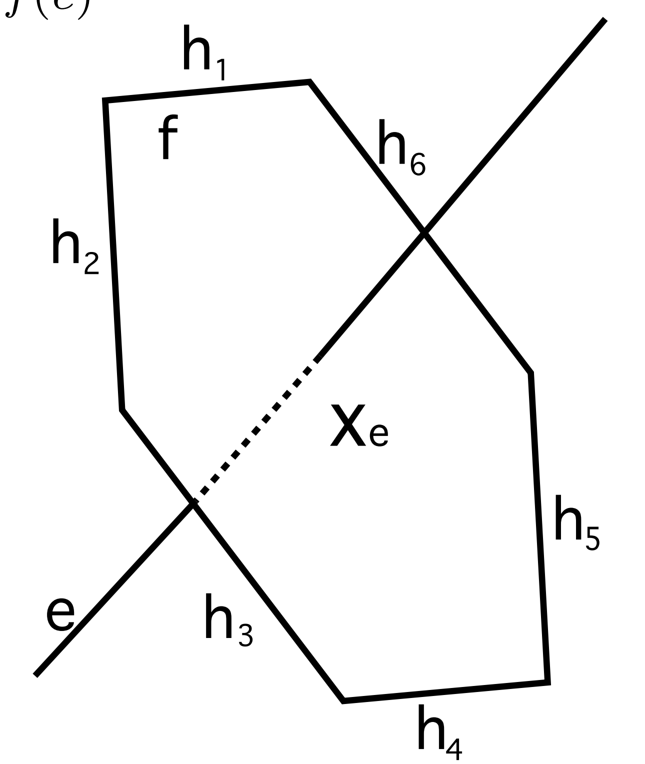

discrete non-commutative path integral
(depending on quantisation map):

$$\mathcal{A}_\Gamma = \int \prod_l [dh_l] \int \prod_e [d^3 x_e]\; e^{i \sum_e Tr\,[x_e H_e]}$$

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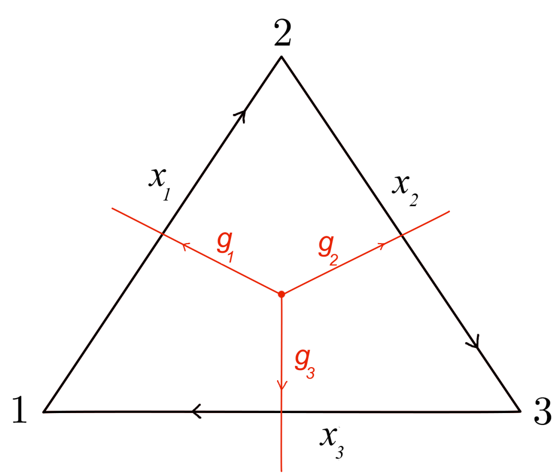

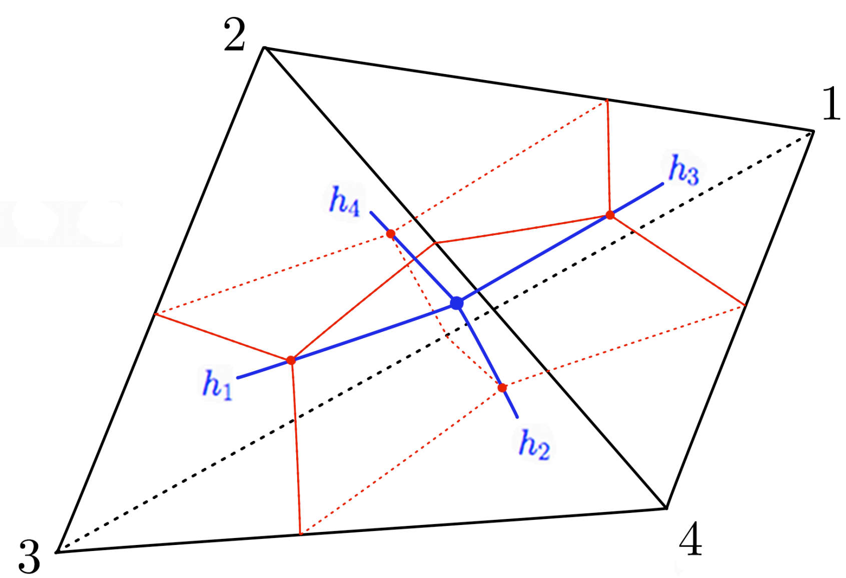

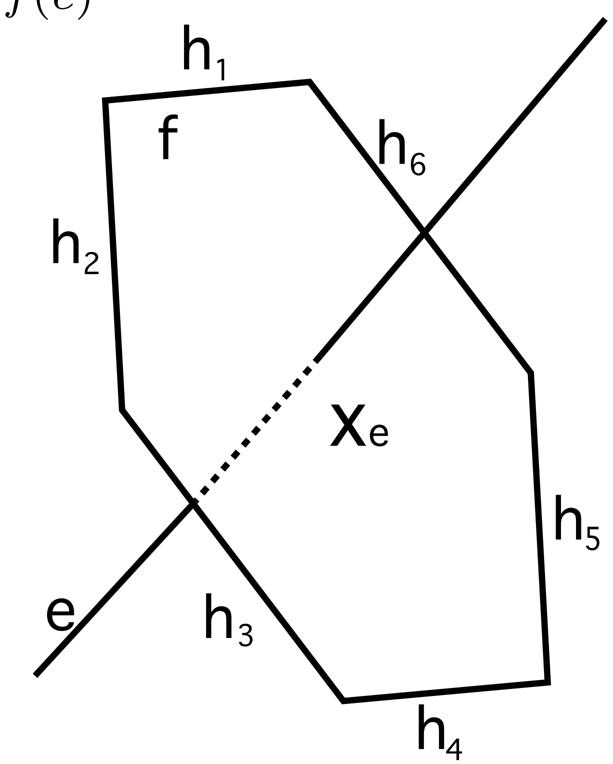

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4d simplicial geometry in group-theoretic data

classical tetrahedron in 4d:

J. Barrett, L. Crane, '97; J. Baez, J. Barrett, '98; L. Freidel, K. Kransov, '07

$$B_i^{IJ} \in \wedge^2\mathbb{R}^4 \simeq \mathfrak{so}(4)\,,\; N^I \in S^3 \subset \mathcal{T}\mathbb{R}^4 \qquad N_I\left(*B_i^{IJ}\right) = 0 \qquad \sum_i B_i^{IJ} = 0$$

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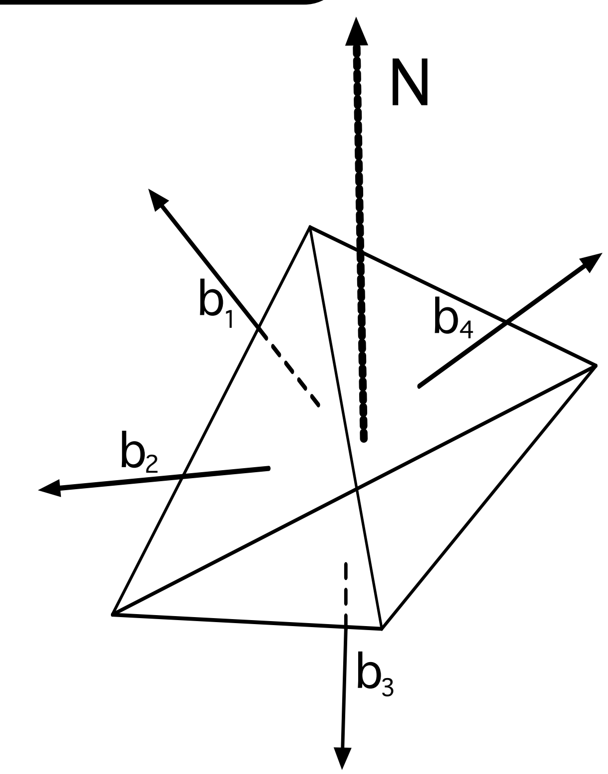

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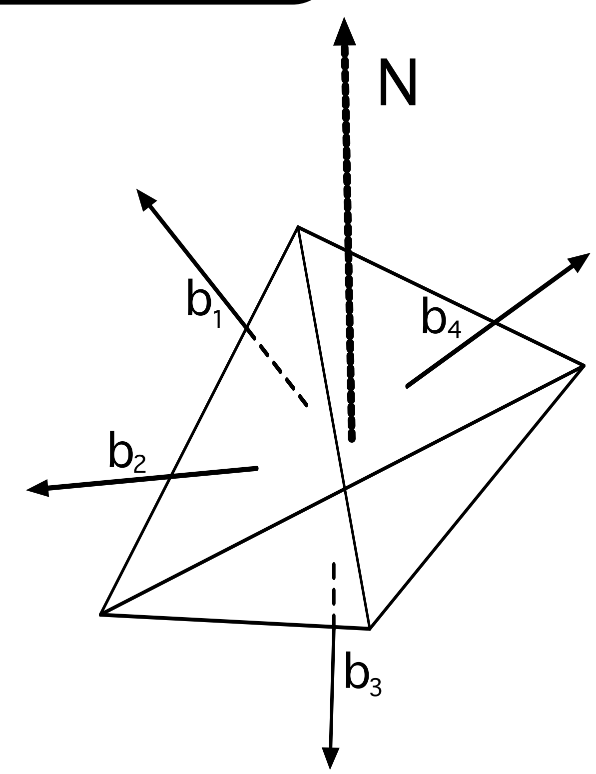

phase space before constraints:

$$[\mathcal{T}^* Spin(4)]^{\times 4} \simeq [\mathcal{T}^* SU(2) \times \mathcal{T}^* SU(2)]^{\times 4}$$

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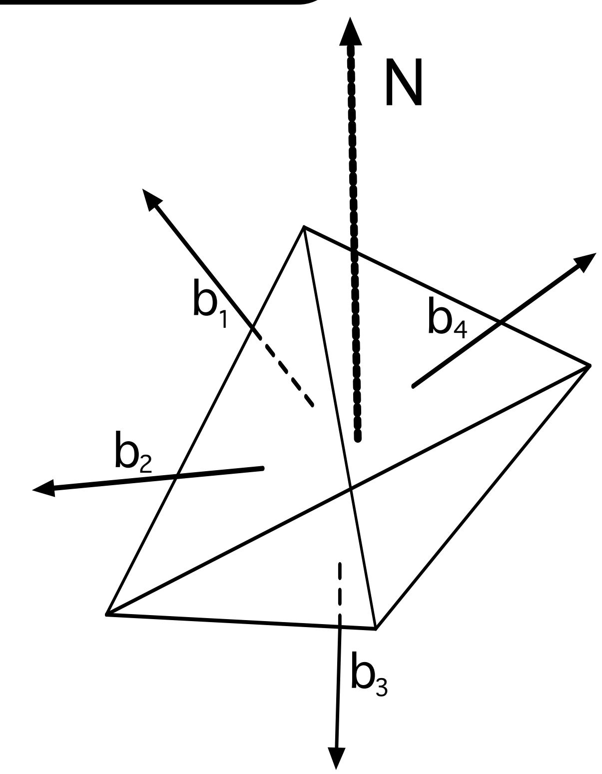

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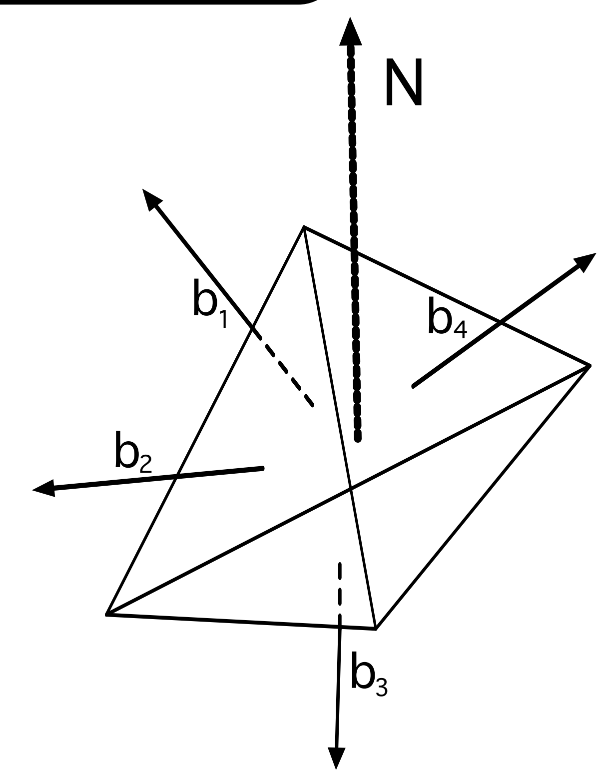

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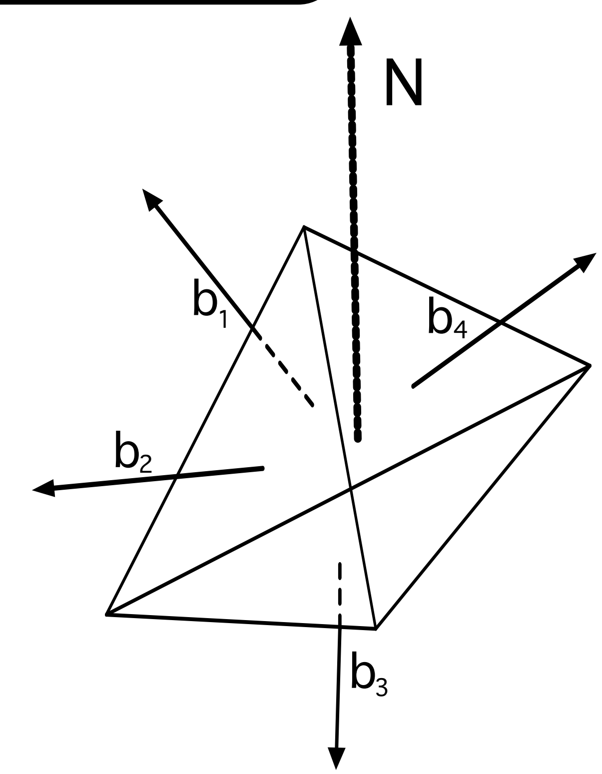

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Plebanski-Holst action (topological BF theory+simplicity constraints)

$$S_{Pleb} = \frac{1}{G}\int_{\mathcal{M}} \left[B \wedge F(\omega) + \frac{1}{\gamma} \star B \wedge F(\omega) + \phi \, B \wedge B \right]$$

$$\delta\phi = 0 \Rightarrow \star B \wedge B = 0 \Rightarrow B \simeq \star e \wedge e$$

$$B \in \mathfrak{so}(3,1)$$

$connection: \quad \omega \in \mathfrak{so}(3,1) \qquad curvature \quad F(\omega) = d_\omega \omega \qquad co-tetrad \quad e^I \qquad (I = 0,1,2,3)$

discretize to get simplicial gravity action and path integral (as in 3d case)

several models in the literature (depending on imposition of geometric constraints)

one construction:

A. Baratin, D. Oriti, '11

(non-commutative) simplicial gravity path integral

$$\mathcal{A}_\Delta = \int [d^6 B_t][dN_\tau]\mathcal{D}_\beta^{B_t, N_\tau}[h_{\tau\sigma}] \star \prod_t \left[e^{i\, tr[B_t\, H_t]} \star \delta_{-N_{\tau_o(t)} B_t^- N_{\tau_o(t)}^{-1}}(\beta B_t^+) \right]$$

can also be expressed in group representation (as lattice gauge theory)
or "spin" representation (spin foam model ~ LQG)

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with group-theoretic ingredients

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need to remove dependence on fixed triangulation and control arbitrary refined ones:

one strategy:

sum over triangulations weighted by simplicial gravity path integral

this defines full theory: candidate path integral of continuum quantum gravity

$$Z = \sum_\Delta w(\Delta)\, \mathcal{A}_\Delta = \sum_\Delta w(\Delta) \int \mathcal{D}g_\Delta \; e^{i\, S_\Delta(g_\Delta)} \equiv \int \mathcal{D}g \; e^{i\, S(g)}$$

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fixing all group theoretic data, eg to equilateral triangulations, gives
purely combinatorial construction ~ euclidean dynamical triangulations

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new questions:

- (in addition to: which discrete variables and amplitudes?)
- which triangulations? which topologies?
- which of them are dominant/suppressed in which regime?
- which combinatorial measure?
- how to control it? numerically? analytically?
- universality classes? which ingredients are really crucial?

defining/computing the sum = defining the continuum gravity path integral

The Group Field theory formulation

of discrete gravity path integrals

Group field theories

(Boulatov, Ooguri, De Pietri, Freidel, Krasnov, Rovelli, Perez, DO, Livine, Baratin, ……)

QFT of spacetime, not defined on spacetime

Quantum field theories over group manifold G (or corresponding Lie algebra) $\varphi : G^{\times d} \to \mathbb{C}$

relevant classical phase space for "GFT quanta": $(\mathcal{T}^* G)^{\times d} \simeq (\mathfrak{g} \times G)^{\times d}$

can reduce to subspaces in specific models

very general framework; interest rests on specific models (e.g. for QG models, G = Lorentz group, d = 4)

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can reduce to subspaces in specific models

very general framework; interest rests on specific models (e.g. for QG models, G = Lorentz group, d = 4)

$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym \left\{ \left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)} \right) \right\}$$

boson statistics is -assumption-

$$\mathcal{H}_v = L^2 \left(G^d ; d\mu_{Haar} \right)$$

$$\left[\hat{\varphi}(\vec{g}) \,,\, \hat{\varphi}^\dagger(\vec{g}') \right] = \mathbb{I}_G(\vec{g}, \vec{g}') \qquad \left[\hat{\varphi}(\vec{g}) \,,\, \hat{\varphi}(\vec{g}') \right] = \left[\hat{\varphi}^\dagger(\vec{g}) \,,\, \hat{\varphi}^\dagger(\vec{g}') \right] = 0$$

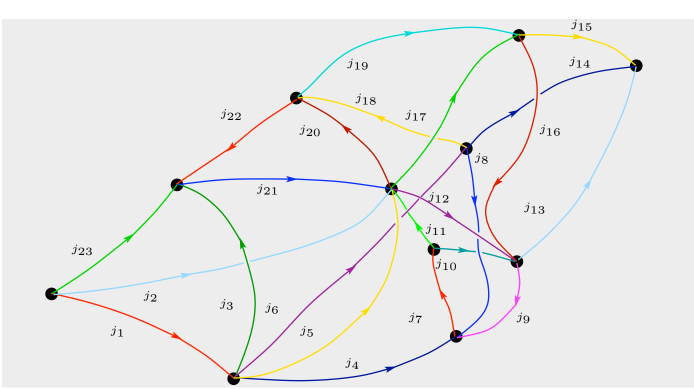
additional conditions (e.g. symmetries) on fields ⟶ restrictions on Hilbert space

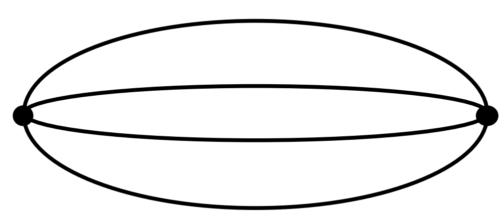
Group field theories

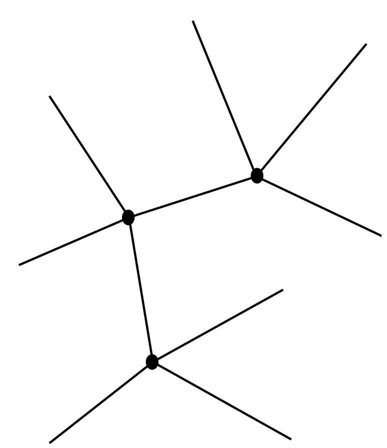
a QFT for the building blocks of (quantum) space

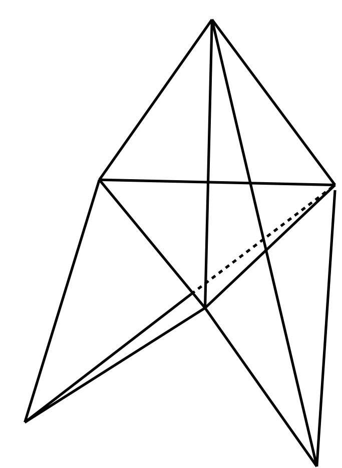
(d=4)

generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)







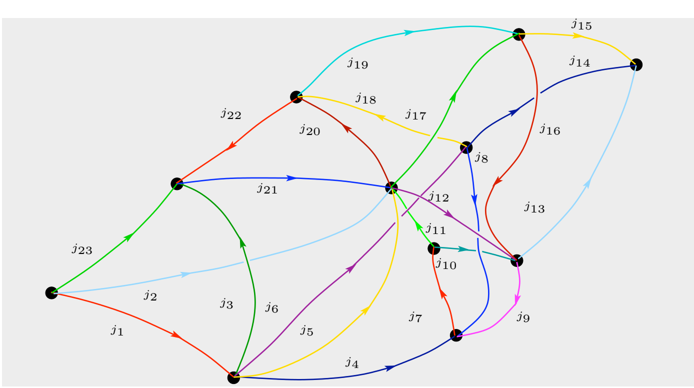


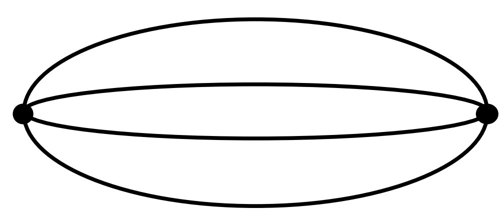
Group field theories

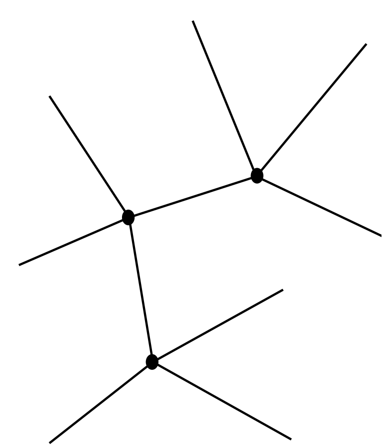
a QFT for the building blocks of (quantum) space

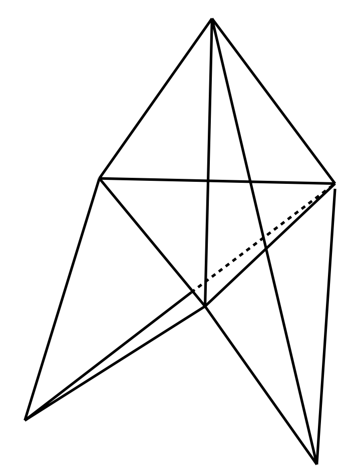
Fock vacuum: “no-space” (“emptiest”) state $|\, 0 >$ (d=4)

generic quantum state: arbitrary collection of spin network vertices (including glued ones) or tetrahedra (including glued ones)









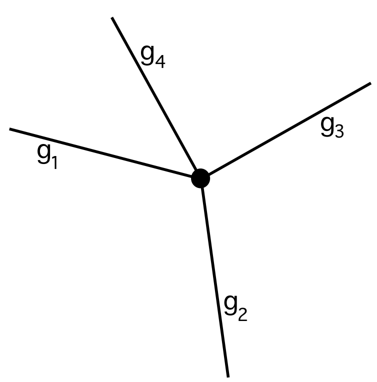
Group field theories

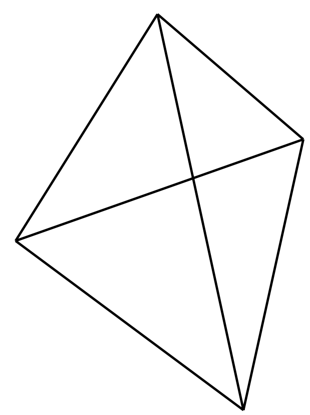
a QFT for the building blocks of (quantum) space

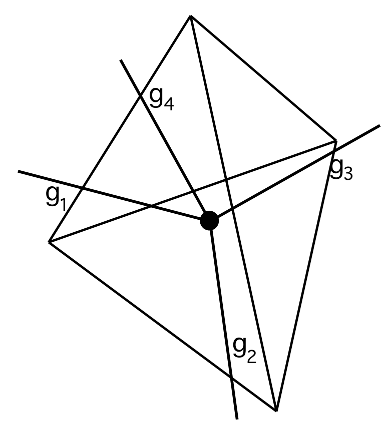
Fock vacuum: “no-space” (“emptiest”) state $|\,0>$ (d=4)

single field “quantum”: spin network vertex or tetrahedron
(“building block of space”)

$$\varphi(g_1, g_2, g_3, g_4) \leftrightarrow \varphi(B_1, B_2, B_3, B_4) \to \mathbb{C}$$







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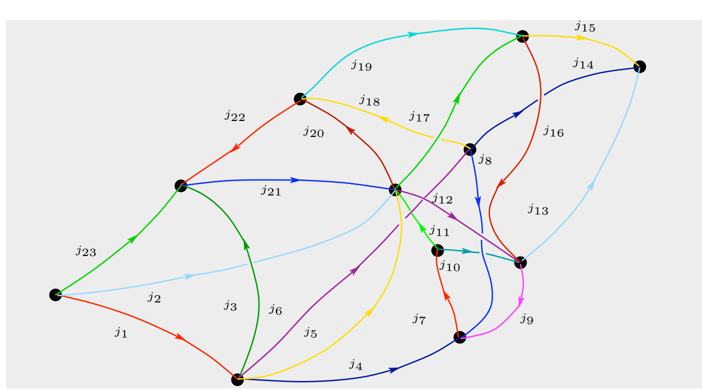

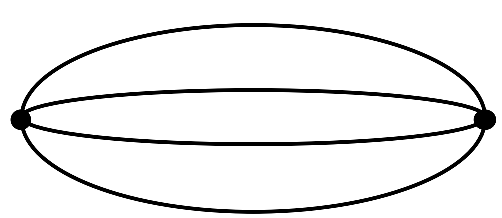

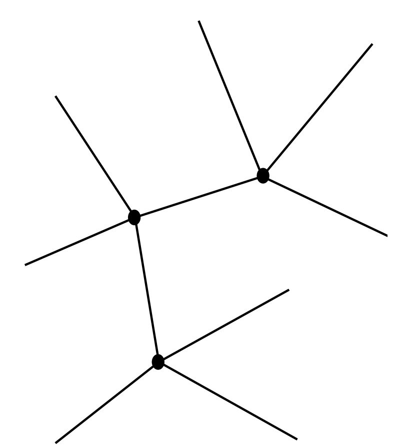

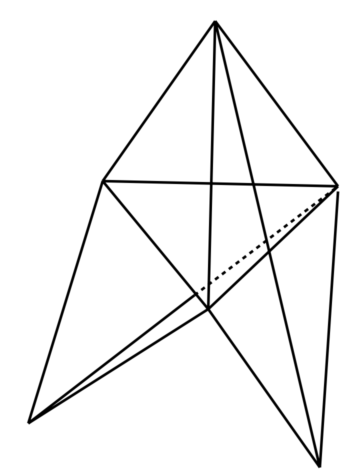

Group field theories

a QFT for the building blocks of (quantum) space

classical action: kinetic (quadratic) term + (higher order) interaction (convolution of GFT fields)

$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

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"combinatorial non-locality"
in pairing of field arguments

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specific combinatorics depends on model

simplest example (case d=4): simplicial setting

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"combinatorial non-locality" in pairing of field arguments

specific combinatorics depends on model

simplest example (case d=4): simplicial setting

combinatorics of field arguments in interaction: gluing of 5 tetrahedra across common triangles, to form 4-simplex ("building block of spacetime")

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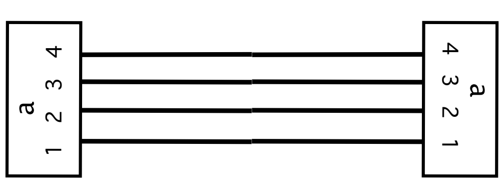
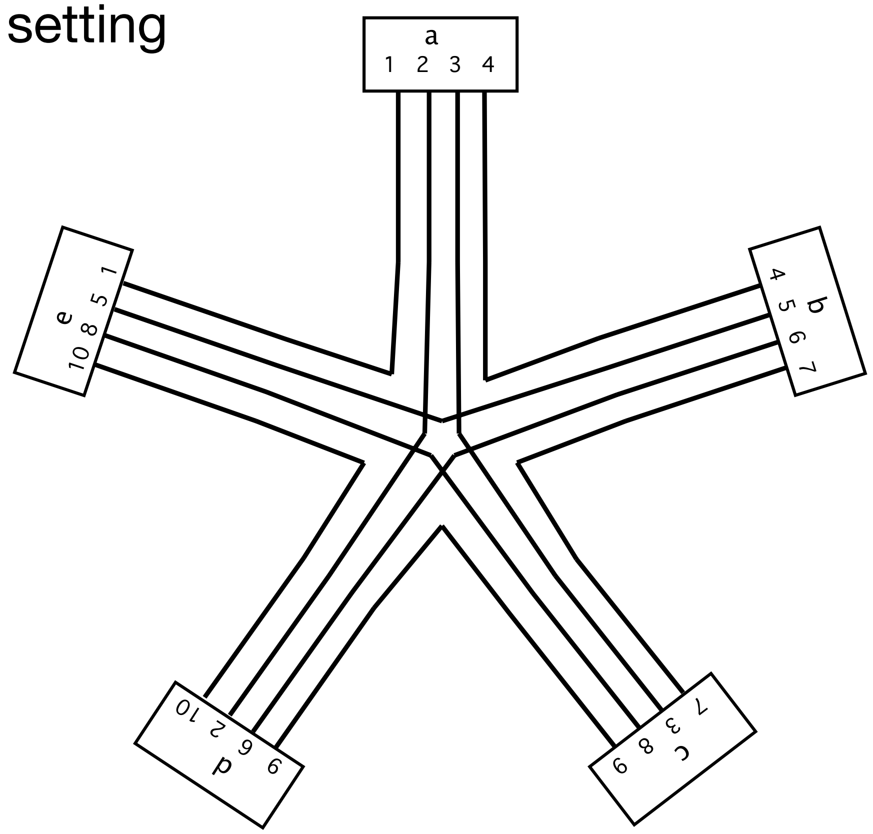
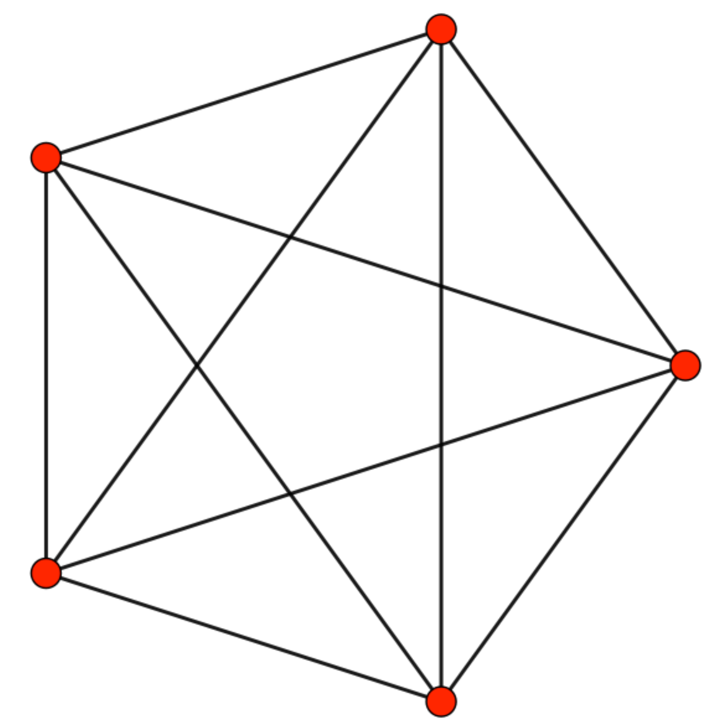






Group field theories

a QFT for the building blocks of (quantum) space

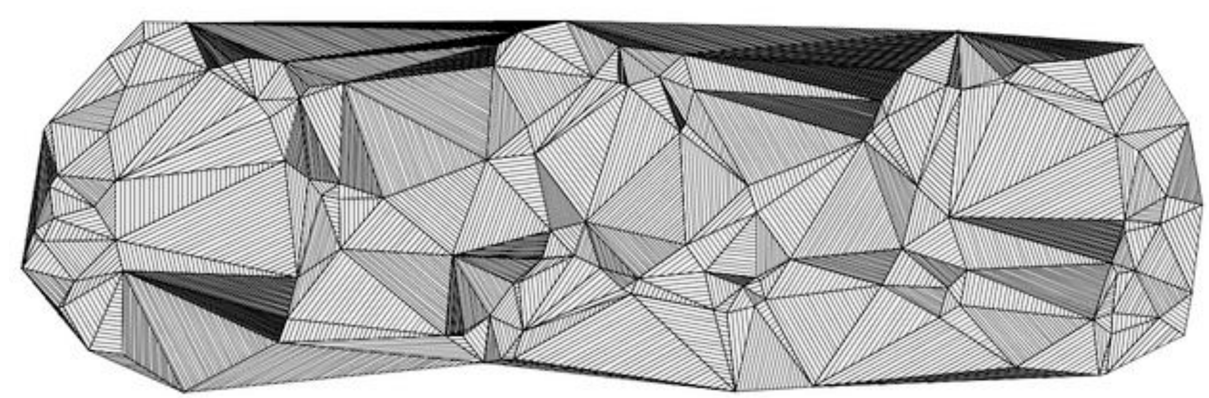
Feynman perturbative expansion around trivial vacuum

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$

Feynman diagrams (obtained by convoluting propagators with interaction kernels) =

= stranded diagrams dual to cellular complexes of arbitrary topology

(simplicial case: simplicial complexes obtained by gluing d-simplices)



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Feynman amplitudes (model-dependent):

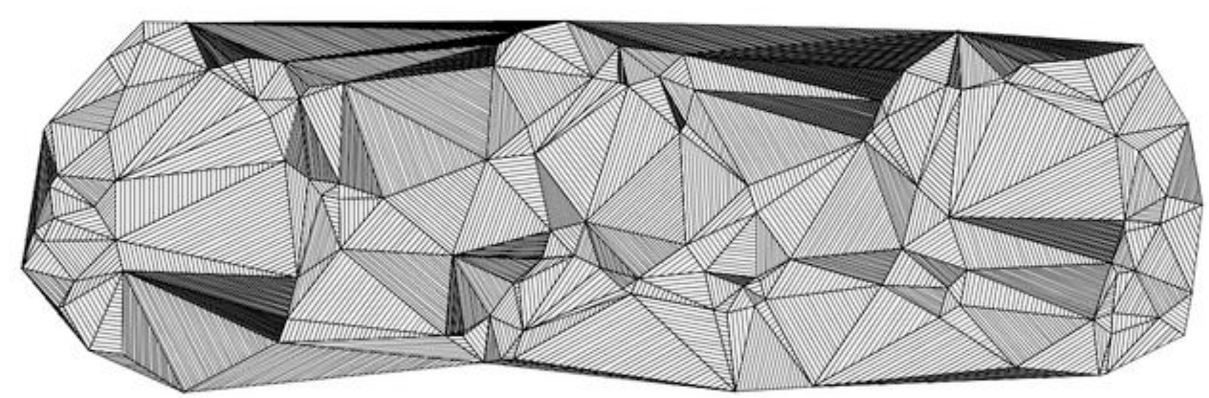
equivalently:

- spin foam models (sum-over-histories of spin networks ~ covariant LQG)

  Reisenberger,Rovelli, '00

- lattice path integrals (with group+Lie algebra variables)

  A. Baratin, DO, '11



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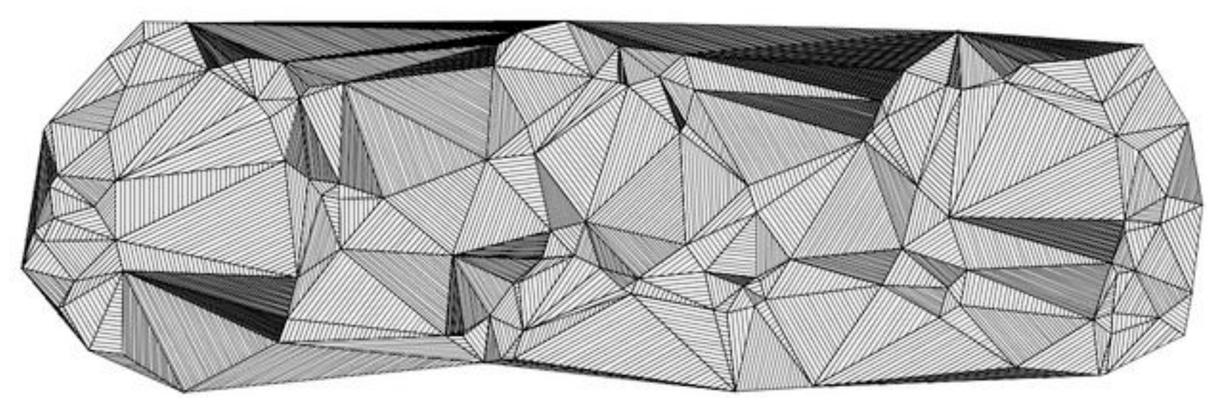


GFT as lattice quantum gravity:

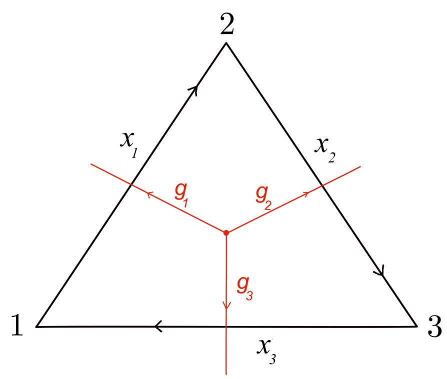
dynamical triangulations + quantum Regge calculus

Group field theory for 3d quantum gravity

$$\varphi : SU(2)^{\times 3} \to \mathbb{C}$$

$$S(\varphi) = \frac{1}{2}\int [dg] \varphi^2(g_1, g_2, g_3) + \frac{1}{4!}\int [dg] \varphi(g_1, g_2, g_3)\, \varphi(g_3, g_4, g_5)\, \varphi(g_5, g_2, g_6)\, \varphi(g_6, g_4, g_1) + \text{cc}$$

for fields satisfying: $\varphi(g_1, g_2, g_3) = \varphi(hg_1, hg_2, hg_3) \qquad \forall h \in SU(2)$



$$\varphi(g_1, g_2, g_3) \leftrightarrow \varphi(x_1, x_2, x_3)$$

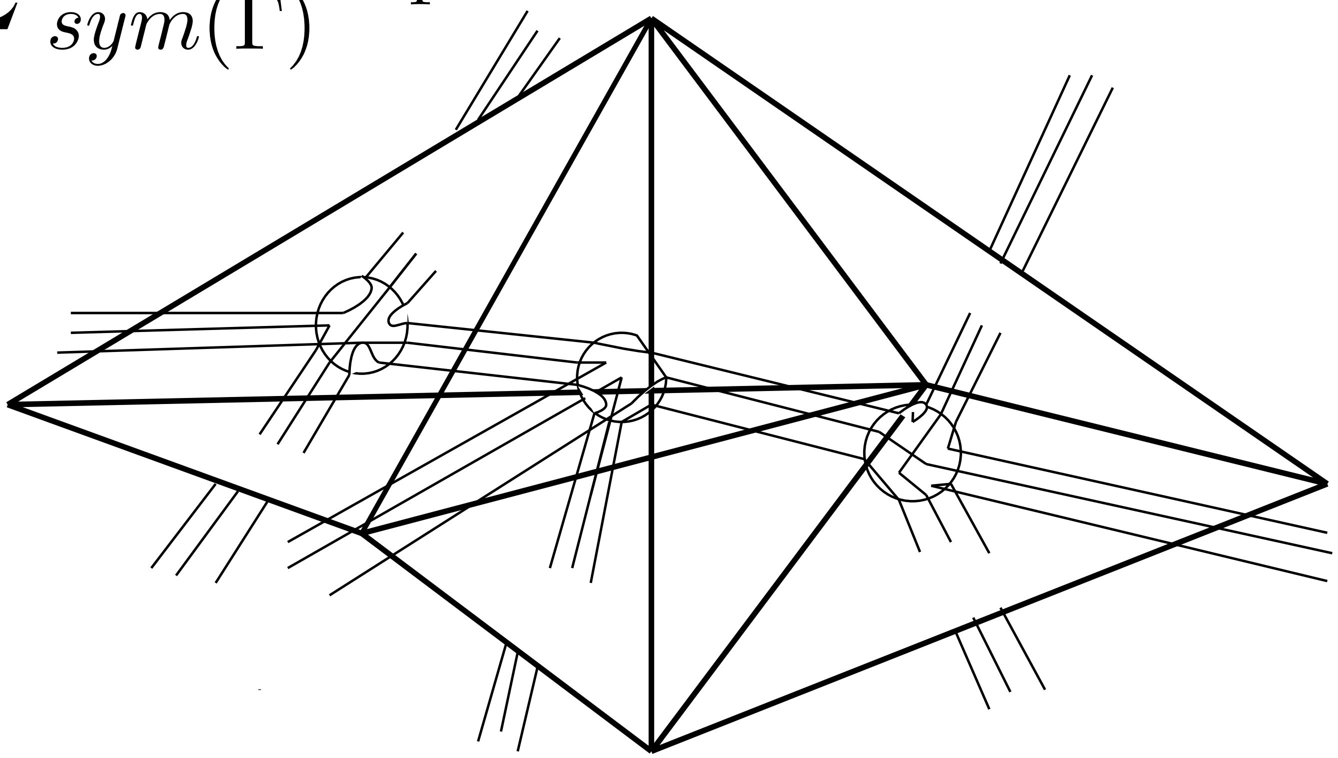
$$\mathcal{F}(\mathcal{H}_v) = \bigoplus_{V=0}^{\infty} sym \left\{ \left(\mathcal{H}_v^{(1)} \otimes \mathcal{H}_v^{(2)} \otimes \cdots \otimes \mathcal{H}_v^{(V)} \right) \right\}$$

$$\mathcal{H}_v = \mathcal{H}_{triangle} = Inv\left(\otimes_i \mathcal{H}_i^{SU(2)} \right)$$

many-body quantum states = quantised triangles (glued to one another)

Group field theory for 3d quantum gravity

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\, e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_{\Gamma} \frac{\lambda^{N_\Gamma}}{sym(\Gamma)}\, \mathcal{A}_\Gamma$$



Feynman amplitudes in different representations:

see talks by A. Riello and B. Dittrich

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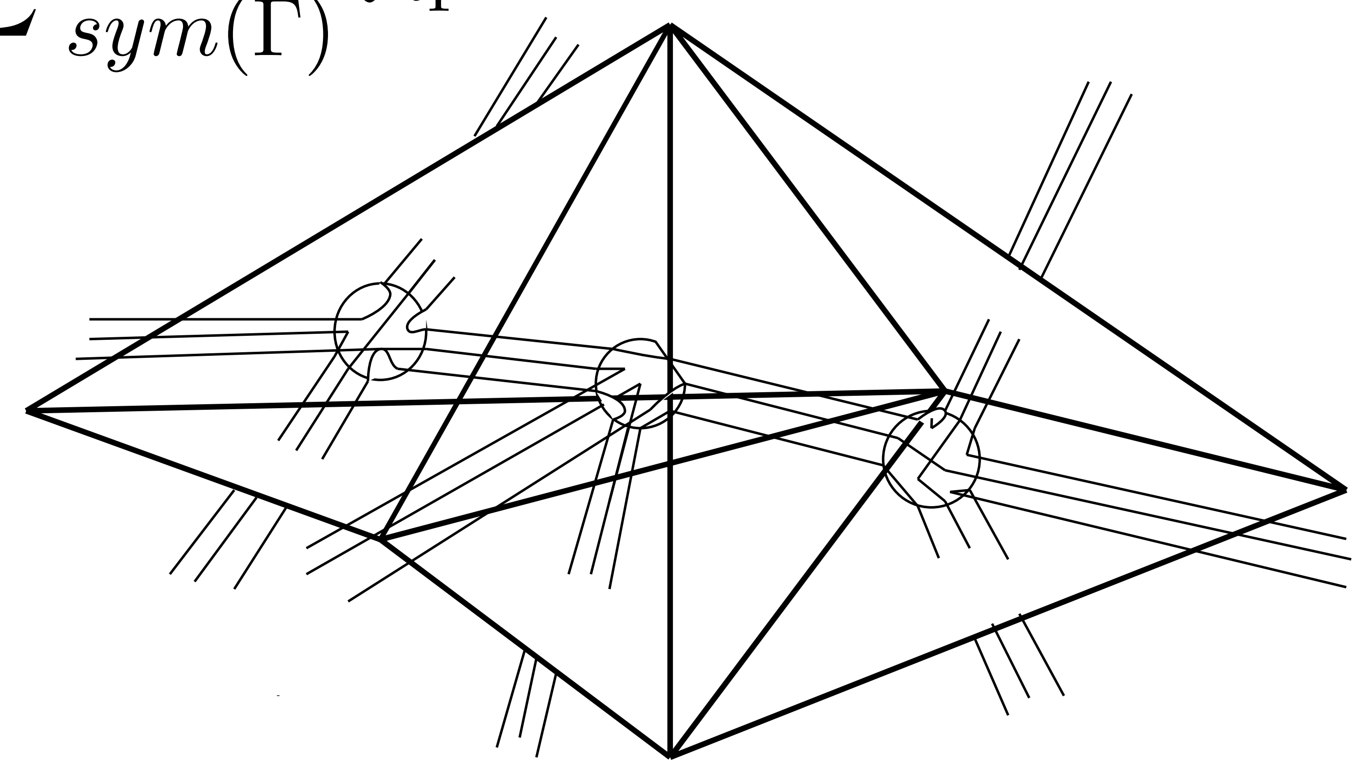

Feynman amplitudes in different representations:

$$\mathcal{A}_\Gamma \quad = \quad \int \prod_l \mathrm{d}h_l \prod_f \delta\left(H_f(h_l)\right) = \int \prod_l \mathrm{d}h_l \prod_f \delta\left(\overrightarrow{\prod}_{l\in\partial f} h_l\right) =$$

$$= \quad \sum_{\{j_e\}} \prod_e d_{j_e} \prod_\tau \left\{ \begin{array}{ccc} j_1^\tau & j_2^\tau & j_3^\tau \\ j_4^\tau & j_5^\tau & j_6^\tau \end{array} \right\} = \int \prod_l [\mathrm{d}h_l] \prod_e [\mathrm{d}^3 x_e]\, e^{i \sum_e \mathrm{Tr}\, x_e H_e}$$

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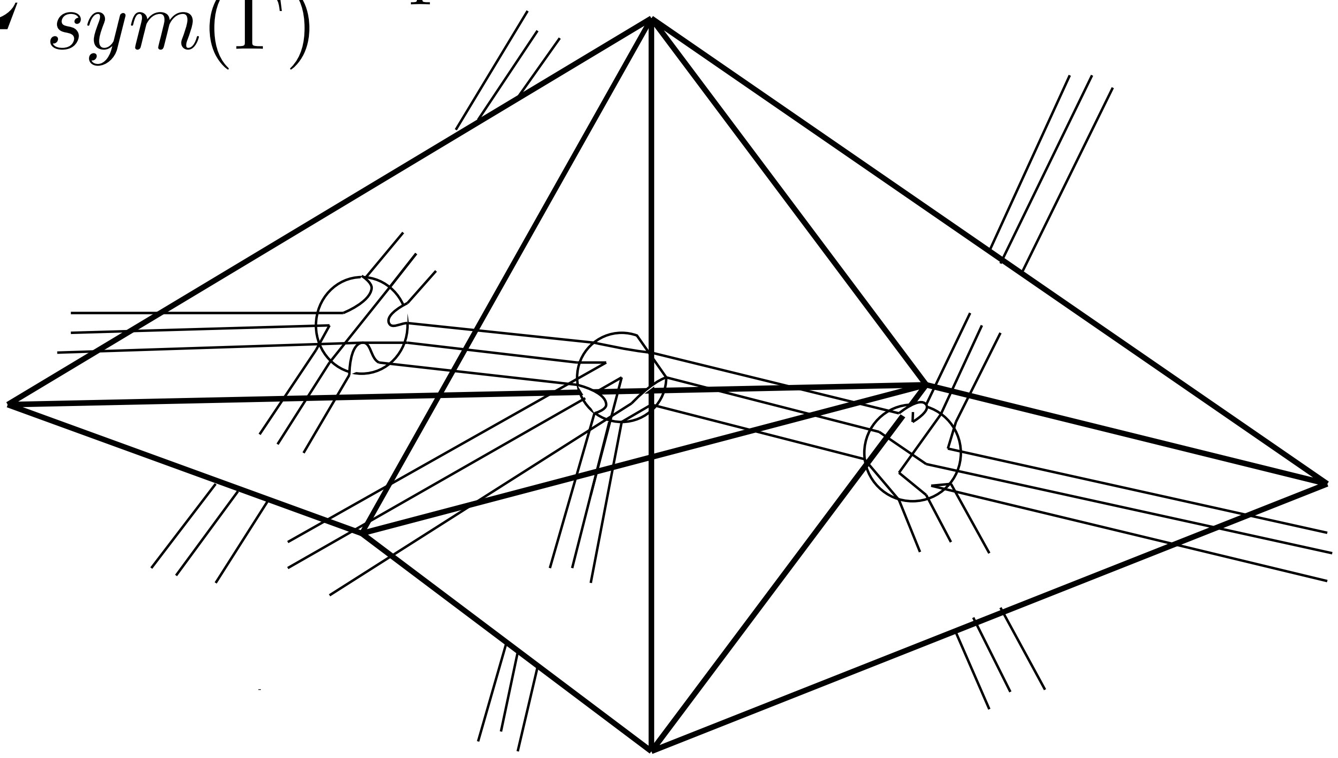

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lattice gauge theory formulation of 3d gravity/BF theory

$$= \sum_{\{j_e\}} \prod_e d_{j_e} \prod_\tau \left\{ \begin{array}{ccc} j_1^\tau & j_2^\tau & j_3^\tau \\ j_4^\tau & j_5^\tau & j_6^\tau \end{array} \right\} = \int \prod_l [\mathrm{d}h_l] \prod_e [\mathrm{d}^3 x_e]\, e^{i \sum_e \mathrm{Tr}\, x_e H_e}$$

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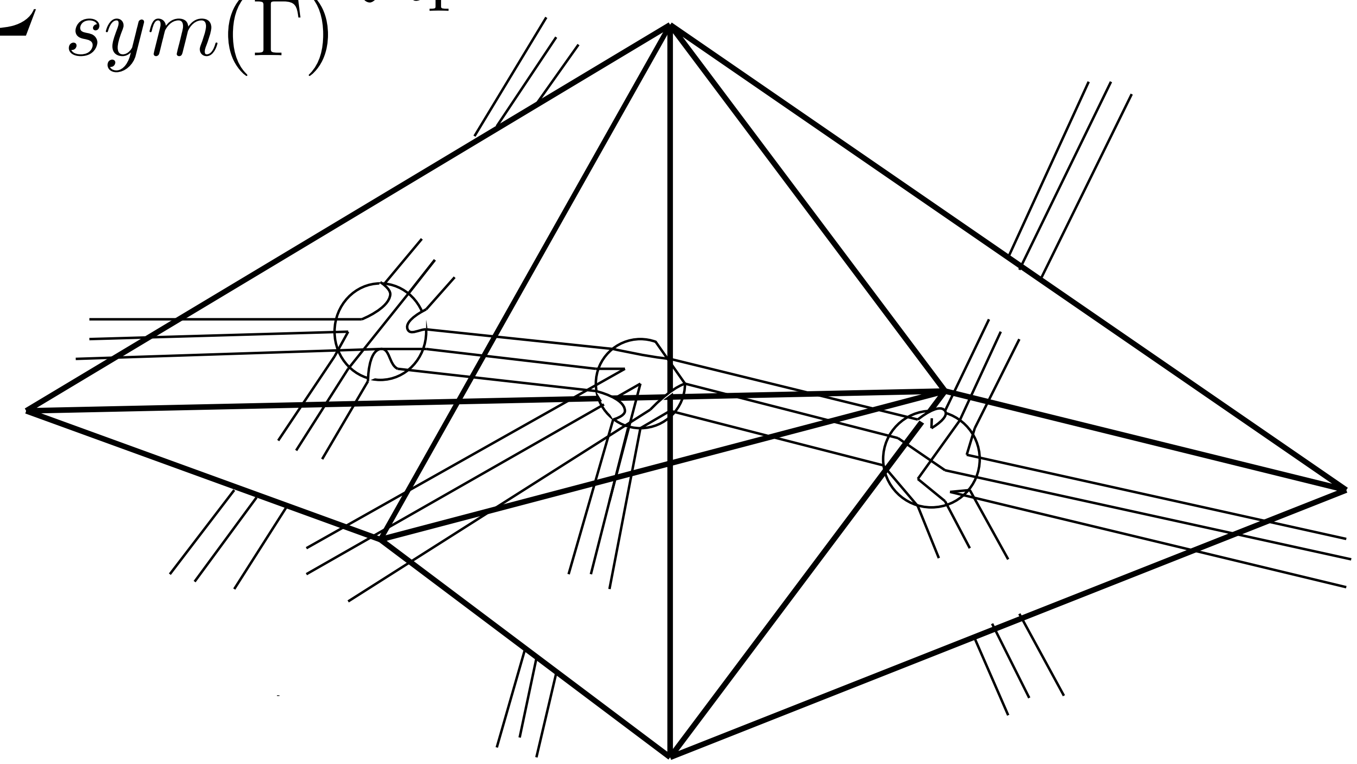

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spin foam formulation of 3d gravity

see talks by A. Riello and B. Dittrich

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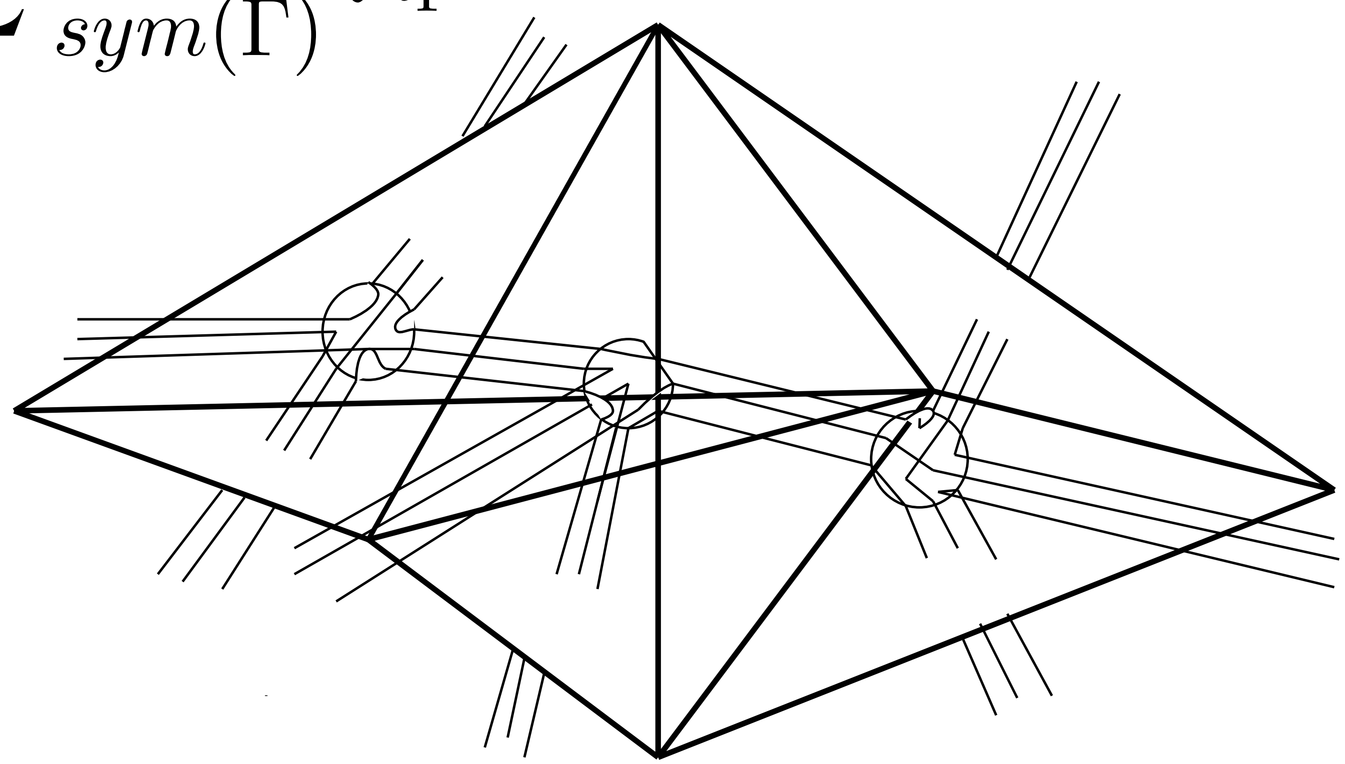

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lattice gauge theory formulation of 3d gravity/BF theory

spin foam formulation of 3d gravity

discrete 1st order path integral for 3d gravity on simplicial complex dual to GFT Feynman diagram

see talks by A. Riello and B. Dittrich

Group field theory for 4d quantum gravity

starting from GFT model for 4d BF theory (here in Lie algebra/bivector variables, with simplicial interactions)

$$S[\phi] = \frac{1}{2}\int dB \int dN \, [\phi(B_1, B_2, B_3, B_4; N)]^{\star 2} + \frac{\lambda}{5!}\int dB \int dN \, [\phi(B_1, B_2, B_3, B_4; N_1) \star \phi(B_4, B_5, B_6, B_7; N_2)$$

$$\star \phi(B_7, B_3, B_8, B_9; N_3) \star \phi(B_9, B_6, B_2, B_{10;N_4}) \star \phi(B_{10}, B_8, B_5, B_1; N_5)]$$

and adding "geometricity constraints" to the dynamics

$$B_i^{IJ} \in \wedge^2 \mathbb{R}^4 \simeq \mathfrak{so}(4)\,,\; N^I \in S^3 \subset \mathcal{T}\mathbb{R}^4 \qquad N_I\,(*B_i^{IJ}) = 0 \qquad \sum_i B_i^{IJ} = 0$$

$$B_i^{IJ} \simeq N^I \wedge b_i^J$$

one can define GFT models whose Feynman amplitudes are simplicial gravity path integrals, e.g.

A. Baratin, DO, '11

$$\mathcal{A}_\Delta = \int [d^6 B_t][dN_\tau] \mathcal{D}_\beta^{B_t, N_\tau}[h_{\tau\sigma}] \star \prod_t \left[e^{i\, tr[B_t\, H_t]} \star \delta_{-N_{\tau_o(t)} B_t^- N_{\tau_o(t)}^{-1}}(\beta B_t^+) \right]$$

equivalently re-written as lattice gauge theory or spin foam model

GFT and holography: first contacts

GFTs and tensor models

(Ambjorn, Durhuus, Sasakura, …, Gurau, Rivasseau, Bonzom, Ryan, …..)

same combinatorics, no group-theoretic data

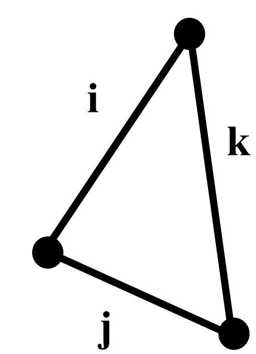
purely combinatorial amplitudes ~ lattice gravity path integrals on equilateral triangulations

example: d=3

dropping group/algebra data
(or restricting to finite group)

$$\varphi(g_1, g_2, g_3) : G^{\times 3} \to \mathbb{C} \quad \longrightarrow \quad \begin{array}{l} T_{ijk} : \mathbb{Z}_N^{\times 3} \to \mathbb{C} \\ T_{ijk} : X^{\times 3} \to \mathbb{C} \end{array} \qquad X = 1, 2, ..., N$$

$$S(T) = \frac{1}{2} \sum_{i,j,k} T_{ijk} T_{kji} - \frac{\lambda}{4! \sqrt{N^3}} \sum_{ijklmn} T_{ijk} T_{klm} T_{mjn} T_{nli}$$



Quantum dynamics:

$$Z = \int \mathcal{D}T \, e^{-S(T,\lambda)} = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} Z_\Gamma = \sum_{\Gamma} \frac{\lambda^{V_\Gamma}}{sym(\Gamma)} N^{F_\Gamma - \frac{3}{2} V_\Gamma}$$

can be recast in terms of Regge action for gravity discretised on equilateral triangulation

recently used also in the context of SYK model and AdS/CFT

Witten, Klebanov, Gurau, Rosenhaus, Verlinde, ……

Tensorial (G)FTs, SYK model and holography

Witten, Klebanov, Gurau, Rosenhaus, Verlinde, ...

- **Sachdev-Ye-Kitaev** models = disordered systems of N Majorana fermions

  [Sachdev, Ye, George, Parcollet '90s...; Kitaev '15, Maldacena, Stanford, Polchinski, Rosenhaus...]

$$H_{\rm int} \sim J_{i_1 i_2 i_3 i_4}\, \psi_{i_1} \psi_{i_2} \psi_{i_3} \psi_{i_4} \,, \qquad \left\langle J_{i_1 i_2 i_3 i_4} \right\rangle \sim 0 \,, \quad \left\langle J^2_{i_1 i_2 i_3 i_4} \right\rangle \sim \frac{J^2}{N^3}$$

many related models have been constructed (bosonic, supersymmetric, different dimension, etc)

  - Many interesting properties:
    - **solvable at large N**
    - emergent **conformal symmetry** at **strong coupling**
    - maximal **quantum chaos**
    - holography in low dimension: "$NAdS_2/NCFT_1$"
- Same **melonic large N limit** as tensor models [Witten '16]

→ **SYK-like quantum-mechanical models**:

- same qualitative properties at large N and strong coupling;
- **no disorder**.

→ New class of **QFTs** with solvable large N limits. tensorial (G)FT models that capture the same physics

[Gurau, Bonzom, Rivasseau,... 10s; Tanasa, SC '15] [Klebanov, Tarnopolsky '16...] [...]

GFT states as generalised tensor networks

Chirco, DO, Zhang, ‘17

Quantum states in many-body systems conveniently encoded in tensor networks Vidal, ‘06
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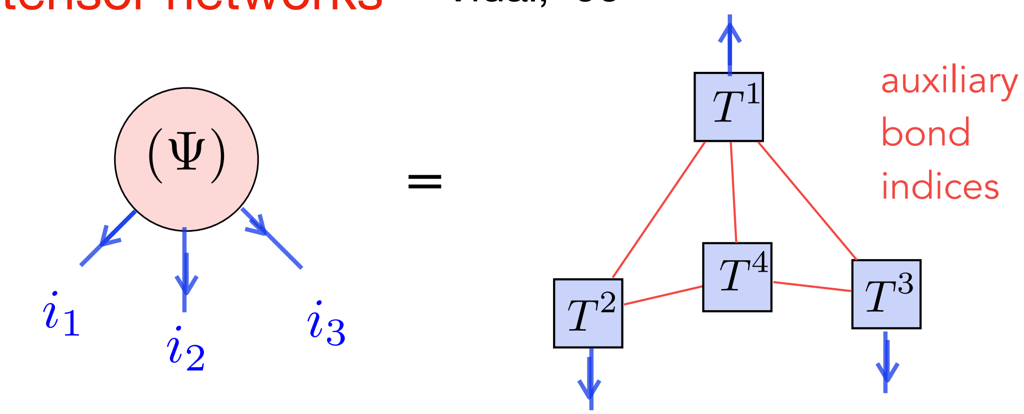

Efficient encoding of entanglement properties

Saturate entropy bounds (RTN, in large bond limit)

Holographic features (use in AdS/CFT, ….)

Swingle, ’09; van Raamsdonk ’09; ……; Hayden et al. ’16

e.g. Ryu-Takanayagi entropy formula

$$S_A = \frac{\text{Area}(\gamma_A)}{4G_N}$$

Ryu-Takanayagi, ’12;
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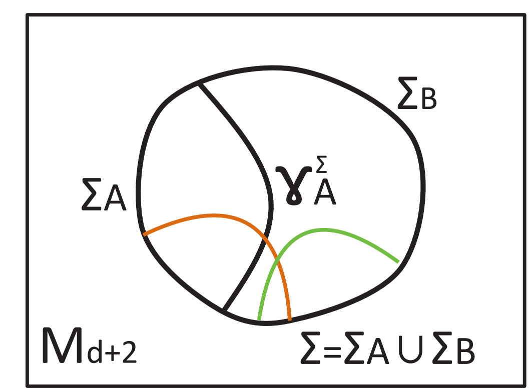

GFT states as generalised tensor networks

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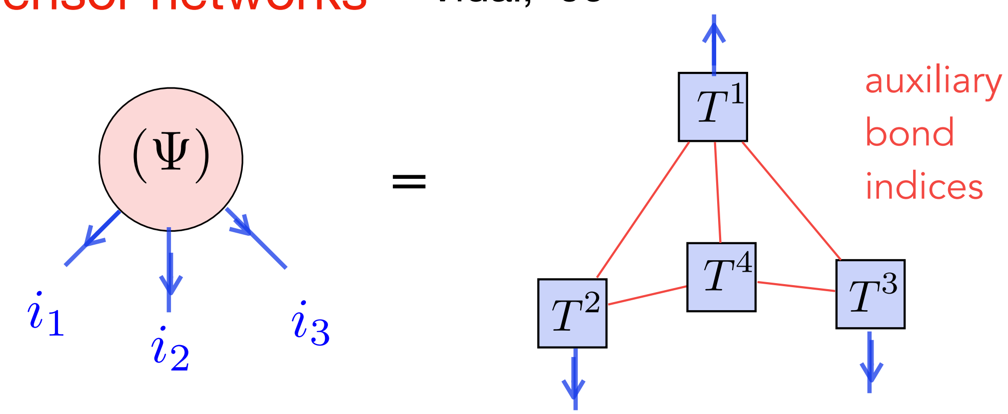

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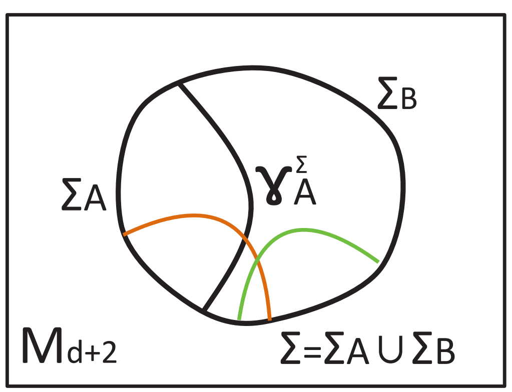

Spin networks (for fixed and equal spins) are a special case of tensor networks (with local gauge symmetry)

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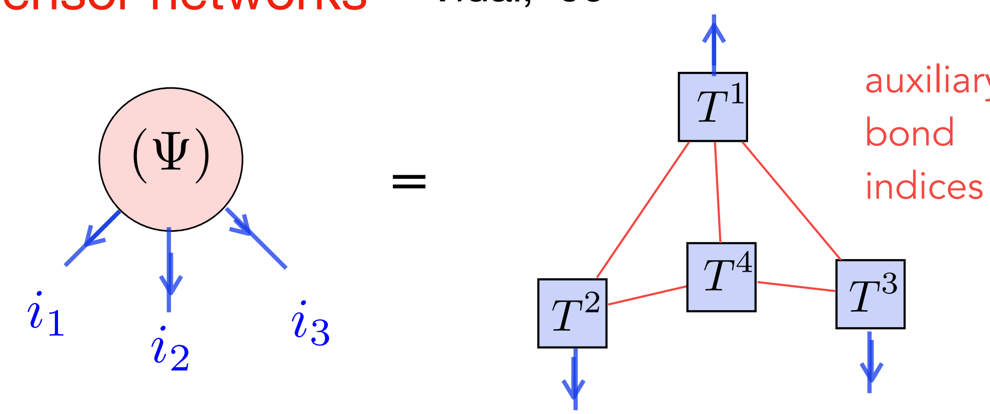

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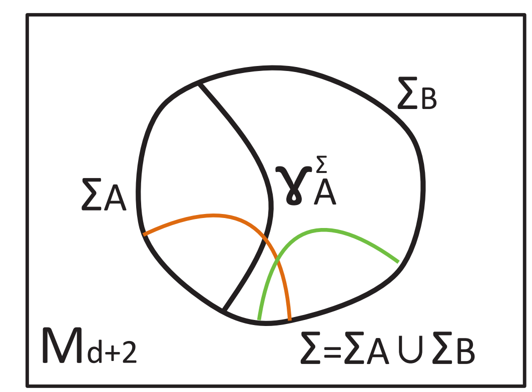

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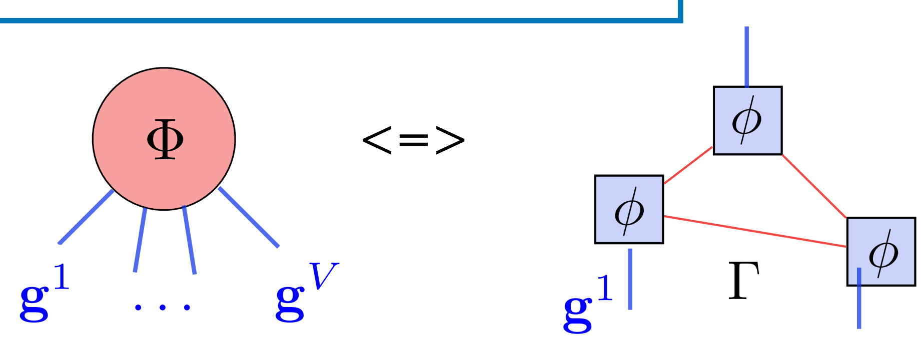
Group field theory states are a field-theoretic generalization of random tensor networks - GFT dynamics defines probability measure

$$|\Phi_{\mathcal{N}}\rangle \equiv \bigotimes_{\ell \in \mathcal{N}} \langle M_\ell| \bigotimes_{n}^{V} |\phi_n\rangle \in \bigotimes_{\ell \in \partial\mathcal{N}} \mathcal{H}_\ell$$

$$\frac{1}{Z} d\nu(\varphi)$$



Towards Ryu-Takanayagi formula in full QG

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(large) open spin network GFT state (written as random tensor network)

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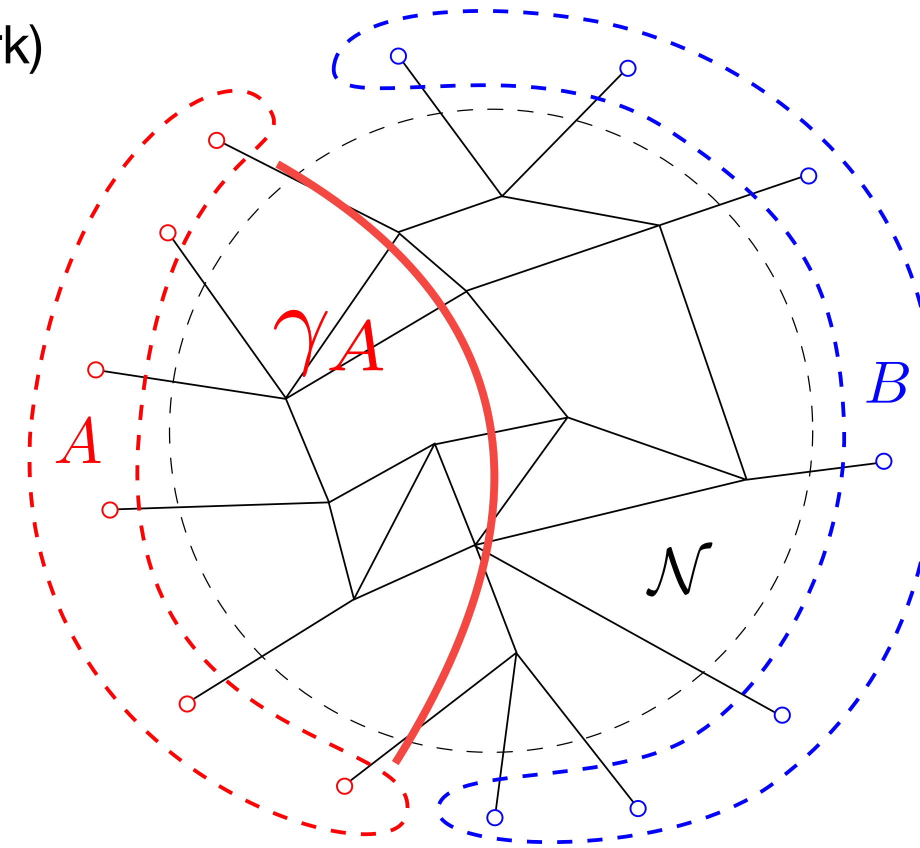
corresponding density operator

$$\rho = \mathrm{tr}_\ell \left[\bigotimes_{\ell\in\Gamma} |M_\ell\rangle\langle M_\ell| \bigotimes_{v}^{V} |\phi_v\rangle\langle\phi_v| \right]$$

reduced density operator associated to boundary sub-region A

$$\hat{\rho}_A = \mathrm{tr}_B[\rho]/\mathrm{tr}[\rho]$$



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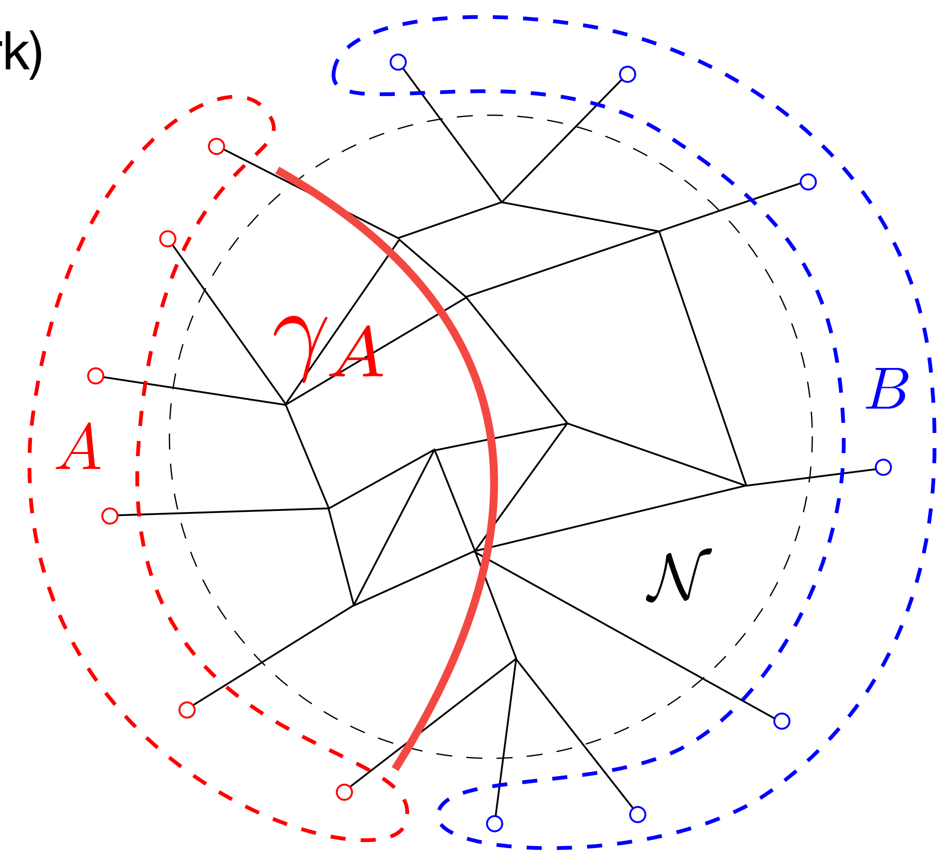

Goal: entanglement entropy between sub-regions A and B

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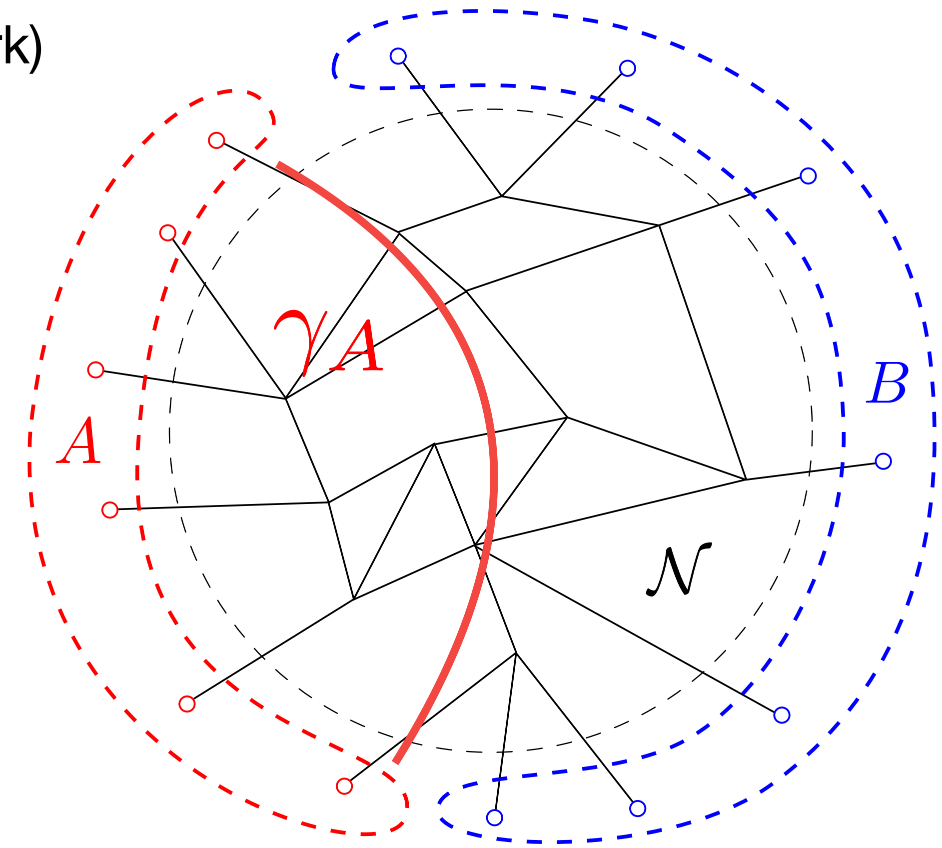

Goal: entanglement entropy between sub-regions A and B

$$S_{EE} = -\mathrm{tr}[\hat{\rho}_A \log \hat{\rho}_A] = \lim_{N\to 1} S_N(A) = \frac{1}{1-N} \log \mathrm{tr}[\hat{\rho}_A^N]$$

from Reny entropy (via replica trick)

computation made easier by:

$$e^{-S_N(A)} = \mathrm{tr}[\rho_A^N]/(\mathrm{tr}[\rho])^N \equiv Z_A/Z_0$$

- random character: calculate expectation value
- large bond approx.: fluctuations are suppressed

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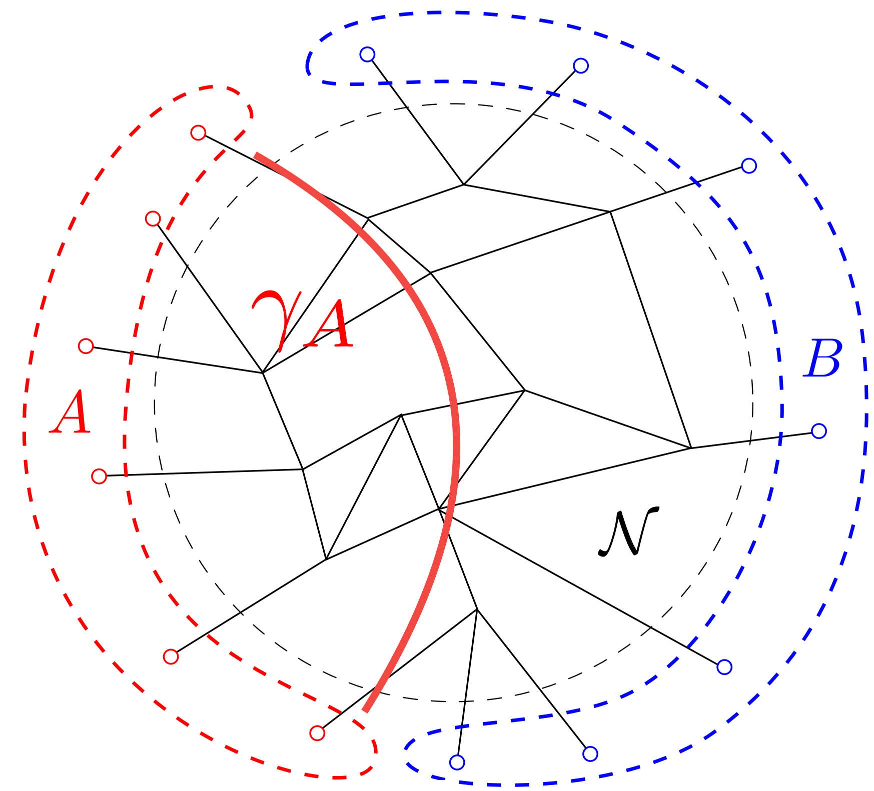

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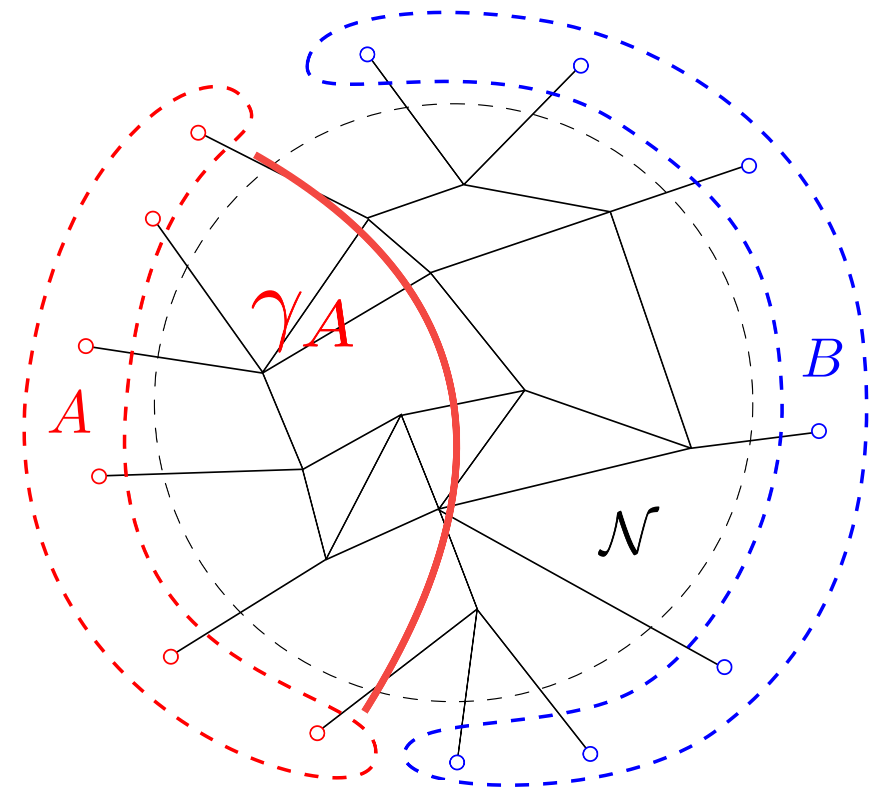

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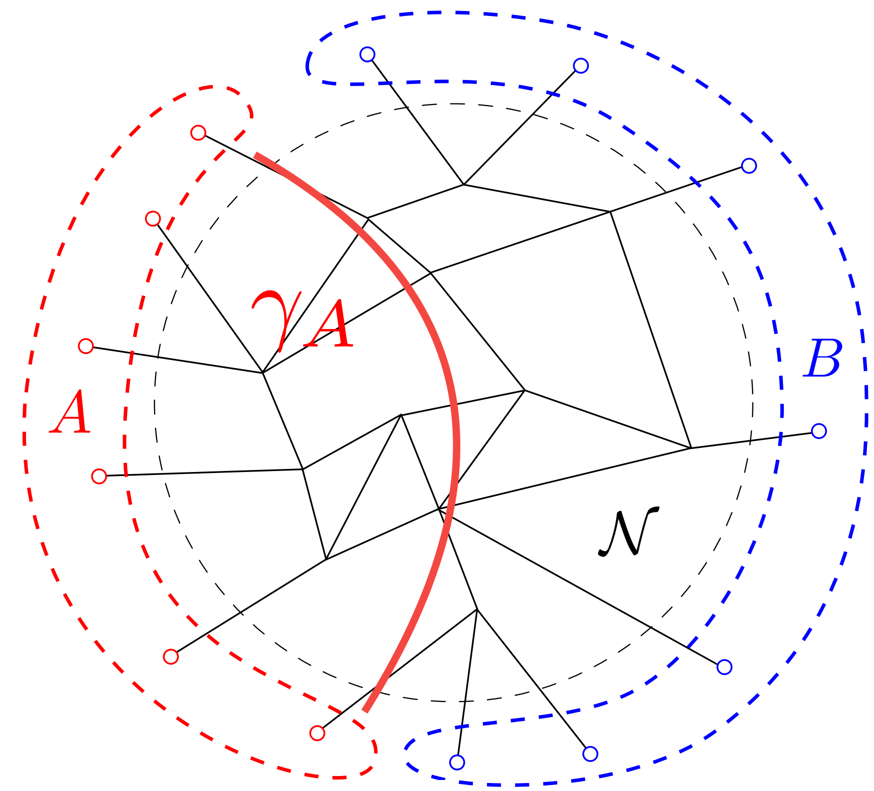

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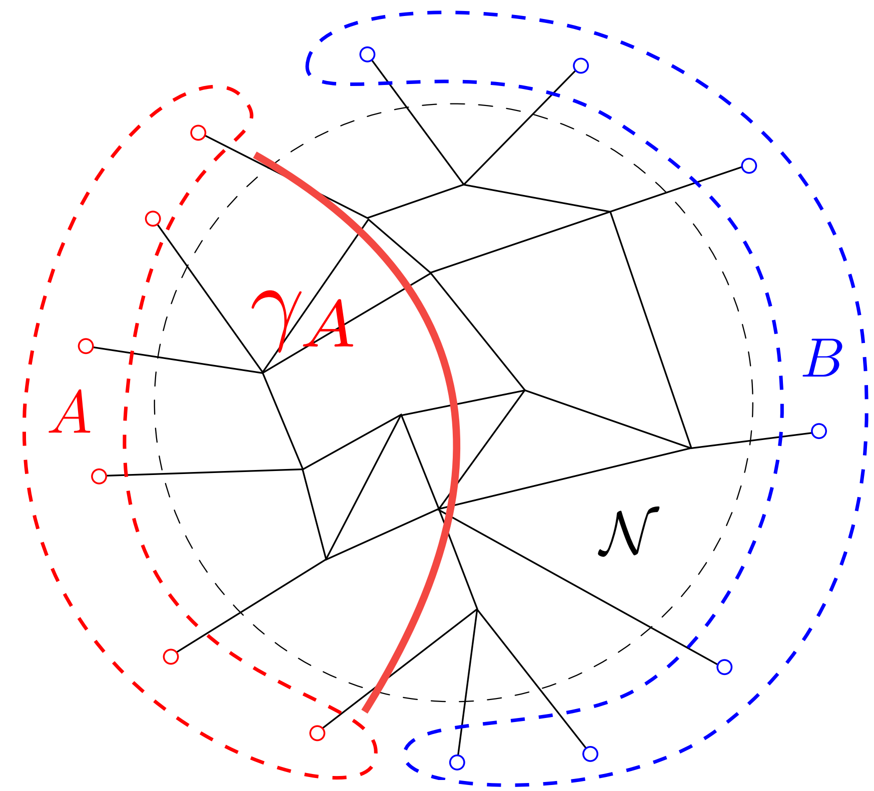

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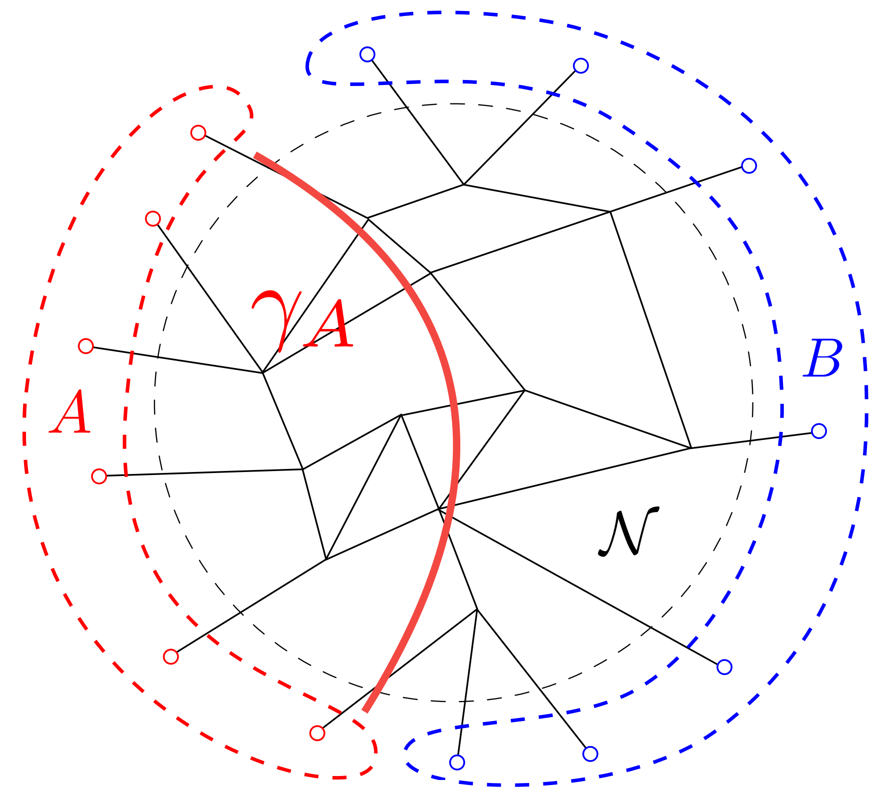

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can compute non-perturbative QG corrections…..

Continuum limit
of discrete quantum gravity

via (functional) GFT renormalization

Simplicial path integral for quantum gravity

$$\mathcal{Z} = \int \mathcal{D}\varphi \mathcal{D}\overline{\varphi}\; e^{i\, S_\lambda(\varphi,\overline{\varphi})} \quad = \quad \sum_\Gamma \frac{\lambda^{N_\Gamma}}{sym(\Gamma)} \mathcal{A}_\Gamma$$

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defining full simplicial path integral for quantum gravity = defining full GFT path integral for suitable model

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- restrictions on triangulations generated as FD?
- how to control it?
- fixed points, continuum phases and phase transitions?
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→ continuum limit of simplicial gravity path integral ~ RG flow of GFT model

Problem of the continuum in QG: role of RG

Renormalization Group is crucial tool

for taking into account the physics of more and more d.o.f.s

need to understand effective dynamics at different “GFT scales”:
RG flow of effective actions & phase structure & phase transitions Koslowski, ’07; DO, ‘07

many results in related formalisms:

- renormalization in SF models (~ lattice gauge theories)
  Dittrich, Bahr, Steinhaus, Martin-Benito, ……
- different (kinematical) phases in LQG Ashtekar-Lewandowski, Koslowski-Sahlmann, Dittrich-Geiller)
- phase diagrams in (causal) dynamical triangulations
  Ambjorn, Loll, Jurkiewicz, …..
- renormalization and phase diagram of tensor models Eichhorn, Koslowski, Ben Geloun, Bonzom, ….

GFT renormalisation - general scheme

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$$S(\varphi,\overline{\varphi}) = \frac{1}{2}\int [dg_i]\overline{\varphi(g_i)}\mathcal{K}(g_i)\varphi(g_i) + \frac{\lambda}{D!}\int [dg_{ia}]\varphi(g_{i1})....\varphi(\bar{g}_{iD})\mathcal{V}(g_{ia},\bar{g}_{iD}) \quad + \quad c.c.$$

general strategy:

treat GFTs as ordinary QFTs defined on Lie group manifold

use group structures (Killing form, topology, etc) to define notion of scale and to set up mode integration

subtleties of quantum gravity context at the level of interpretation

scales:

defined by propagator: e.g. spectrum of Laplacian on G = indexed by group representations

$$\sum_{\ell=1}^{d} j_\ell(j_\ell+1) \lesssim \Lambda^2$$

- need to have control over “theory space” (e.g. via symmetries) A. Kegeles, DO, ’15,’16
- main difficulty:

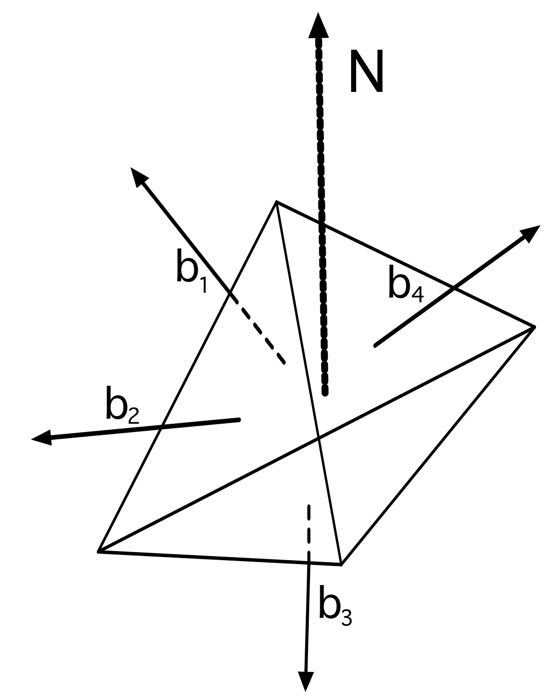
  controlling the combinatorics of GFT Feynman diagrams

  need to adapt/redefine many QFT notions: connectedness, subgraph contraction, Wick ordering, …..

GFT Renormalization: geometric interpretation?

arguments of GFT field: $b_i \in \mathfrak{su}(2)$ gravity case: d=4

I b I ~ J = irrep of SU(2) ~ “area of triangles”



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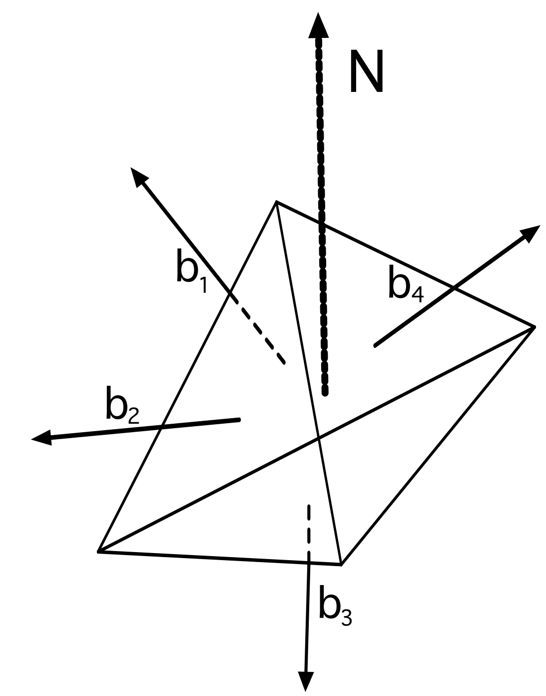

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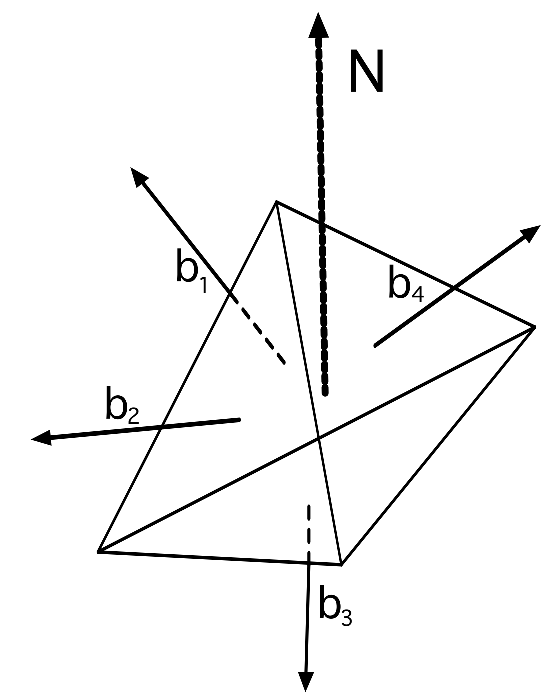

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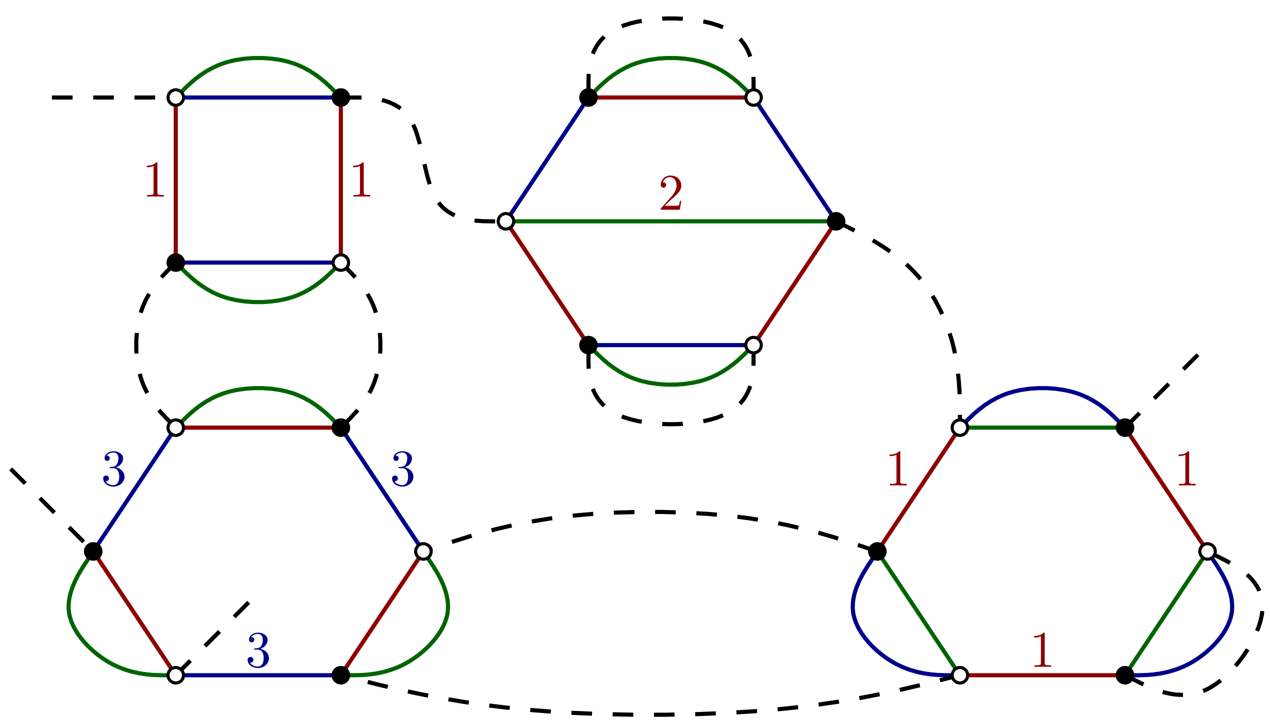
geometric intepretation?

from LQG
from Regge calculus

- CAUTION in interpreting things geometrically outside continuum geometric approx.
- expect "physical" continuum areas $A \sim < J > < n >$
- expect proper continuum geometric interpretation (and effective metric field)

  for $< J >$ small, $< n >$ large, A finite (not too small)
- from continuum geometric perspective, large areas are result of coarse graining of microscopic dofs

GFT Renormalization: combinatorics of FDs

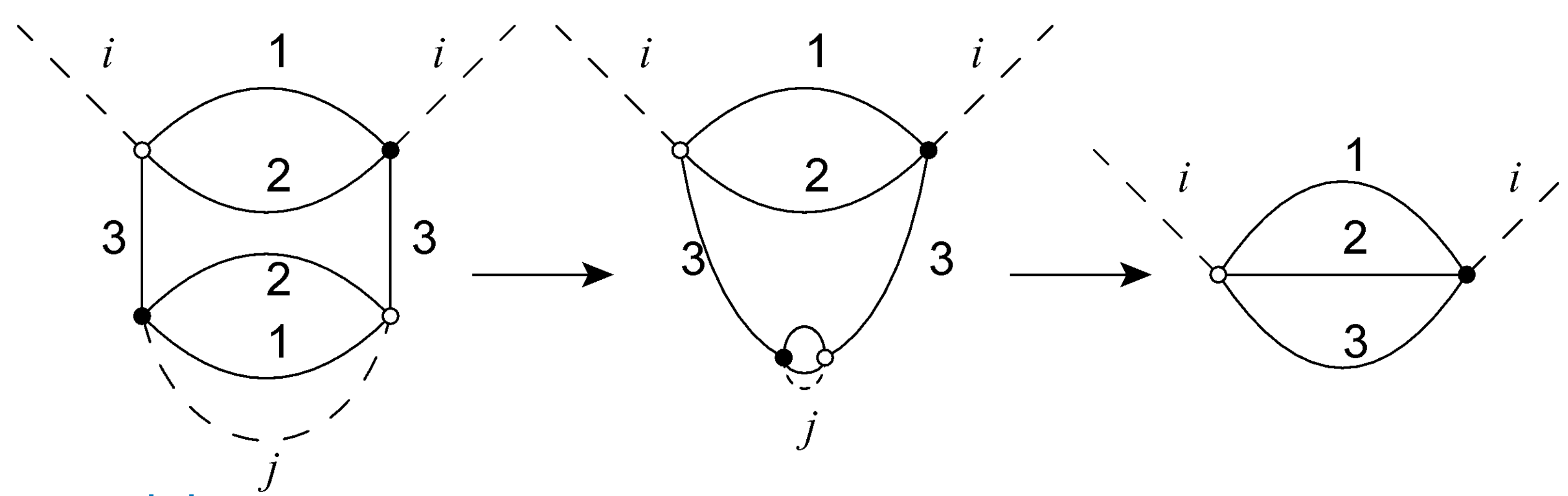


example of Feynman diagram in 4d (interaction process of tetrahedra ~ 4d simplicial complex)

Example: when $\mathrm{internal\ scales}\ j \gg \mathrm{external\ scales}\ i$

contraction of (divergent) subgraphs + absorption in effective vertices is coarse-graining of simplicial lattices

(perturbative) GFT renormalization = renormalization of lattice gravity path integral



spin foam amplitude consistency under coarse graining = RG consistency of GFT Feynman amplitudes

see Bianca's talk

FRG analysis of GFT models

D. Benedetti, J. Ben Geloun, DO, '14

regularised path integral: $$\mathcal{Z}_k[J,\overline{J}] = e^{W_{k'}[J,\overline{J}]} = \int d\phi d\overline{\phi}\; e^{-S[\phi,\overline{\phi}]-\Delta S_{k'}[\phi,\overline{\phi}]+\mathrm{Tr}(J\cdot\overline{\phi})+\mathrm{Tr}(\overline{J}\cdot\phi)}$$

regulator cutting off IR modes (UV well-defined with appropriate choice of IR regulator)

$$\Delta S_k[\phi,\overline{\phi}] = \mathrm{Tr}(\overline{\phi}\cdot R_k\cdot\phi) = \sum_{\mathbf{P},\mathbf{P}'} \overline{\phi}_{\mathbf{P}}\, R_k(\mathbf{P};\mathbf{P}')\,\phi_{\mathbf{P}'}$$

effective action: $$\Gamma_k[\varphi,\overline{\varphi}] = \sup_{J,\overline{J}}\left\{\mathrm{Tr}(J\cdot\overline{\varphi}) + \mathrm{Tr}(\overline{J}\cdot\varphi) - W_k[J,\overline{J}] - \Delta S_k[\varphi,\overline{\varphi}]\right\}$$

Wetterich equation: $$\boxed{\partial_t\Gamma_k = \mathrm{Tr}[\partial_t R_k\cdot(\Gamma_k^{(2)} + R_k)^{-1}]} \qquad t = \log k$$

boundary conditions: $$\Gamma_{k=0}[\varphi,\overline{\varphi}] = \Gamma[\varphi,\overline{\varphi}]\,, \qquad \Gamma_{k=\Lambda}[\varphi,\overline{\varphi}] = S[\varphi,\overline{\varphi}] \qquad \varphi = \langle\phi\rangle$$

computing the effective action solving the Wetterich equation amounts to solving the GFT path integral

Renormalization flow of GFT models - 3d example

- consider GFT model for 3d gravity:

$$S(\varphi) = \frac{1}{2}\int [dg]|\varphi|^2(g_1,g_2,g_3) + \frac{1}{4!}\int [dg]\varphi(g_1,g_2,g_3)\,\varphi(g_3,g_4,g_5)\,\varphi(g_5,g_2,g_6)\,\varphi(g_6,g_4,g_1) \text{ + cc}$$

$\varphi : SU(2)^{\times 3} \to \mathbb{C}$ for fields satisfying: $\varphi(g_1,g_2,g_3) = \varphi(hg_1,hg_2,hg_3) \qquad \forall h \in SU(2)$

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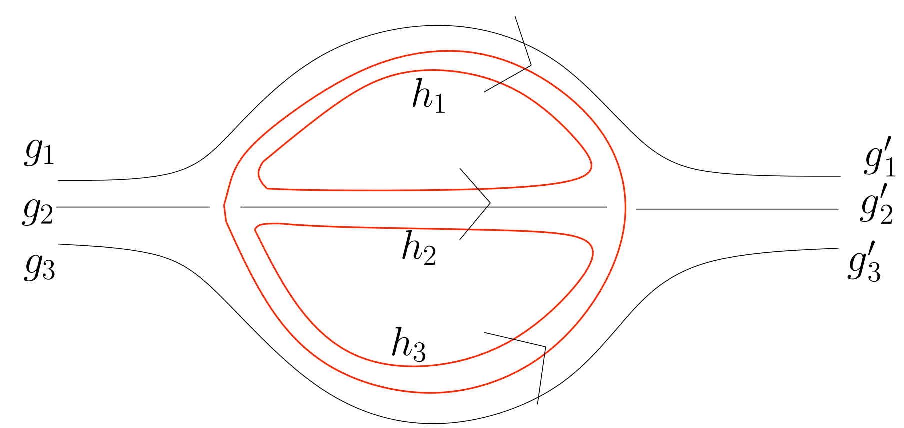
- radiative corrections generate non-trivial kinetic term

Ben Geloun, Bonzom, '11; Ben Geloun, '13



kinetic term = e.g. Laplacian on SU(2)

propagator $$\left(m^2 - \sum_{\ell=1}^{d} \Delta_\ell\right)^{-1}$$

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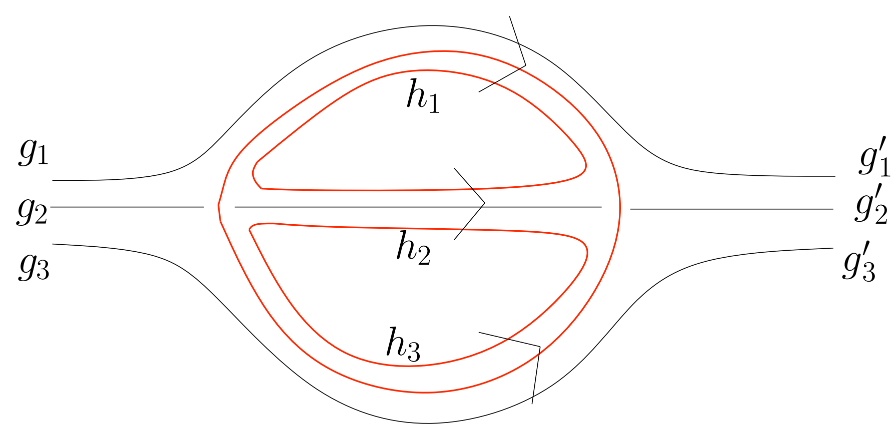

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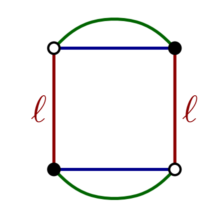
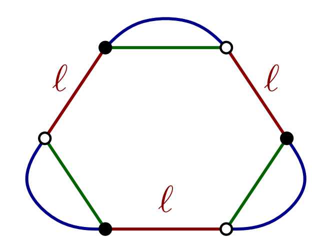


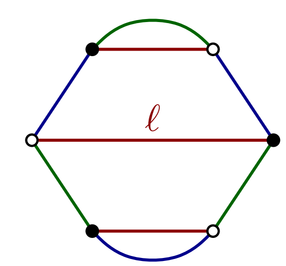
- interactions generate effective terms associated to "bubbles"



"tensor invariants" $S(\varphi, \overline{\varphi}) = \sum_{b \in \mathcal{B}} t_b I_b(\varphi, \overline{\varphi})$

large "tensorial" theory space

indexed by bipartite 3-colored graphs ("bubbles") ~
dual to 3-cells with triangulated boundary



Renormalization flow of GFT models - 3d example

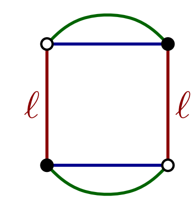
S.Carrozza, D. Oriti, V. Rivasseau, ’13; S. Carrozza, V. Lahoche, ‘16

this suggests to consider general models of “tensorial” type - example: d=3, G=SU(2)

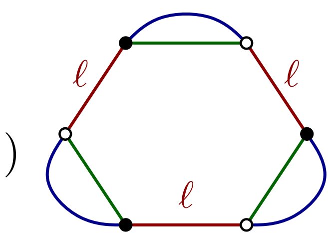
$$\mathcal{Z}_\Lambda := \int \mathrm{d}\mu_{C_\Lambda}[\overline{\psi},\psi]\, e^{-S_\Lambda[\bar{\psi},\psi]}$$

Gaussian measure

$$S_\Lambda[\psi,\overline{\psi}] = \frac{\lambda_4(\Lambda)}{2}\sum_{\ell=1}^{3}\int[\prod_{j=1}^{4}\mathrm{d}\mathbf{g}_j]\,\mathcal{W}^{(\ell)}(\mathbf{g}_1,\mathbf{g}_2,\mathbf{g}_3,\mathbf{g}_4)\psi(\mathbf{g}_1)\bar{\psi}(\mathbf{g}_2)\psi(\mathbf{g}_3)\bar{\psi}(\mathbf{g}_4)$$



$$+\frac{\lambda_{6,1}(\Lambda)}{3}\sum_{\ell=1}^{3}\int[\prod_{j=1}^{6}\mathrm{d}\mathbf{g}_j]\,\mathcal{X}^{(\ell)}(\mathbf{g}_1,\mathbf{g}_2,\mathbf{g}_3,\mathbf{g}_4,\mathbf{g}_5,\mathbf{g}_6)\psi(\mathbf{g}_1)\bar{\psi}(\mathbf{g}_2)\psi(\mathbf{g}_3)\bar{\psi}(\mathbf{g}_4)\psi(\mathbf{g}_5)\bar{\psi}(\mathbf{g}_6)$$



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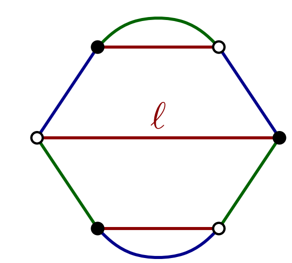

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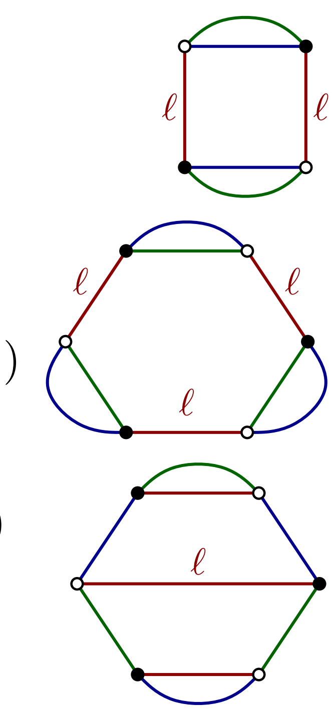

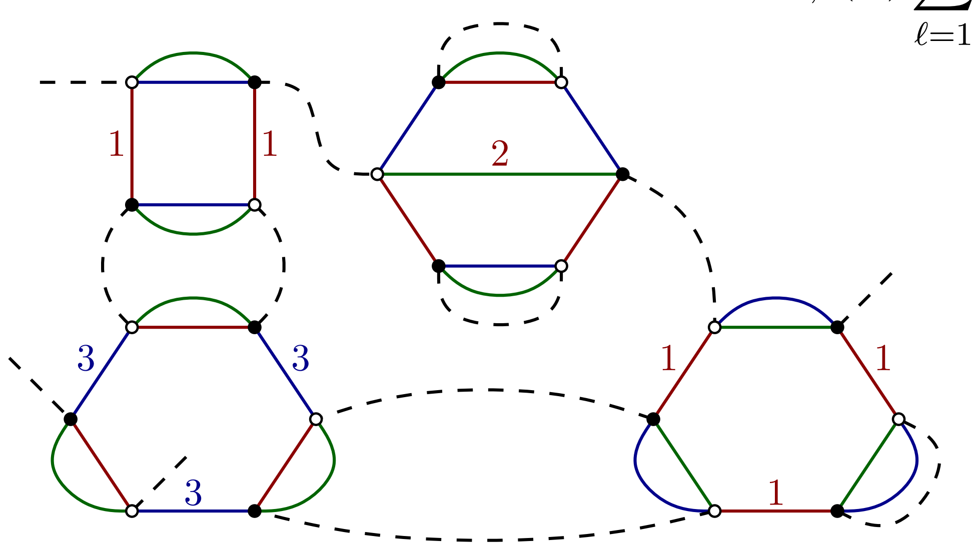

example of Feynman diagram - amplitudes are lattice gauge theories

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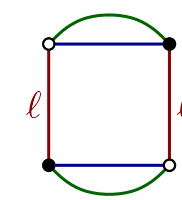

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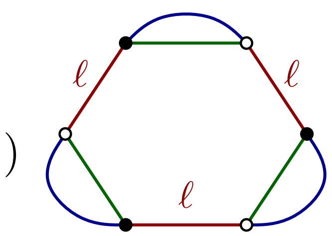

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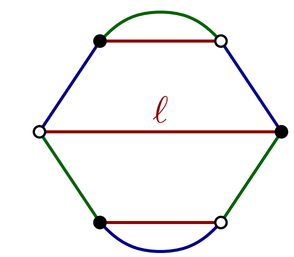

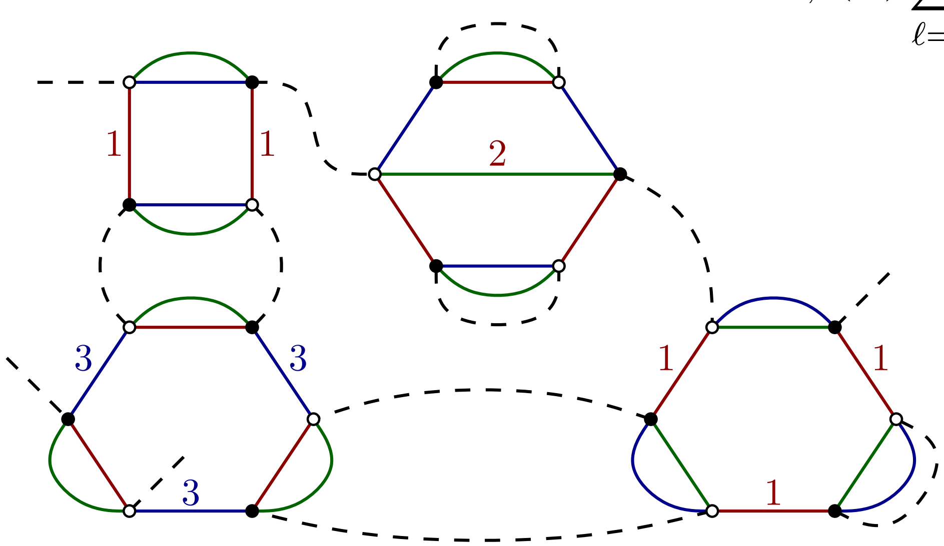

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key (most divergent, renormalizable) diagrams: melonic diagrams

most divergent configurations: flat connections

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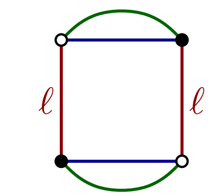

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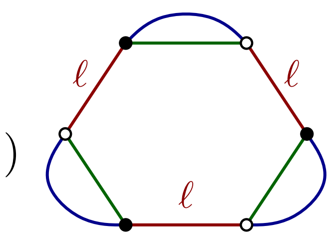

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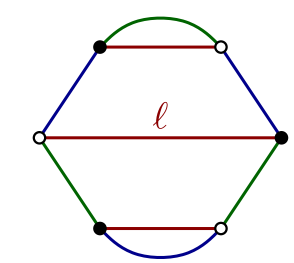

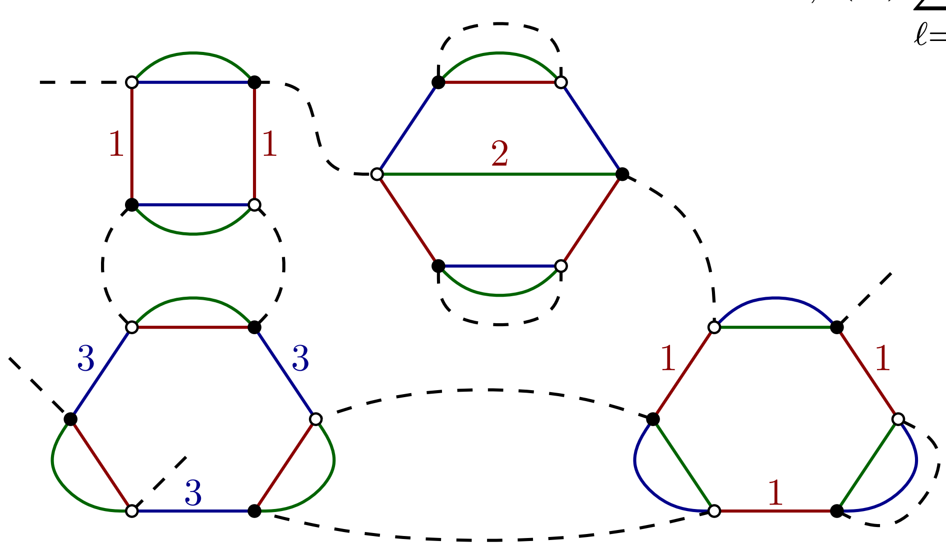

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key (most divergent, renormalizable) diagrams: melonic diagrams

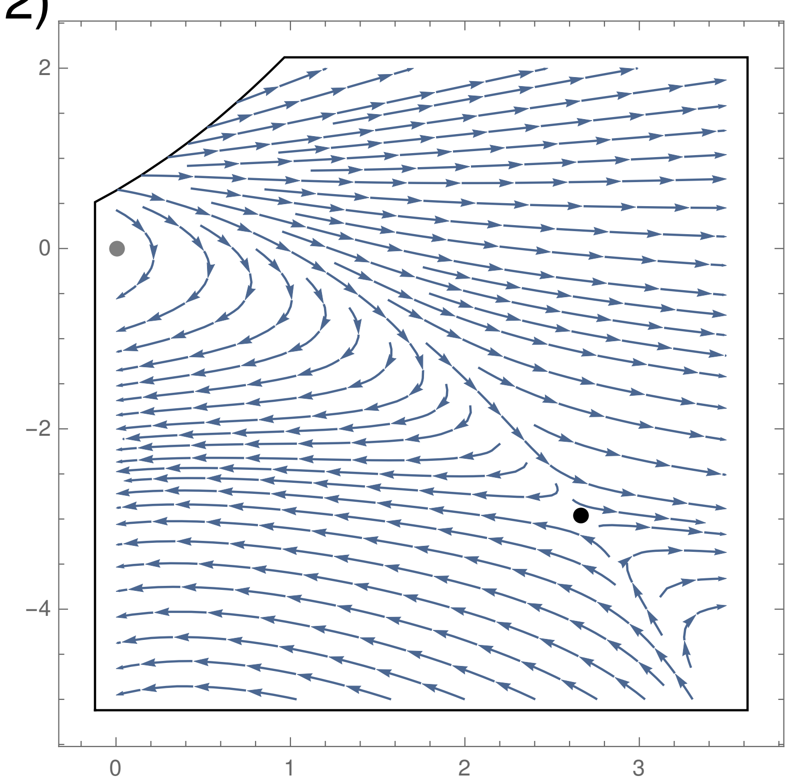
most divergent configurations: flat connections

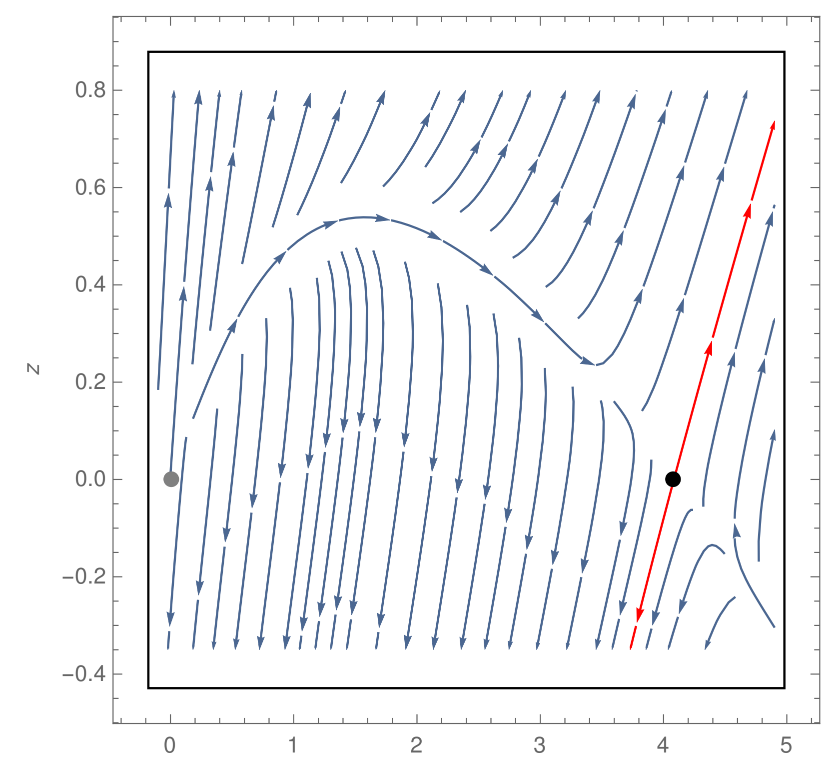
use FRG techniques to explore nature of perturbative UV fixed points
and to search for non-perturbative and IR fixed points

Renormalization flow of GFT models - 3d example

Tensorial GFT - d=3 , G=SU(2)   S. Carrozza, V. Lahoche, '16

- FRG analysis at order 6 truncation:
  - in the UV (large spins): Gaussian fixed point - two relevant + two marginal repulsive directions
  - in the UV: 1-parameter family of non-Gaussian fixed points - probably artefact of truncation
  - in the UV: 2 isolated non-Gaussian fixed points: one with three irrelevant directions and one relevant direction (FP1); one with three relevant, one irrelevant directions (FP2 - IR fixed point?)
- improvement of truncation (order 8, order 12 for subclass of interactions) suggest that FP1 is stable UV fixed point (less evidence for FP2)
- this supports:
  - asymptotic safety in UV
  - hints for condensation in IR





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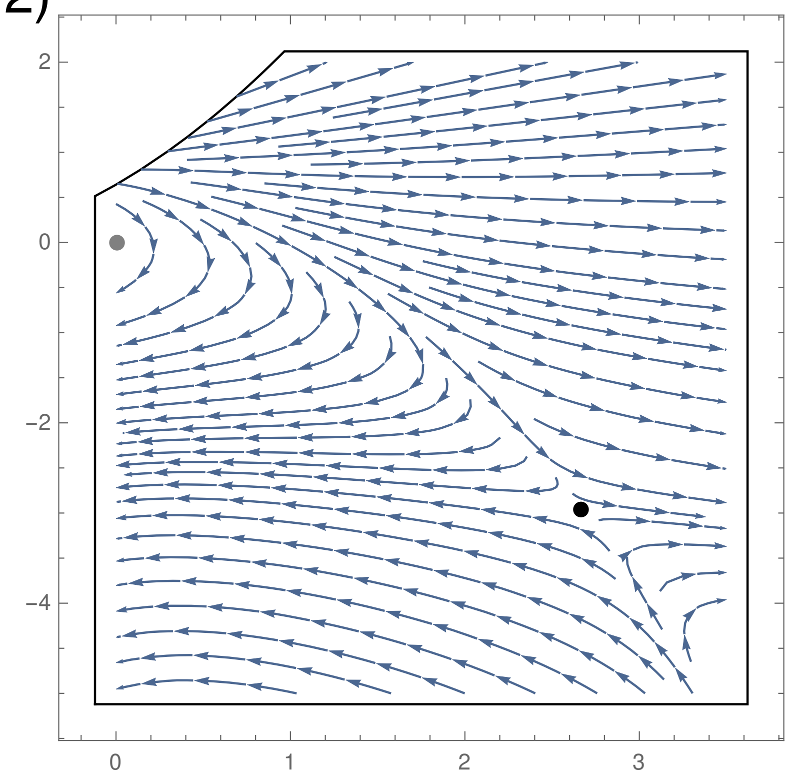
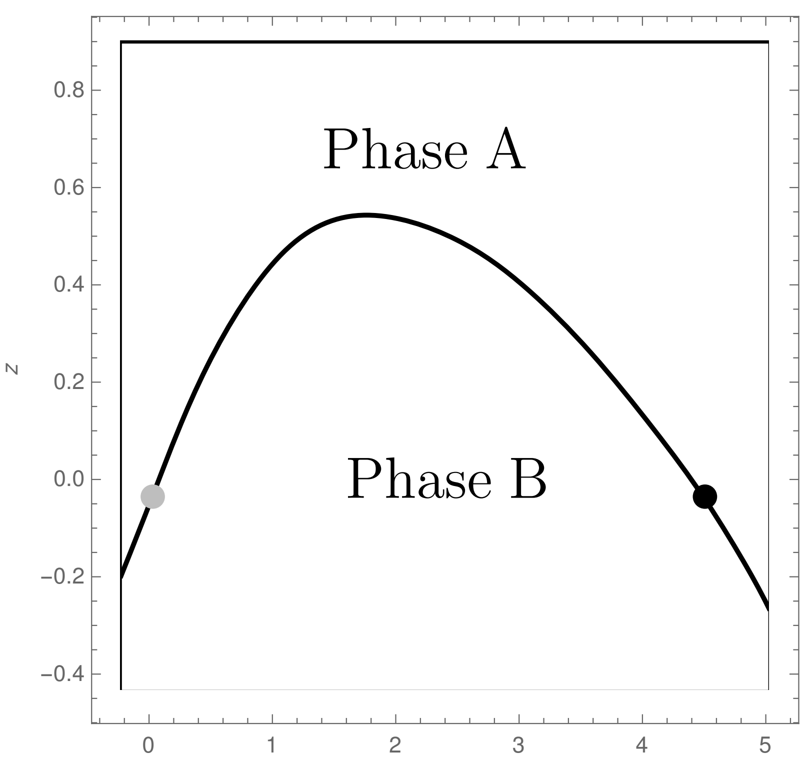




GFT renormalization:

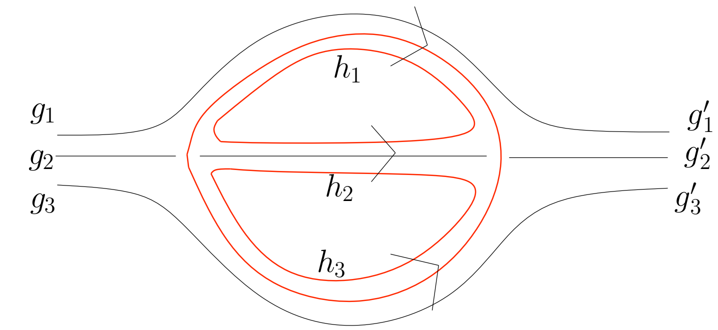
a brief survey of results

GFT perturbative renormalisation

towards renormalizable 4d gravity simplicial GFT models:

- calculation of some radiative corrections

see poster by Finocchiaro



T. Krajewski et al., '10; A. Riello, '13; V. Bonzom, B. Dittrich, '15; P. Dona', '17 ; M. Finocchiaro, to appear

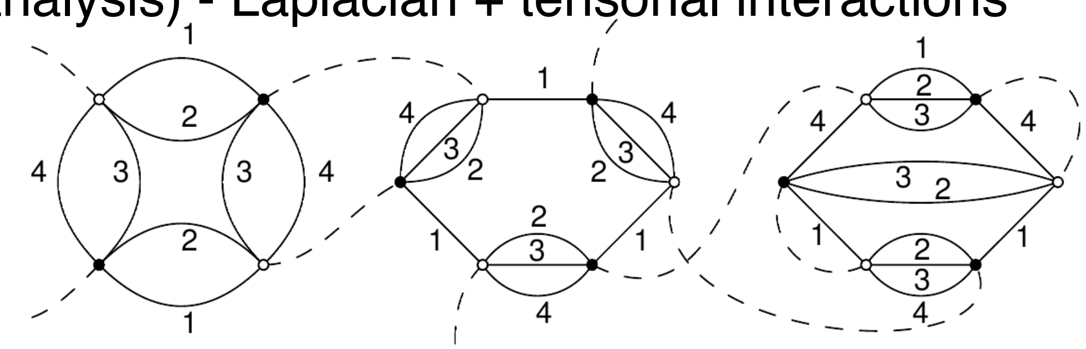
- finiteness results in 3d simplicial models (Boulatov with Laplacian kinetic term)

Ben Geloun, Bonzom, '11; Ben Geloun, '13

- renormalizable TGFT models (3d, 4d, and higher - multi scale analysis) - Laplacian + tensorial interactions

Ben Geloun, Rivasseau, '11
Carrozza, DO, Rivasseau, '12. '13

$$S(\varphi,\overline{\varphi}) = \sum_{b\in\mathcal{B}} t_b I_b(\varphi,\overline{\varphi})$$



—> with gauge invariance

—> non-abelian (SU(2))

—> on homogeneous spaces (towards TGFTs for 4d QG): first steps

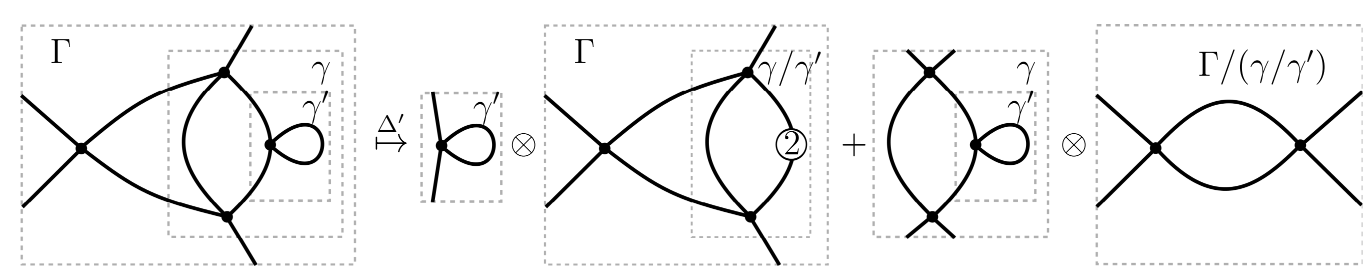
Lahoche, DO, '15

————> generic asymptotic freedom/safety

Ben Geloun, '12; Carrozza, '14; Carrozza, Lahoche, '16

- Hopf algebra methods in TGFT renormalization

M. Raasakka, A. Tanasa, '13; R. Cochou, V. Rivasseau, A. Tanasa, '17



GFT (and friends) non-perturbative renormalisation

- GFT constructive analysis

  Freidel, Louapre, Noui, Magnen, Smerlak, Gurau, Rivasseau, Tanasa, Dartois, Delpouve, …..

  non-perturbative resummation of perturbative (SF) series

  variety of techniques:

  - intermediate field method (loop-vertex expansion)
  - Borel summability

- FRG analysis of (discrete gravity) tensor models and SYK-like tensor models/QFTs

  Eichhorn, Koslowski, Duarte Pereira, ….

  Benedetti, Ben Geloun, Carrozza, Gurau, Rivasseau, Sfondrini, Tanasa, Wulkenhaar, ….

  ↓

  comparison with results from resummation of matrix models (FRG counterpart of double scaling limit)

  see talks by Koslowski, Carrozza, Ben Geloun

  see posters by Castro, Duarte Pereira, Lumma, Perez Sanchez

- TGFT non-perturbative renormalization (e.g. FRG analysis ala Wetterich-Morris)

  Benedetti, Ben Geloun, DO, Martini, Lahoche, Carrozza, Ousmane-Samary, Duarte Pereira, ….

GFT non-perturbative renormalisation

recent results:

FRG for (tensorial) GFT models

(similar to matrix/tensor models but distinctively field-theoretic)

Eichhorn, Koslowski, ‘14

- Polchinski formulation based on SD equations   Krajewski, Toriumi, ‘14

- general set-up for Wetterich-Morris formulation based on effective action
    - RG flow and phase diagram (in simple truncations) for: Benedetti, Ben Geloun, DO, ’14 ; Ben Geloun, Martini, DO, ’15, ’16, Benedetti, Lahoche, ’15; Lahoche, Ousmane-Samary, ’16; ……
        - TGFT on compact U(1)^d (with gauge invariance)
        - TGFT on non-compact R^d (with gauge invariance)
        - TGFT on SU(2)^3 (with gauge invariance).   Carrozza, Lahoche, ’16
        - models/truncations beyond melonic sector

          J. Ben Geloun, T. Koslowski, A. Duarte Pereira, DO, ‘18

          S. Carrozza, V. Lahoche, DO, ‘17

- epsilon-expansion   Carrozza, ‘14

key challenges:

- scaling dimensions of couplings (depend on combinatorics of corresponding interactions)
- non-autonomous systems of flow equations
- more subtle thermodynamic limit
- combinatorics

GFT non-perturbative renormalisation

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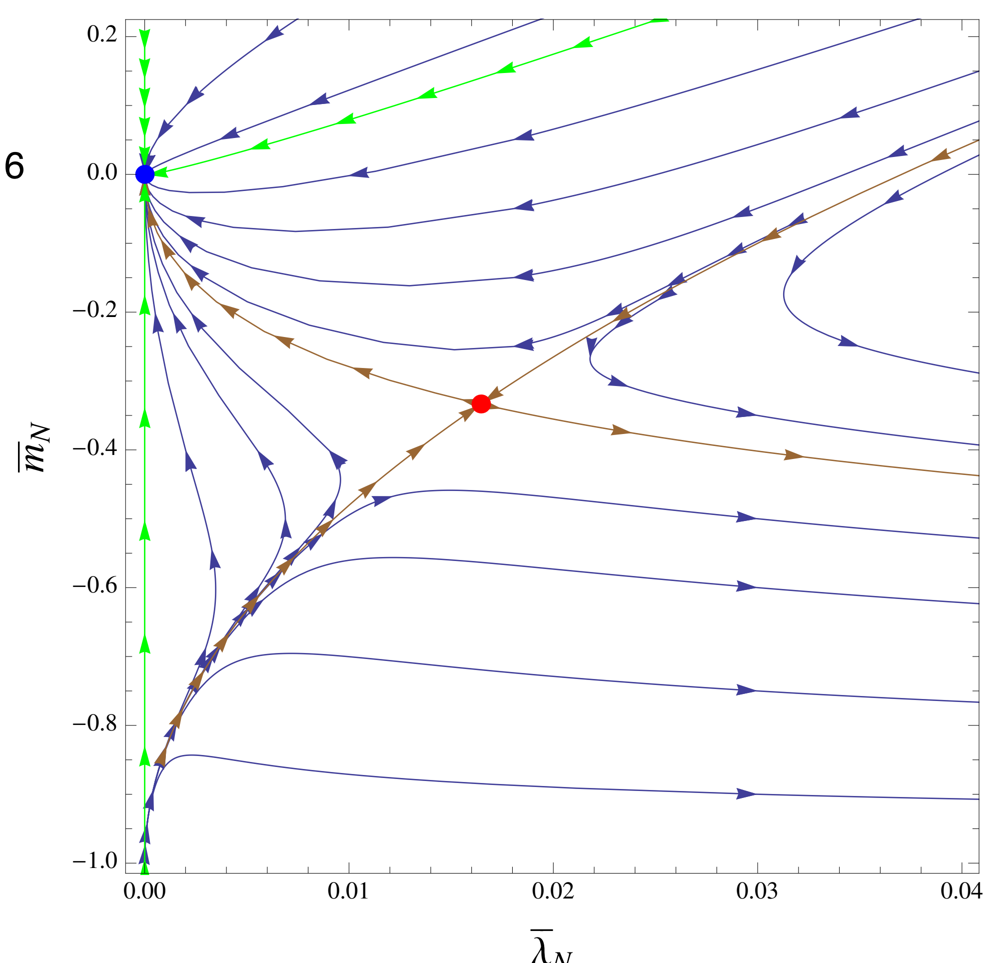

generically (so far):

- asymptotic freedom/safety
- hints of broken or condensate phase

(non-trivial minimum of classical potential)

GFT non-perturbative renormalisation

recent results:

FRG for (tensorial) GFT models

(similar to matrix/tensor models but distinctively field-theoretic)

- Polchinski formulation based on SD equations

Eichhorn, Koslowski, '14

Krajewski, Toriumi, '14

- general set-up for Wetterich-Morris formulation based on effective action
  - RG flow and phase diagram (in simple truncations) for:
    - TGFT on compact U(1)^d (with gauge invariance)
    - TGFT on non-compact R^d (with gauge invariance)
    - TGFT on SU(2)^3 (with gauge invariance). Carrozza, Lahoche, '16
    - models/truncations beyond melonic sector

Benedetti, Ben Geloun, DO, '14 ; Ben Geloun, Martini, DO, '15, '16, Benedetti, Lahoche, '15; Lahoche, Ousmane-Samary, '16; ……

J. Ben Geloun, T. Koslowski, A. Duarte Pereira, DO, '18

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- Landau approach to phase transitions A. Pithis, J. Thurigen, '18
- inequivalent condensate representations of quantum GFT algebra

A. Kegeles, DO, '17

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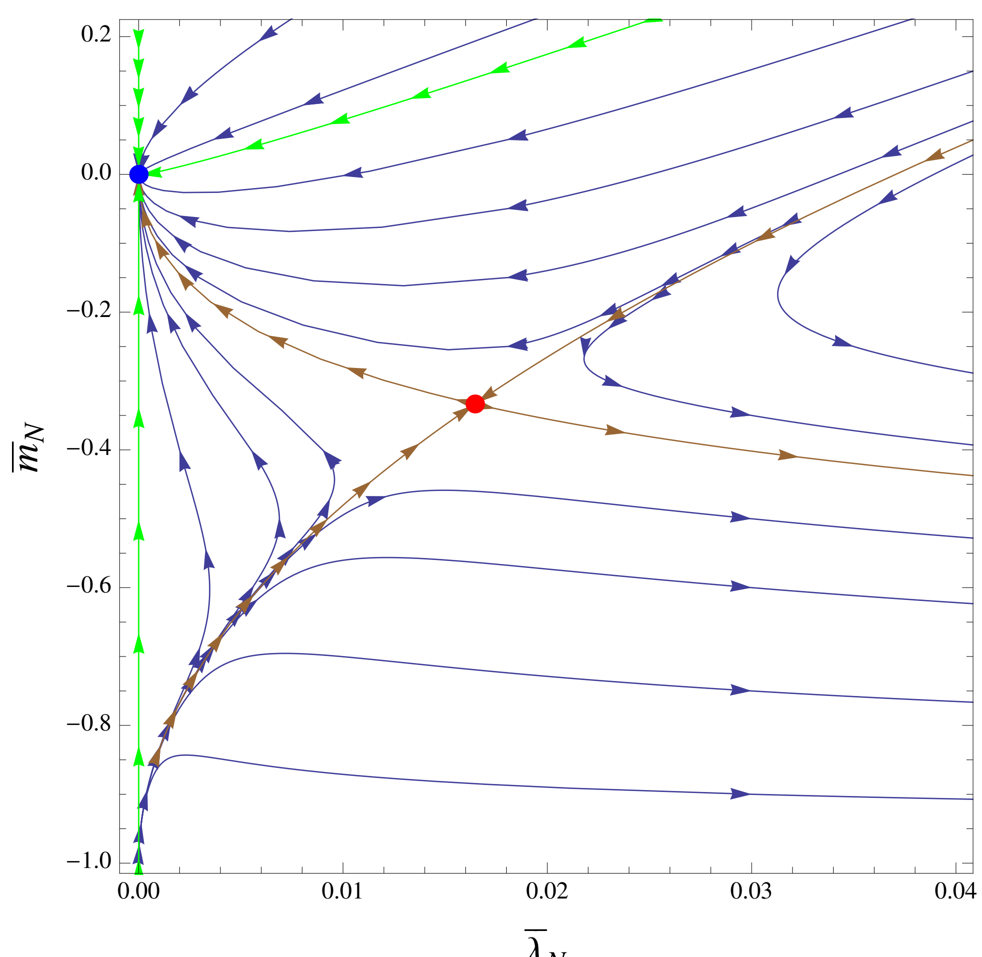


GFT renormalization:

key open issues and new directions

Key open issues: RG flow of more (T)GFT models

(T)GFT Renormalization with simplicity constraints

Simplicity constraints, imposed on topological BF models on Spin(4) or SL(2,C), ensure "geometricity"

GFT amplitudes become 4d simplicial gravity path integrals - various models

- Simple group structure is lost; symmetries are broken; amplitudes much more involved
- No complete power counting of divergences - results on various classes of diagrams
  see Finocchiaro's poster
  Riello, Bonzom, Dittrich, Finocchiaro, Dona, …
- Main difficulty: dominant configurations are not just flat connections (richer simplicial geometry, related to Regge geometries found in semi-classical spin foam amplitudes)
  Barrett, Williams, Freidel, Conrady, Pereira, Hellmann, Han, Zhang, …
- Main difficulty 2: do not know what is relevant (large enough) theory space

rely on (and extend) work on GFT symmetries — A. Kegeles, DO, '15, '16

Key open issues: RG flow of more (T)GFT models

(T)GFT Renormalization with additional local directions

DO, Sindoni, Wilson-Ewing, '16; Y. Ling, DO, M. Zhang, '17; S. Gielen, DO, '17

Coupling simplicial geometry with (minimally coupled) scalar fields (simpler for free, massless case):

$$K = \sum_{n=0}^{\infty} \int \mathrm{d}g_v \mathrm{d}g_w \mathrm{d}\phi \, \bar{\varphi}(g_v, \phi) K_2^{(2n)}(g_v, g_w) \frac{\partial^{2n}}{\partial \phi^{2n}} \varphi(g_w, \phi)$$

$$V = \int \left(\prod_{a=1}^{5} \mathrm{d}g_{v_a} \right) d\phi \, \mathcal{V}_5(g_{v_1}, \ldots, g_{v_5}; \phi) \prod_{a=1}^{5} \varphi(g_{v_a}, \phi)$$

scalar fields used as "embedding coordinates" —> similar to standard QFTs in flat space with additional "internal" (tensorial) non-local data (quantum geometric)

very similar to SYK-like tensorial models

- What is the RG flow of these "mixed models"? dominant diagrams? fixed points? phases?
- scalar field momentum (energy) as running scale —> different from usual (T)GFTs
- issue: how do matter fields modify the RG flow of "pure gravity" GFTs?

Key open issues: GFT vs lattice SF renormalization

recall: Feynman amplitudes = spin foam models/lattice gauge theories

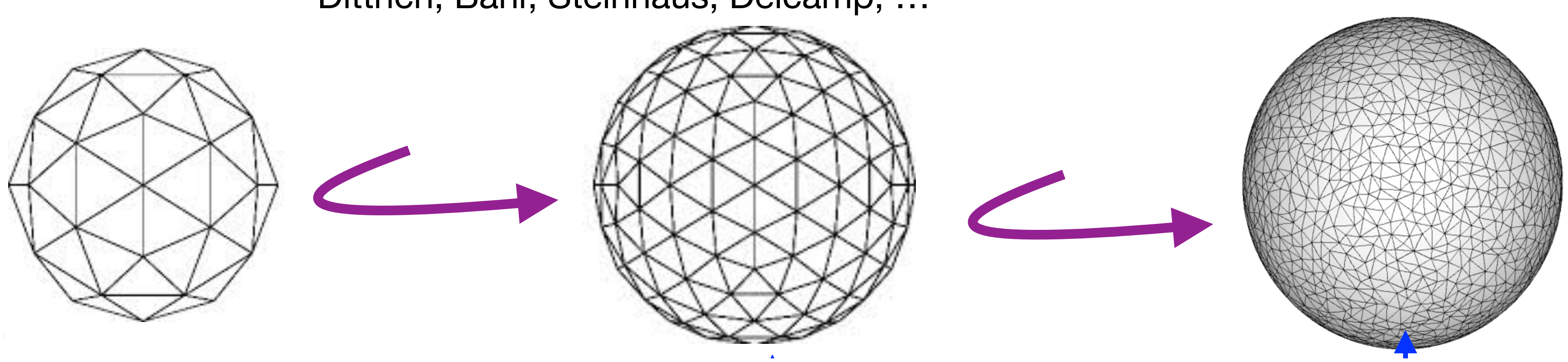
RG flow of spin foam models can be studied with LGT methods

Dittrich, Bahr, Steinhaus, Delcamp, …

cut-off in representations

RG scale is “complexity of lattice”, flow is driven by refinement

see Bianca’s talk



Amplitudes flow by requiring consistency under restriction to coarser boundary states

$$\mathcal{A}^{\text{low com}}(\psi_{\text{low com}}) \longleftarrow \mathcal{A}^{\text{med com}}(\psi_{\text{med com}}) \longleftarrow \mathcal{A}^{\text{high com}}(\psi_{\text{high com}})$$

restricts to

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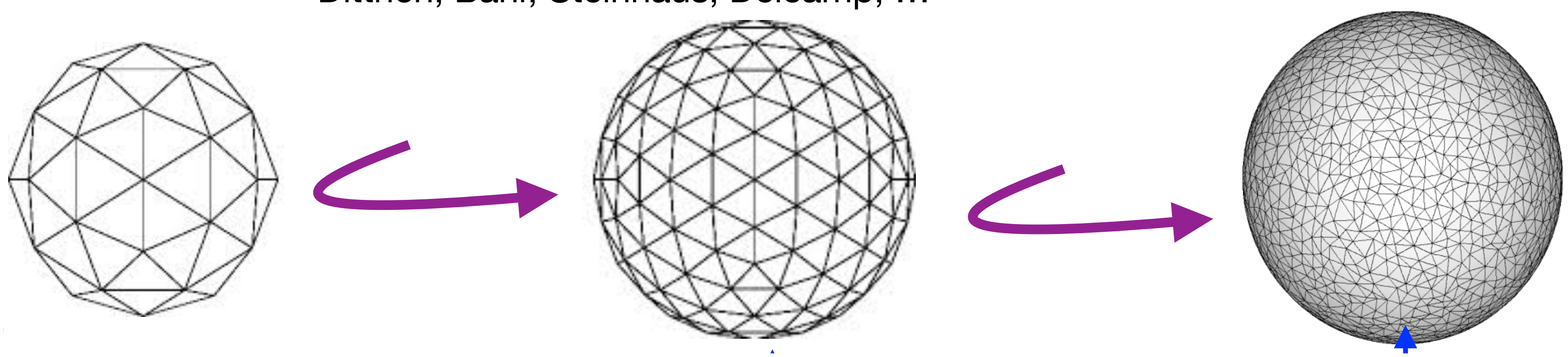

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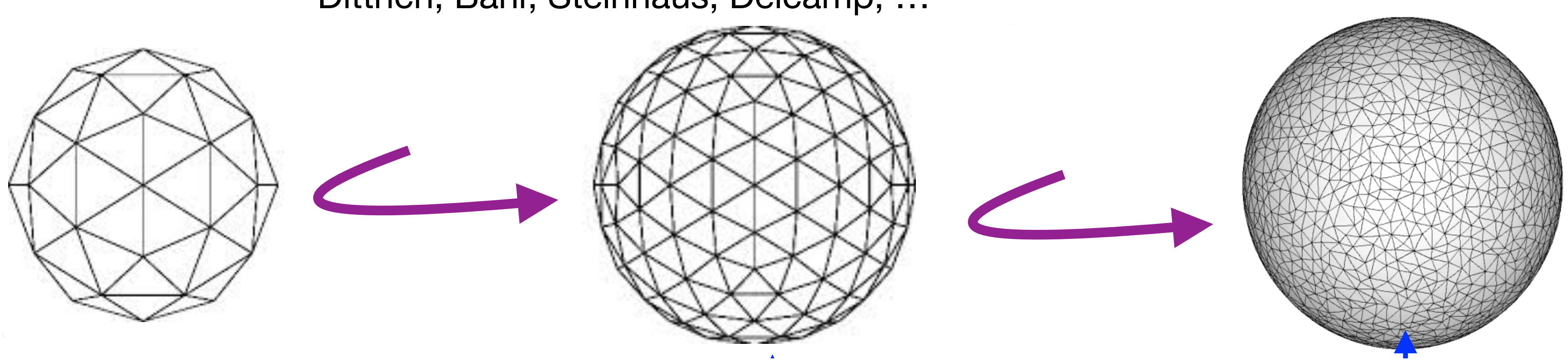

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contraction of (divergent) GFT subgraphs + absorption in effective vertices ~ coarse-graining of SF lattices

spin foam amplitude consistency under coarse graining = RG consistency of GFT Feynman amplitudes

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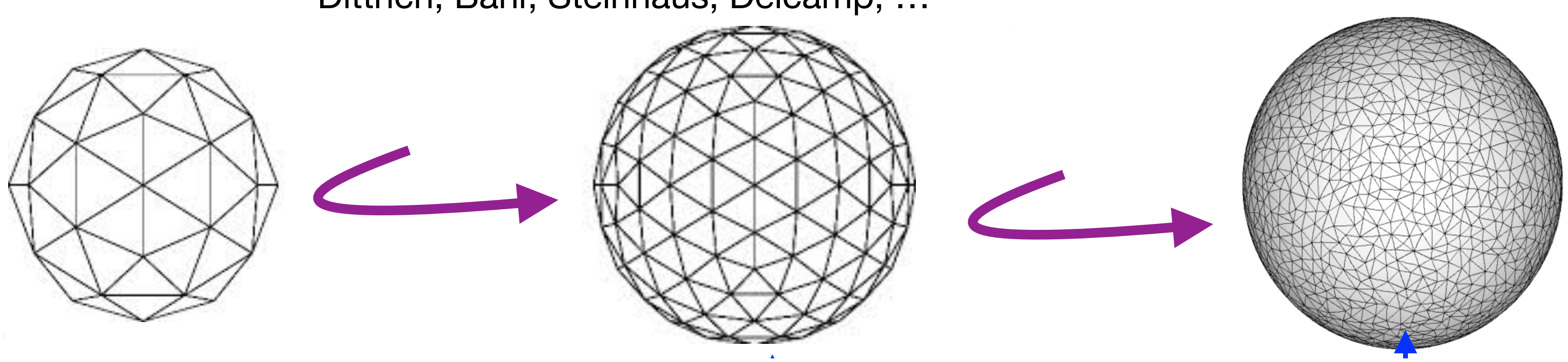

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spin foam amplitude consistency under coarse graining = RG consistency of GFT Feynman amplitudes

Two aspects should become more central in (T)GFT RG analysis (e.g. using tensor network methods):

- combinatorial structure of boundary states and effects on RG flow
- combinatorial complexity as co-determining the “scale” of the RG flow

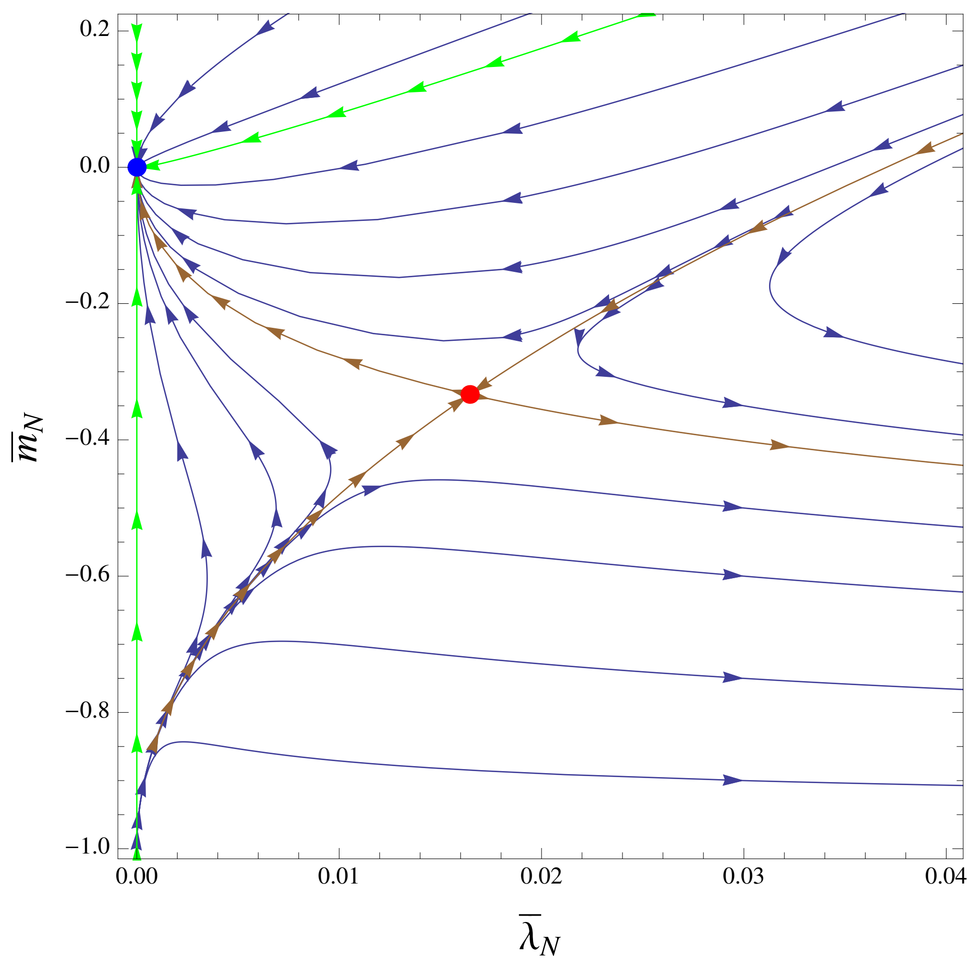
Key open issues: how to extract physics?

suppose we have the full RG flow of a TGFT model (full continuum theory); how do we interpret it, physically? how do we translate it in the language of gravity, geometry, effective field theory?

many related questions:

order parameters? which observables should we focus on?

what is a spacetime metric, from (T)GFT perspective?



- useful insights from (causal) dynamical triangulations
- insights from LQG
- comparison with SYK-like tensorial GFTs

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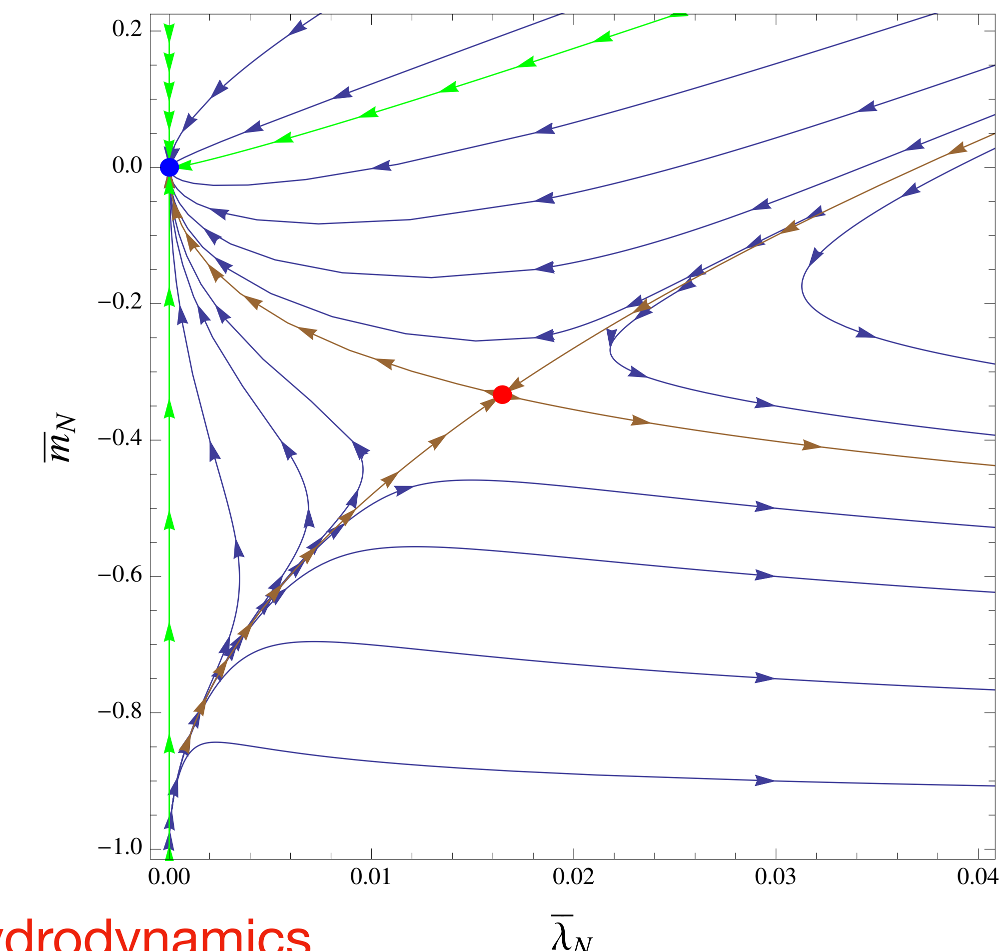

- recently developed strategy: cosmology from GFT (condensate) hydrodynamics

Gielen, DO, Sindoni, Wilson-Ewing, De Cesare, Pithis, Sakellariadou, …., ’13 - …

- interpret GFT hydrodynamic equations as non-linear version of “quantum cosmology”

$$A_j \partial_\phi^2 \sigma_j(\phi) - B_j \sigma_j(\phi) + w_j \bar{\sigma}_j(\phi)^4 = 0$$

+ use group-data and simplicial geometry

- compute collective observables, e.g. “total volume” + obtain dynamical equations for them

Thank you for your attention!