Holographic Signatures of Resolved Cosmological Singularities: Numerical Investigations

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based on

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with N. Bodendorfer & J. Münch

Quantum Gravity meets Lattice QFT
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Introduction: QG and Holography

Holographic aspects of QG actively investigated.

Main research directions (in LQG)

1. **Dual statistical models from 3+0 QG partition function**

   [Costantino '11; Dittrich, Hnybida '13; Bonzom, Costantino, Livine '15; Dittrich, Goeller, Livine, Riello '17]

2. **Spin networks, tensor networks and holographic entanglement entropy**

   [Han, Hung '16; Han, Huang '17; Chirco, Oriti, Zhang '17]

3. **Holography in symmetry-reduced models**

   [Ashtekar, Wilson-Ewing '08; Bodendorfer, Schäfer, Schliemann '16; Bodendorfer, Mele, Münch '18]

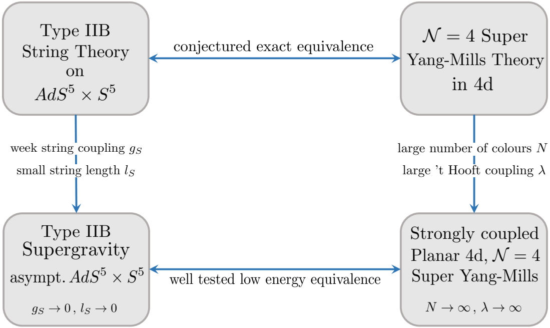
Related work: [Freidel '08; Bodendorfer, Thiemann, Thurn '11; Bodendorfer '15; Freidel, Perez, Pranzetti '16; Smolin '16; Livine '17, . . .]

In this talk we will focus on the third direction.

Gauge/Gravity Duality in a Nutshell

[Maldacena '97; Gubser, Klebanov, Polyakov '98; Witten '98]

Maldacena's original proposal:



Holographic Principle ['t Hooft '93; Susskind '95; Bousso '02; ...]

Duality between gauge and (quantum) gravity theories

- Tools for QFT (e.g, condensed matter, QCD)
- Quantum gravity models

QG for AdS/CFT

- Gauge/Gravity mainly understood in the classical (super)gravity regime
- Singularities in classical gravity
  - limitations in application of perturbative string theory
  - no consensus on the fate of singularities in dual QFT

    [Hertog, Horowitz '04, '05; Das, Michelson, Narayan, Trivedi '07; Barbón, Rabinovici '11; Craps, Hertog, Turok '12; Smolkin, Turok '12; Engelhardt, Horowitz '15, '16; . . .]
- Progress in singularity resolution from non-perturbative QG

Question

QG resolution of bulk singularity $\rightarrow$ signatures/improvements in dual QFT?

Prototype calculation: effective bulk quantum geometry in a cosmological setting

Plan of the Talk

1. Setup and strategy

2. Holographic signatures of cosmological singularities

   bulk singularity $\longleftrightarrow$ pole in 2-point correlator

3. Resolved singularities in quantum corrected effective geometries

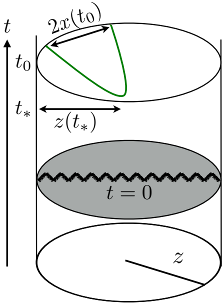
   - Simplifications needed for analytic treatment
   - Otherwise numerics necessary

   singularity resolution $\longleftrightarrow$ pole resolution

4. Conclusions and outlook

Setup

[Engelhardt, Horowitz '14; Engelhardt, Horowitz, Hertog '15]



Bulk Kasner-AdS: $ds_5^2 = \frac{1}{z^2}\left(dz^2 + ds_4^2(t)\right)$

Boundary: $ds_4^2(t) = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2 \quad , \quad p_i \in \mathbb{R}$

Kasner conditions: $\sum_i p_i = 1 = \sum_i p_i^2$

$p_i < 0$ geodesics bent towards the singularity

Equal-time 2-point correlator

Geodesic approximation [Balasubramanian, Ross '00]

$$\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle = \exp\left(-\Delta L_{ren}\right)$$

$\Delta \equiv$ conformal weight of massive scalar operator $\mathcal{O}$
$L_{ren} \equiv$ renormalized length of bulk geodesic connecting x and -x

Renormalized Geodesic Length

fixed z metric — boundary metric

$$\frac{1}{z^2}\left(-dt^2 + a^2(t)\, dx^2 + \dots\right) \quad - - - - \rightarrow \quad dt^2 + a^2(t)\, dx^2 + \dots$$

- Two-point correlator:

$$\langle \mathcal{O}\mathcal{O} \rangle_{CFT} = z^{-2\Delta} \, \langle \phi\phi \rangle_{bulk}$$

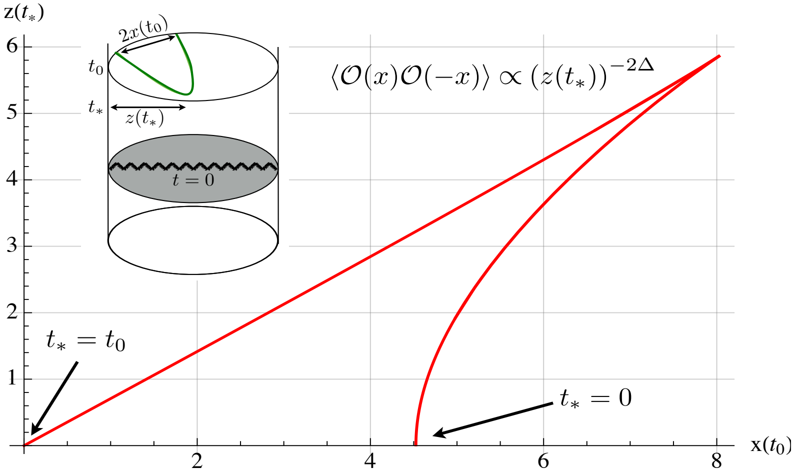
- Geodesic approximation:

$$\langle \phi\phi \rangle_{bulk} = \exp\left(-\Delta\, L\right) \qquad , \qquad L \overset{\epsilon \to 0}{=} 2 \log\left(2z(t_*)\right) - 2 \log \epsilon$$

$$\langle \mathcal{O}(-x)\mathcal{O}(x) \rangle_{CFT} = \exp\left(-\Delta\, L_{ren}\right) = \left(2z(t_*)\right)^{-2\Delta}$$

Holographic Signature of Cosmological Singularity

[Engelhardt, Horowitz '14; Engelhardt, Horowitz, Hertog '15]



Finite distance pole in two-point correlator

Quantum Corrected Kasner-AdS

Classical metric

$$ds_5^2 = \frac{1}{z^2}\left(dz^2 + ds_4^2(t)\right) \quad , \quad ds_4^2(t) = -dt^2 + \sum_{i=1}^{3} t^{2p_i} dx_i^2$$

- no large curvatures associated with z-direction
- curvature singularity in t-direction

$$\downarrow$$

Quantum corrected metric

$$ds_5^2 = \frac{1}{z^2}\left(dz^2 + ds_4^2(t,z)\right) \quad , \quad ds_4^2(t,z) = -dt^2 + a(t,z)^2 dx^2 + \ldots$$

- z-direction classical
- effective QG for 4d part s.t. singularity resolved

Quantum Corrected Kasner-AdS

In principle effective metric from full 5d quantum Einstein eqs. $\rightarrow$ here ansatz

- singularity resolution in $ds_4^2 \quad \longrightarrow \quad$ singularity-free ds_5^2

  (possible z-dependence of ds_4^2 must be small!)

- solution of 5d Einstein eqs. up to quantum corrections

- bulk QG effects relevant at 5d-Planck scale

- classical Kasner boundary metric

- quantum bounce with Kasner transitions

  [Gupt, Singh '12; Ashtekar, Wilson-Ewing '08; Wilson-Ewing '17]

Quantum Corrected Metric: A First Example

Quantum corrected metric [Bodendorfer, Schäfer, Schliemann '16]

$$ds_5^2 = \frac{1}{z^2}\left(dz^2 + ds_4^2(t)\right) \quad , \quad a(t) = \frac{a_{ext}}{\lambda^p}\left(t^2 + \lambda^2\right)^{p/2}$$

Two main simplifications:

1. λ related to 4d bulk Planck scale
2. No Kasner transitions

Renormalized length and two-point correlator

Geodesic eqs. analytically solvable:

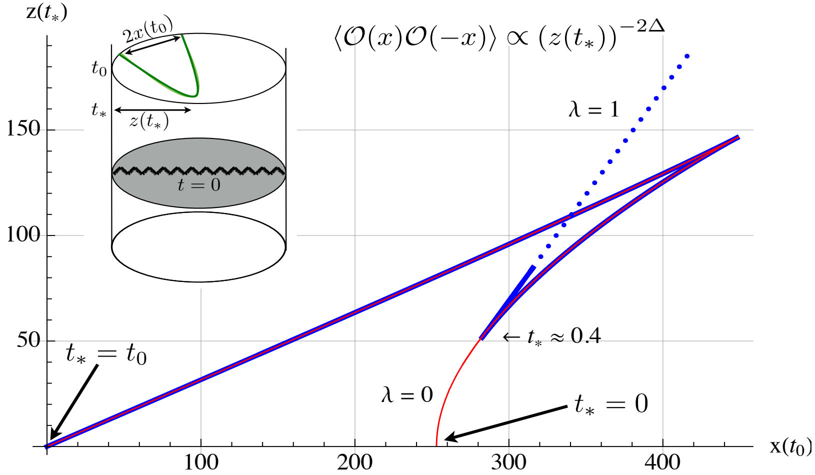
$$\pm s(z = \epsilon) \stackrel{\epsilon \to 0}{=} \log\left(2z(t_*)\right) - \log\left(\epsilon\right) \qquad \epsilon = \text{boundary regulator}$$

and

$$L_{ren} = 2\log\left(2z(t_*)\right) \quad \Rightarrow \quad \boxed{\langle \mathcal{O}(x)\mathcal{O}(-x)\rangle = \left(2z(t_*)\right)^{-2\Delta}}$$

Holographic Signature of Resolved Cosmological Singularity

[Bodendorfer, Schäfer, Schliemann '16]



No finite distance pole in two-point correlator!

Drop Simplification 1: 5d Planck Scale

Quantum corrected metric [Bodendorfer, Mele, Münch '18]

$$ds_5^2 = \frac{1}{z^2}\left(dz^2 - dt^2 + \sum_i a_i^2(t,z)dx_i^2\right) \quad , \quad a_i^2(t,z) = \frac{a_{ext}^2}{\lambda^{2p_i}}\left(t^2 + z^2\lambda^2\right)^{p_i}$$

- ds_5^2 not singular at $t = 0$

$$R^{(5)}_{\text{Kretschmann}} = z^4 R^{(4)}_{\text{Kretschmann}} + \dots \quad , \quad R^{(4)}_{\text{Kretschmann}} \sim \lambda^{-4}$$

$\Rightarrow$ effective 5d scale: $\lambda \to z\lambda$ s.t. $R^{(5)}_{\text{Kretschmann}} = \mathcal{O}(\lambda_{5d}^{-4})$

- z-dependence of ds_4^2 small (order of λ) and $\partial_z a \to 0$ in zeroth order in λ
- classical Kasner-AdS boundary metric recovered ($\frac{a_{ext}}{\lambda^p} \to 1$, $\lambda, z \to 0$)
- symmetries:
  - translation (e.g., for us in x-direction)
  - scaling: $t \longmapsto \Lambda t$, $z \longmapsto \Lambda z$, $x \longmapsto \Lambda^{1-p} x$ $(\Lambda \in \mathbb{R})$
- no Kasner transitions

Boundary Pole Resolution

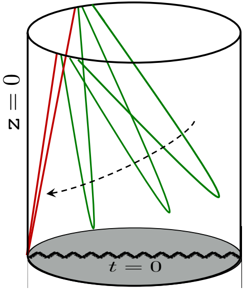
Boundary metric is singular

$$a_i^2(t,z) = \frac{a_{ext}^2}{\lambda^{2p_i}} \left(t^2 + \lambda^2 z^2\right)^{p_i} \xrightarrow{z\to 0} \frac{a_{ext}^2}{\lambda^{2p_i}}\, t^{2p_i}$$

Classical Kasner-AdS: [Engelhardt, Horowitz, Hertog '15]

$z_* \to 0$ as $t_* \to 0$

family of space-like geodesics approaches a light-like boundary geodesic probing high-curvature regime



Quantum corrected bulk: **boundary geodesics isolated** (more later).

Solving Geodesic Equations

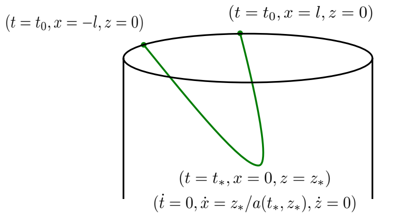
geodesic Eqs. not decoupled $\longrightarrow$ **numerics necessary!**

Strategy

boundary-value problem

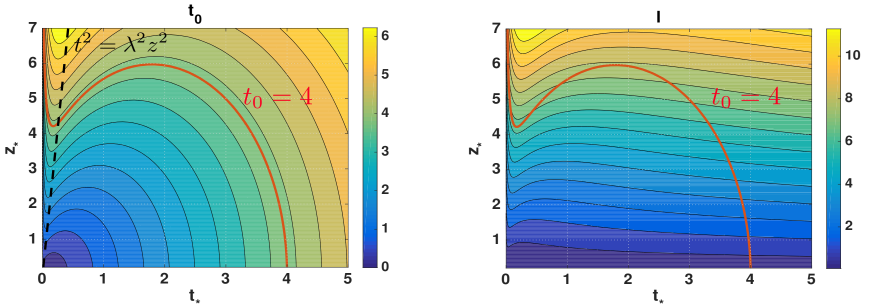
$\downarrow$

initiual-value problem



- initial values at the turning point only in terms of t_* and z_*
- solve IVP numerically and find $t_0(t_*, z_*)/l(t_*, z_*)$
- we are interested in fixed t_0-values and vary $l \rightarrow$ find relation $z_*(l)$

Key point: relate turning point data (t_*, z_*) with boundary data (t_0, l)



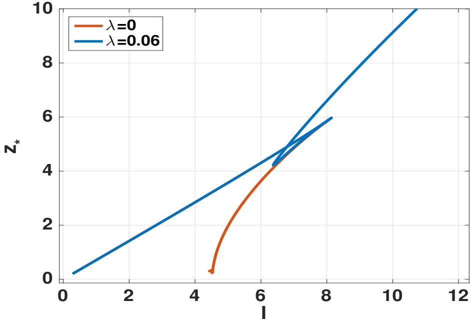
$$p = -1/4 \ , \ \lambda = 0.06 \ , \ a_{ext} = \lambda^p$$

Note

- Classical case: bulk geodesics approaching a null boundary geodesic ($z_* \to 0$ for $t_* \to 0$)
- Quantum case: z_* grows as $t_* \to 0 \Rightarrow$ null boundary geodesic not limit of bulk geodesics

Solving Geodesic Equations

Reading off the values of l along t_0-level line, we get the relation $z_*(l)$:



$p = -1/4 \quad , \quad a_{ext}/\lambda^p = 1$

$\lambda = 0$

$\lambda = 0.06$

last step: check $L_{ren} \sim \log\left(z(t_*)\right) \quad \Rightarrow \quad \langle \mathcal{O}(-x)\mathcal{O}(x)\rangle \propto \left(z(t_*)\right)^{-2\Delta}$

Renormalized Geodesic Length & Two-point Correlator

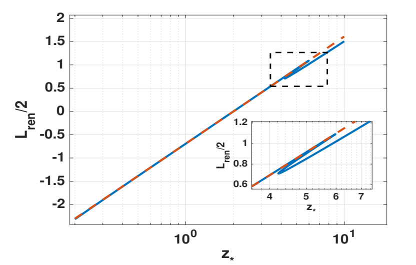
Renormalization of geodesic length

- Conformal boundary ($z = 0$) lies at infinity $\rightarrow$ geodesic length diverges
- Renormalization procedure needed:

  1. Evaluate geodesic up to $\epsilon = z_{UV}/t << 1$
  2. Subtract divergent contribution ($\epsilon \rightarrow 0$)

$$L_{ren} = L - L_0 \qquad , \qquad L_0 = 2\log\left(\left|\frac{z_{UV}}{t}\right|\right)$$

Two-point correlator



$$\langle \mathcal{O}(-l)\mathcal{O}(l)\rangle \propto e^{-\Delta L_{ren}} = (z_*)^{-2\Delta}$$

$$+$$

$z_* \neq 0$ for finite non-zero l

$$\Downarrow$$

pole resolved

Drop Simplification 2: Inclusion of Kasner Transitions

Inclusion of Kasner transitions highly non trivial:

- Find a metric satisfying all requirements discussed before not straightforward
- We provided two examples of quantum corrected metrics partially satisfying them

$$\downarrow$$

pole resolution qualitatively the same!

Conclusion

Summary

- Bulk singularity resolution $\overset{dual}{\longleftrightarrow}$ boundary finite-distance pole resolution
- Application of LQG in Gauge/Gravity-framework possible
- New tools for holography/improvements in dual field theory

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Future work

- 5d-Loop quantization
- Look for independent field theory calculations, e.g. via lattice QFT
- Generalize to other spacetimes, e.g. black holes (work in progress)
- Long term goal: finite N gauge theory

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Thank you for your attention!

Inclusion of Kasner Transitions

Quantum corrected metric

$$ds_5^2 = \frac{1}{z^2}\left(dz^2 - dt^2 + a(t,z)^2 dx^2 + \ldots\right)$$

with

$$a(t,z)^2 = \frac{a_{ext}^2}{\lambda^{2p}}\left(t^2 + \lambda^2 z^2\right)^p \exp\left[2\Delta p \sinh^{-1}\left(\frac{t}{z\lambda}\right)\right] \quad , \quad \Delta p \in \mathbb{R}$$

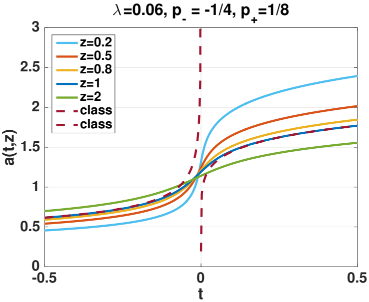
- ds_4^2: $\quad a(t,z) = \frac{a_{ext}}{\lambda^p}\left(t^2 + \lambda^2\right)^{p/2} \exp\left[2\Delta p \sinh^{-1}\left(\frac{t}{\lambda}\right)\right]$
  - limiting curvature mimetic gravity [Chamseddine, Mukhanov '16]
  - toy model effective homog. and isotr. LQG [Bodendorfer, Schäfer, Schliemann '17]
- $\lambda \to z\lambda$ (quantum effects at 5d Planck scale)
- implement transition $p_- = p - \Delta p \ \to \ p_+ = p + \Delta p$
- $\Delta p = 0 \ \to$ 5d Planck scale quantum corrected metric with no transitions

Inclusion of Kasner Transitions

For $\left|\frac{t}{z\lambda}\right| >> 1$:

$$a(t,z)^2 \simeq \frac{a_{ext}^2}{\lambda^{2p}} \left(\frac{2}{\lambda z}\right)^{\pm 2\Delta p} t^{2p_\pm} \quad , \quad p_\pm = p \pm \Delta p$$

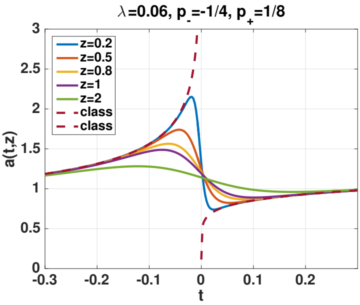
$\Rightarrow$ **transition** $p_- \to p_+$ at the bounce (t=0).



boundary 4d classical Kasner	✘
solution of 5d Einstein eqs. up to $\mathcal{O}(\lambda)$	✘
Kretschmann condition $R^{(5)}_{\text{Kretsch.}} = \mathcal{O}(\lambda_{5d}^{-4})$	✔
Qualitative LQC behavior	✔
symmetries (translation, scaling)	✔

Alternative Metric for Kasner Transitions

$$a(t,z)^2 = \frac{a_{ext}^2}{\lambda^{2p}} \left(t^2 + \lambda^2 z^2\right)^p \exp\left[2\Delta p \sinh^{-1}\left(\frac{t}{z\lambda}\right)\right] \cdot$$
$$\cdot \left(\lambda z\right)^{2\Delta p \tanh\left(\frac{t}{z\lambda}\right)}$$

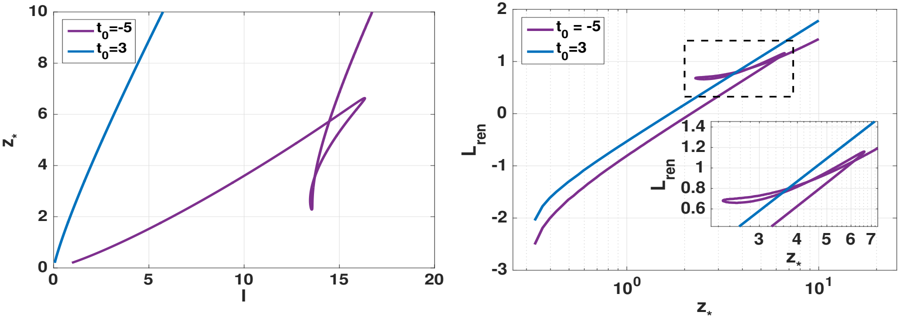


boundary 4d classical Kasner	✔
solution of 5d Einstein eqs. up to $\mathcal{O}(\lambda)$	✔
qualitative LQC behavior	✘
Kretschmann condition $R^{(5)}_{\text{Kretsch.}} = \mathcal{O}(\lambda_{5d}^{-4})$	✘

Numerical Results with Kasner Transitions

Two kinds of solutions:

- starting at $t_0 < 0$, bent towards the resolved singularity and passing it
- starting at $t_0 > 0$, bent away from the resolved singularity



Finite-distance pole in two-point correlator resolved!

Numerical Checks

cut-off dependence ?

No Kasner Transitions

z_{UV}	$\exp\left(-\mathbf{L}_{\text{ren}}\right)$	$\exp\left(-\mathbf{L}_{\text{ren}}\right)$
0.05	0.761810	0.917559
0.06	0.761841	0.917597
0.07	0.761860	0.917661
0.08	0.761896	0.917716
0.09	0.761943	0.917783
0.1	0.761972	0.917897

$\lambda = 0.06\,,\ p = -\frac{1}{4}$

Kasner Transitions

z_{UV}	$\exp\left(-\mathbf{L}_{\text{ren}}\right)$	$\exp\left(-\mathbf{L}_{\text{ren}}\right)$
0.05	0.367113	0.308506
0.06	0.367149	0.308554
0.07	0.367185	0.308602
0.08	0.367229	0.308661
0.09	0.367267	0.308714
0.1	0.367314	0.308774

$\lambda = 0.06\,,\ p = -\frac{1}{16}\,,\ \Delta p = \frac{3}{16}$

Renormalization procedure independent of z_{UV} within 0.1%

Comparison with Analytical Results: No z-dependence in ds_4^2

Analytic | Numeric