Finite coupling corrections to gauge fields in AdS/CFT

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Content

Gravity

Type IIb superstring theory with string length $l_s = \sqrt{\alpha'}$, coupling g_s and $2g_{\text{YM}}^2 N = \frac{L^4}{\alpha'}$

→

Field Theory

$\mathcal{N} = 4$ super Yang-Mills-theory (SYM) with coupling constant $g_{\text{YM}} = 2\pi g_s^2$ and 't Hooft coupling $g_{\text{YM}}^2 N = \lambda$

↓ $\frac{\alpha'}{L^2} \to 0,\ g_s \to 0$

↓ $N \to \infty,\ \lambda \to \infty$

Gravity

Type IIb supergravity in $\mathcal{O}(\alpha'^0)$

→

Field Theory

Large 't Hooft coupling, planar diagrams

Gravity

Type IIb superstring theory with string length $l_s = \sqrt{\alpha'}$, coupling g_s and $2 g_{\mathrm{YM}}^2 N = \frac{L^4}{\alpha'}$

→ **Field Theory**

$\mathcal{N} = 4$ super Yang-Mills-theory (SYM) with coupling constant $g_{\mathrm{YM}} = 2\pi g_s^2$ and 't Hooft coupling $g_{\mathrm{YM}}^2 N = \lambda$

↓ $g_s \to 0$ (Gravity) ↓ $N \to \infty$ (Field Theory)

Gravity

Type IIb supergravity, $\alpha' > 0$

→ **Field Theory**

$\mathcal{N} = 4$ SYM, λ finite

↓ $\frac{\alpha'}{L^2} \to 0$ (Gravity) ↓ $\lambda \to \infty$ (Field Theory)

Gravity

Type IIb supergravity in $\mathcal{O}(\alpha'^0)$

→ **Field Theory**

Large 't Hooft coupling, planar diagrams

Comparison with lattice results

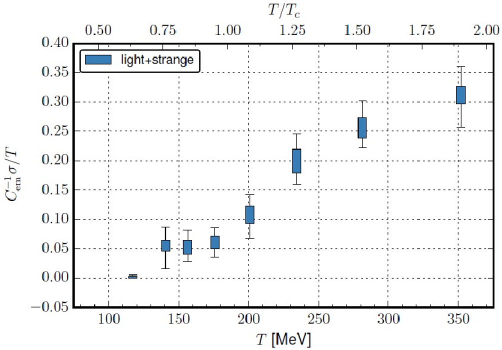


Figure: Temperature dependence of the conductivity to temperature ratio $e^{-2}\sigma/T$ computed on the lattice. The vertical boxes reflect systematic the whiskers statistical errors. [G. Aarts et. al., 2014].

Comparison with lattice results

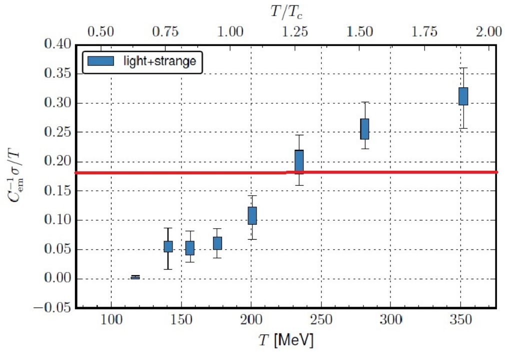


Figure: The conductivity computed via holography in the $\lambda \to \infty$ limit (red line) compared with results of hot QCD lattice calculations.

Type IIb SUGRA action

For $N \to \infty$, $\lambda \to \infty$ the 10 dimensional effective action obtained from closed massless strings is given by

$$\mathcal{S}_{\text{IIB}} = \frac{1}{\kappa_{10}} \int d^{10}X\sqrt{-g}\Big(e^{-2\phi}(R + 4\partial_\mu\phi\partial^\mu\phi - \frac{|H_3|^2}{2}) - \frac{|F_1|^2}{2}$$
$$- \frac{|F_3|^2}{2} - \frac{|F_5|^2}{4}\Big) - \frac{1}{4\kappa_{10}} \int C_4 \wedge H_3 \wedge F_3$$

up to $\mathcal{O}(\alpha'^0)$. The solutions we are interested in imply $H_3 = F_1 = F_3 = 0$, such that

$$S_{10} = \frac{1}{2\kappa_{10}} \int d^{10}x\sqrt{-g}\left[R_{10} - \partial_\mu\phi\partial^\mu\phi - \frac{|F_5|^2}{4 \times 5!}\right].$$

Higher derivative corrections

Either

- compute higher α' corrections to string theory β functions
- Or compute higher string corrections to graviton scattering amplitudes

and model the effective action accordingly.

Defining $\gamma = \frac{\zeta(3)}{8}\lambda^{-\frac{3}{2}}$ with $\lambda \propto \frac{1}{\sqrt{\alpha'}}$ gives

$$S = S_{10} + \gamma S_{10}^{\gamma} + \mathcal{O}(\gamma^{\frac{4}{3}}).$$

$$S_{10}^{\gamma} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{-g} \Big[C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4 \Big].$$

The terms C^4, $C^3\mathcal{T}$, ..., are schematical. They represent several tensor contractions of the form

$$\ldots, C_{abcd} C^{a}{}_{e}{}^{f}{}_{g} C^{b}{}_{fhi} \mathcal{T}^{cdeghi}, C_{abc}{}^{d} C^{abc}{}_{e} \mathcal{T}_{dfghij} \mathcal{T}^{efhgij}, \ldots.$$

Einstein-Maxwell-AdS/CFT in the $\lambda \to \infty$ limit

The type IIb SUGRA action:

$$S_{10} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{-g}\left[R_{10} - \frac{1}{4 \times 5!} F_5^2\right].$$

- To obtain Maxwell-terms $F_{\mu\nu}F^{\mu\nu}$ twist the five sphere S_5 in a maximally symmetric manner. (Chemblin, Emparan, Johnson, 1999)
- We treat this twist as a tiny perturbation of our background geometry.
- Their propagator corresponds to the current-current correlation function in the SYM on the boundary of the corresponding $U(1)$ current.

The perturbed metric

The Schwarzschild-AdS$\times S_5$-solution of the geometry

$$ds_{10}^2 = ds_{\text{AdS}}^2 + \sum_{i=1}^{3} (d\mu_i^2 + \mu_i^2 (d\phi_i + \frac{2}{\sqrt{3}} A_\mu dx^\mu)^2)$$

$$\mu_1 = \sin(y_1),\ \mu_2 = \cos(y_1)\sin(y_2),\ \mu_3 = \cos(y_1)\cos(y_2), \phi_i = y_{i+2}$$

$$ds_{AdS}^2 = -r_h^2 \frac{1-u^2}{u} dt^2 + \frac{1}{4u^2(1-u^2)} du^2 + \frac{r_h^2}{u}(dx^2 + dy^2 + dz^2)$$

is perturbed by a vector field A_μ, whose EoM will be obtained from linearizing the resulting Einstein equations.

- The Ricci scalar becomes

$$R_{10} = R_{10}^{A_\mu \to 0} - \frac{1}{3} F_{\mu\nu} F^{\mu\nu}.$$

- The dilaton field's EoM decouple such that we can focus on the F_5^2-part of the action.

The EoM of the five form

- The EoM of the 5-form F_5

$$d * F_5 = 0,$$

is obtained by varying $\mathcal{S}_{10}$ with respect to C_4, with $dC_4 = F_5$.

- The full solution of the five form in order $\mathcal{O}(\alpha'^0)$ including gauge fields is

$$F_5 = (1 + *)((F_5^0)^{el} + (F_5^1)^{el})$$

$$(F_5^0)^{el} = -4\epsilon_5, \qquad (F_5^1)^{el} = \frac{1}{\sqrt{3}} \sum_{i=1}^{3} d(\mu_i^2) \wedge d\phi_i \wedge \bar{*} F_2.$$

- The EoM for A_μ can be obtained both by varying the action with respect to A_μ and from the $tuyzy_1y_3$, $tuyzy_2y_4$, $tuyzy_2y_5$, $tuyzy_1y_5$ and $tuyzy_1y_4$-directions of $d * dC_4 = 0$:

$$\partial_u^2 A_x - \frac{2u}{1-u^2}\partial_u A_x + \frac{\hat{\omega}^2 - \hat{q}^2(1-u^2)}{u(1-u^2)^2} A_x = 0.$$

Higher derivative corrections to the geometry

From the variation of the action with finite coupling corrections

$$S = \frac{1}{2\kappa_{10}} \int d^{10}x\sqrt{-g}\left[R_{10} - \frac{1}{4\times 5!}F_5^2\right] + \gamma \int d^{10}x\sqrt{-g}\,C^4$$

with respect to the metric and the four form one obtains EoM whose solutions of the form

$$ds_{10}^2 = -r_h^2 U(u)dt^2 + \tilde{U}(u)du^2 + r_h^2 e^{2V(u)}(dx^2+dy^2+dz^2)$$
$$+ L(u)^2 d\Omega_5^2.$$
$$F_5 = -\Big(1+*\Big)\frac{4}{L(u)^5}\epsilon^{\gamma}_{\mathrm{AdS}_5}$$

Here U^2, $\tilde{U}^2$, e^{2V}, and L^2 are the α'-corrected metric components, whose $\lambda \to \infty$ limit gives the AdS-Schwartzschild metric.

Twisted metric ansatz

As in the coupling correction free case we introduce gauge fields by twisting the 5-sphere.

$$ds_{10}^2 = -r_h^2 U(u)dt^2 + \tilde{U}(u)du^2 + r_h^2 e^{2V(u)}(dx^2 + dy^2 + dz^2) +$$
$$L(u)^2 \sum_{i=1}^{3} (d\mu_i^2 + \mu_i^2(d\phi_i + \frac{2}{\sqrt{3}} A_\mu dx^\mu)^2).$$

- We consider transverse fields $A_\mu(u, z, t)$.
- Without loss of generality we may assume that only the x-component is non-vanishing.
- Here U^2, $\tilde{U}^2$, e^{2V}, and L^2 are the α'-corrected metric components, whose $\lambda \to \infty$ limit gives the AdS-Schartzschild metric.

Higher derivative EoM

The EOMs, obtained by varying with respect to the 4-form components ($dC_4 = F_5$) can be shown to be equivalent to

$$d\left(* F_5 - * \frac{2\gamma}{\sqrt{-g}} \frac{\delta S_{10}^{\gamma}}{\delta F_5} \right) = 0, \text{ which yields } F_5 = * \left(F_5 - \frac{2\gamma}{\sqrt{-g}} \frac{\delta S_{10}^{\gamma}}{\delta F_5} \right).$$

where $*$ is the 10-dimensional Hodge star operator. The variation of the action with respect to A_μ can be split in several parts

- The Riemann-part is straightforward again, since

$$R_{10} = \left(R_{10}\big|_{A_\mu \to 0} \right) - \frac{L(u)^2}{3} F_{\mu\nu} F^{\mu\nu}.$$

  as is the C^4-part of the coupling correction terms.
- The calculation of $\frac{\delta \gamma \sqrt{-g} C^2 \mathcal{T}^2}{\delta A_\mu}$ and $\frac{\delta \gamma \sqrt{-g} C^3 \mathcal{T}}{\delta A_\mu}$. is simplified by the fact that

$$\mathcal{T}\big|_{A_\mu = 0} = 0.$$

Higher derivative EoM

Since the only components of the twisted metric, which are first order in A_μ, are $g^{xy_3}, g^{xy_4}, g^{xy_5}$, the only γ-corrected 5−form directions relevant in

$$\frac{\delta\sqrt{-g}F_5^2}{\delta A_\mu}$$

are

$$(F_5)_{tuxyz},(F_5)_{tuyzy_3},(F_5)_{tuyzy_4},(F_5)_{tuyzy_5},(F_5)_{y_1y_2y_3y_4y_5},$$
$$(F_5)_{y_1y_2xy_4y_5},(F_5)_{y_1y_2y_3xy_5},(F_5)_{y_1y_2y_3y_4x}.$$

This heavily restricts the directions of

$$d\left(*F_5 - *\frac{2\gamma}{\sqrt{-g}}\frac{\delta S_{10}^{\gamma}}{\delta F_5}\right) = 0,$$

that have to be considered.

Higher derivative EoM

With the ansatz

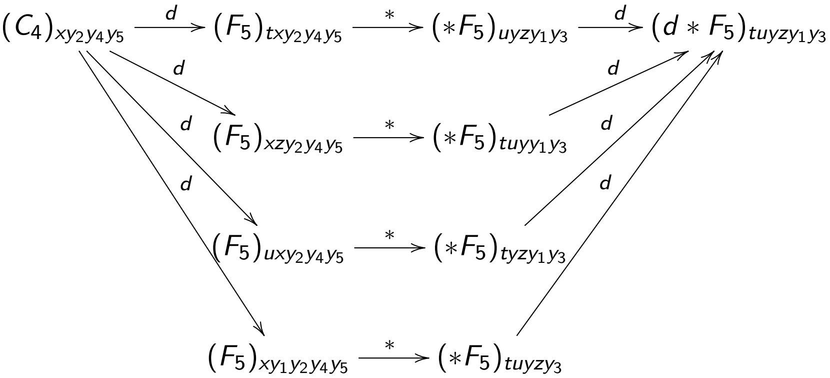
$$(C_4)_{xy_2y_4y_5} = \cos(y_1)^4 \sin(2y_2) \frac{A_x + \gamma C(u,q,\omega)}{\sqrt{3}}.$$

for $(C_4)_{xy_2y_4y_5}$ one can show that

$$-\frac{1}{4\cdot 5!}\frac{\partial\sqrt{-g}F_5^2}{\partial A_x} = \frac{16\gamma C(u,q,\omega)}{3u^2} + \frac{4(F_5)_{tuyzy_3}}{\sqrt{3}\sin(y_1)^2}.$$

Higher derivative EoM

The right hand side of the diagram has to be equal to $\left(d\left(*\frac{2\gamma}{\sqrt{-g}}\frac{\delta S_{10}^{\gamma}}{\delta F_5}\right)\right)_{tuyzy_1y_3}$.



Higher derivative EoM

Applying

$$F_5 = *\left(F_5 - \frac{2\gamma}{\sqrt{-g}}\frac{\delta S_{10}^{\gamma}}{\delta F_5}\right)$$

gives

$$(F_5)_{tuyzy_3} = \sqrt{-g}\, g\, g^{xx} g^{y_1y_1} g^{y_2y_2} g^{y_4y_4} g^{y_5y_5} \Bigg(4\sin(y_1)\cos(y_1)^3 \sin(2y_2)\frac{\gamma C(u,q,\omega)}{\sqrt{3}} - \frac{2\gamma}{\sqrt{-g}}\left(\frac{S_{10}^{\gamma}}{\delta F_5}\right)_{xy_1y_2y_4y_5}\Bigg),$$

which is the final piece in our puzzle.

Higher derivative EoM

- We end up with a coupled differential equation of fourth order for the function $C(u, q, \omega)$, which encodes the α'-correction of the $xy_2y_4y_5$, $xy_2y_3y_5$, $xy_2y_3y_4$-direction of C_4, and the gauge fields A_μ.
- After considering that the relation between the temperature T and the horizon radius r_h get also γ-corrected, one obtains an indicial exponent of $\pm\frac{i\hat{\omega}}{2}$, with $\hat{\omega} = \frac{\omega}{2\pi T}$.
- From the near boundary expansion of these equations and the requirement that $A_\mu/(1-u)^{\frac{-i\hat{\omega}}{2}}$ should be a regular function, one obtains the missing boundary condition $C|_{u=0} = 0$.

$$\partial_u^2 (A_x^1)_k + \frac{2u}{-1+u^2}\partial_u (A_x^1)_k + \frac{(\tilde{q}^2(-1+u^2)+\tilde{\omega}^2)}{u(-1+u^2)^2}(A_x^1)_k + \frac{1}{(48u^2(-1+u^2)^2)}(u^3(-9216\tilde{q}^4u^3(-1+u^2)+\tilde{q}^2(-3900+73507u^2- 145342u^4+75735u^6)+15(520-1061u^2+435u^4)\tilde{\omega}^2)(A_x^0)_k - 2(-1 +u^2)(96C(u,q,\omega)+u^3(-1+u^2)(3900-23846u^2-23040\tilde{q}^2u^3+ 675u^4)\partial_u(A_x^0)_k)) = 0.$$

$$\partial_u^2 (A_x^1)_k + \frac{2u}{-1+u^2}\partial_u (A_x^1)_k + \frac{\tilde{q}^2(-1+u^2)+\tilde{\omega}^2}{u(-1+u^2)^2}(A_x^1)_k + \frac{1}{48u^2(-1+u^2)^2}\big(u^3(-9216\tilde{q}^4u^3(-1+u^2)+\tilde{q}^2(-3900+116931u^2- 260414u^4+147383u^6)+3(2600-10969u^2+7839u^4)\tilde{\omega}^2)(A_x^0)_k + 2(24 (-2+2u^2+\tilde{q}^2u(-1+u^2)+u\tilde{\omega}^2)C(u,q,\omega) - u^2(-1+u^2)(u(-1+u^2) (3900-36702u^2-32480\tilde{q}^2u^3+20895u^4)\partial_u(A_x^0)_k - 24(2u\partial_u C(u,q,\omega)+ (-1+u^2)\partial_u^2 C(u,q,\omega))))\big) = 0.$$

Minkowski prescription and correlators

Solving these EOMs allows us to compute for instance the electric conductivity. The retarded Green's function can be computed following the (Minkowski) prescription:

$$\Pi_\perp = -\frac{N^2 T^2}{8} \lim_{u \to 0} \frac{(A_x)'_k}{(A_x)_k},$$

such that

$$i \int d^4x e^{-iqx} \theta(x_0) \langle [J_\mu(x), J_\nu(0)] \rangle = C^{ret}_{\mu\nu} = P^T_{\mu\nu} \Pi_\perp + P^L_{\mu\nu} P_\parallel,$$

with $P_{\mu\nu} = \eta_{\mu\nu} - \frac{k_\mu k_\nu}{k^2}$, $P^T_{ij} = \delta_{ij} - \frac{k_i k_j}{k^2}$ and zero elsewhere, $P^L_{\mu\nu} = P_{\mu\nu} - P^T_{\mu\nu}$.

Finite coupling corrections to low energy limit

We can determine the higher derivative corrections to the low energy limit of the spectral function, the photoemission rate and the conductivity at once:

$$\chi^{\mu}_{\ \mu} = -4\mathrm{Im}(\Pi_{\perp}) - 2\mathrm{Im}(\Pi_{||}),\ \Pi^{<}_{\mu\nu} = \frac{\chi_{\mu\nu}}{e^{\omega\beta}-1},\ \frac{d\Gamma}{dQ^4} = -\frac{\alpha(\Pi^{<})^{\mu}_{\ \mu}}{24\pi^4 Q^2}$$

$$\chi_{\perp}^{\omega=q} = -4\mathrm{Im}(\Pi_{\perp}) = \frac{N^2T^2}{2}\left((1+\mathbf{125}\gamma)\hat{q} + \mathcal{O}(\hat{q}^2)\right),$$

$$\sigma = -\lim_{\omega\to 0}\frac{e^2}{\omega}\Pi_{xx}(\omega=q)$$

such that we can already read off the correction to the conductivity and the photoemission rate for small frequencies $(1+125\gamma)$. An anlogous computation in the spin $=2$-channel gives a correction of $(1+135\gamma)$ to the transport coefficient there.

- The resummation technique is an approximation using the assumption that the correction terms to the EoM of the gauge fields of higher order are small.
- From the level of the first order EoM we proceed computing exactly in the 't Hooft coupling.
- Applying this to the conductivity for $\lambda = 11.3 \approx 4\pi N \alpha_s|_{N=3,\alpha_s=0.3}$ we obtain a resummed value of
$$\sigma = 0.29082 e^2 T.$$
- Without any coupling corrections the conductivity is given by
$$\sigma_\infty = \frac{9}{16\pi} e^2 T \approx 0.179 e^2 T.$$

This can be compared to results of hot QCD lattice calculations for $T > 1.75 T_c$: $\sigma \approx e^2 T (0.31 \pm 0.05)$

Comparison with lattice results

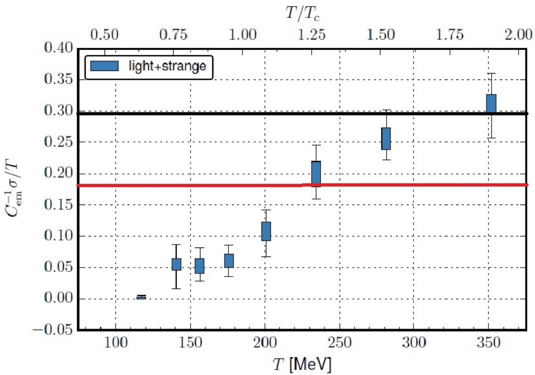


Figure: The conductivity computed via holography in the $\lambda \to \infty$ limit (red line) and the coupling corrected and resummed results for σ/e^2T for $\lambda \approx 11.3$ (black line) compared with results of hot QCD lattice calculations.

Both the low and the high frequency limit of the spectral functions/ photoemission rate coincide with the qualitative **expectations of weak-coupling calculations**:

- In the high energy limit we have

$$\chi_{\perp}^{\omega=q,q\gg 1} = \frac{N^2 T^2}{4} \frac{3^{5/6}\Gamma(\frac{2}{3})}{\Gamma(\frac{1}{3})}\Big((1-80.39\gamma)\hat{q}^{2/3}+\dots\Big)+\mathcal{O}(\gamma^2),$$

- whereas in the low energy limit

$$\chi_{\perp}^{\omega=q} = \frac{N^2 T^2}{2}\Big((1+125\gamma)\hat{q}+\mathcal{O}(\hat{q}^2)\Big)+\mathcal{O}(\gamma^2).$$

α'^3-corrections to the QNM-spectrum

- Quasinormal modes (QNM) correspond to tiny perturbations of the system
- QNM frequencies ω encode the frequency and the decay rate of the response of the system to these perturbations
- The inverse of their negative imaginary part is proportional to the thermalization time τ
- They can be found as the poles of the propagator $\Pi_\perp$
- or by formulation of the differential equations as a generalized eigenvalue problem applying by spectral methods.

α'^3-corrections to the QNM-spectrum

$\hat{q} = 0$	$\gamma = 0$	$\mathcal{O}(\gamma^1)$-correction
1. QNM	$1 - i$	$\gamma(646.132 - 207.258i)$
2. QNM	$2 - 2i$	$\gamma(4896 + 495.5i)$

$\hat{q} = 1$	$\gamma = 0$	$\mathcal{O}(\gamma^1)$-correction
1. QNM	$1.54719 - 0.84972i$	$\gamma(298.289 + 208.678i)$
2. QNM	$2.39890 - 1.87434i$	$\gamma(2357 + 1916i)$

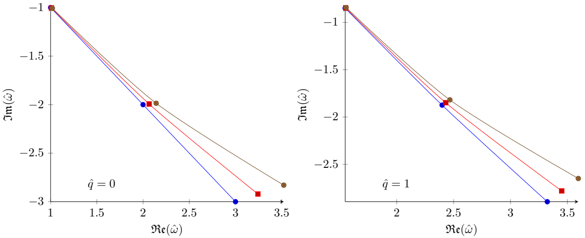


Figure: The first QNM frequencies at $q = 2\pi T$ (right) and $q = 0$ (left) normalized by $2\pi T$ for $\lambda = \infty$ (blue) and their $\mathcal{O}(\gamma)$-corrections for $\lambda = 500$ (red) and $\lambda = 300$ (brown).

The resummed spectrum

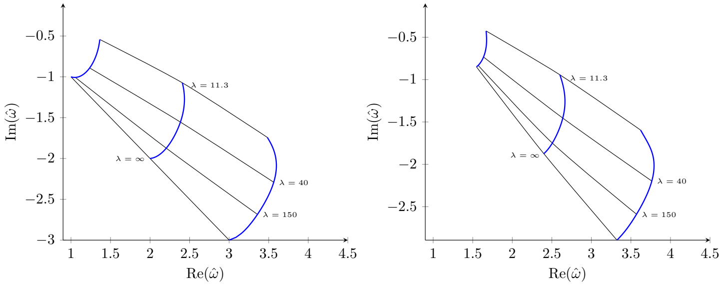


Figure: The flow of the first 3 QNM frequencies, normalized by $2\pi T$, with the 't Hooft coupling between $\lambda = \infty$ and $\lambda = 11.3 \approx 4\pi\alpha_s N$, with $N = 3$ and $\alpha_s = 0.3$ computed in the resummation scheme with $\hat{q} = 0$ (left) and $\hat{q} = 1$ (right). The slopes of the curves at $\gamma = 0$ give the first order corrections.

Off-equilibrium spectral density

The coupling corrected off-equilibrium spectral density is given by

$$\chi(\hat{\omega}, u_s) = \frac{N^2 T^2}{2}(1 - \frac{265}{8}\gamma)\text{Im}\left(\frac{\partial_u A_+}{A_+}\right)\Big|_{u=0},$$

with

$$\text{Im}\left(\frac{A'_+}{A_+}\right)\Big|_{u=0} = \text{Im}\left(\frac{\frac{c_{\text{out}}}{c_{\text{in}}}\partial_u A_{\text{out}} + \partial_u A_{\text{in}}}{\frac{c_{\text{out}}}{c_{\text{in}}}A_{\text{out}} + A_{\text{in}}}\right)\Big|_{u=0}.$$

We compare the cases $u_s = 1$ and $u_s = \frac{r_h^2}{r_s^2}$ with $r_s > r_h$ by calculating the quantity

$$R(\hat{\omega}, u_s) = \frac{\chi(\hat{\omega}, u_s) - \chi(\hat{\omega}, 1)}{\chi(\hat{\omega}, 1)}.$$

Off-equilibrium spectral density

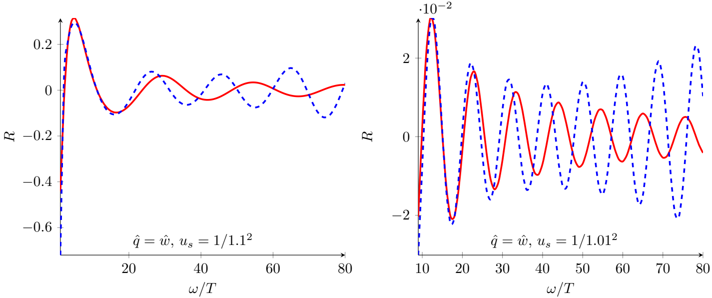


Figure: The function $R_\perp$ plotted for $r_h = 1.1$ on the left side and $r_h = 1.01$ on the right side. In both pictures the solid red line represents the $\lambda = \infty$ limit, whereas the dashed blue line shows the $\mathcal{O}(\gamma^1)$ corrected results at $\lambda = 300$.

A collection of coupling corrections

Quantity	$\mathcal{O}(\gamma^0)$	$\mathcal{O}(\gamma^1)$
$s\,(\frac{1}{2}\pi^2 N_c^2\,T^3)^{-1}$	1	$15\,\gamma$
$\eta\,(\frac{1}{8}\pi N_c^2\,T^3)^{-1}$	1	$135\,\gamma$
$4\pi\,\eta/s$	1	$120\,\gamma$
$\sigma\,(\frac{1}{4}\alpha_{\mathrm{EM}} N^2\,T)^{-1}$	1	$125\,\gamma$
$\omega_2^{\mathrm{shear}}(q=0)\,(2\pi T)^{-1}$	$2.585 - 2.382\,i$	$(1.029 + 0.957\,i)\,10^4\,\gamma$
$\omega_2^{\mathrm{EM}}(q=0)\,(2\pi T)^{-1}$	$2 - 2\,i$	$(4.896 + 0.495\,i)\,10^3\,\gamma$

Table: A collection of results for the zeroth and first order terms in the expansion of various thermal observables in powers of $\gamma = \frac{1}{8}\,\zeta(3)\lambda^{-3/2}$.

Summary and Outlook

- Coupling corrections to the photoemmission rate behave as predicted by weak coupling perturbative calculations in the high and the low energy limit.
- The corrections to the QNM-spectrum, the transport coefficients and spectral functions are of the same order of magnitude as in the spin 2 channel.
- If resummed, the corrections are under control even for λ values close to the QCD-limit.
- The resummed coupling corrected conductivity comes close to hot-QCD-lattice results for realistic λ values.
- Outlook: Compute higher derivative corrections to dynamical processes like shockwave collisions/ isotropization etc.

Backup Slides

Backup Slides

The coupling correction part of the $\mathcal{O}(\alpha'^3)$-action:

$$S_{10}^{\gamma} = \frac{1}{2\kappa_{10}} \int d^{10}x \sqrt{-g} \left[C^4 + C^3\mathcal{T} + C^2\mathcal{T}^2 + C\mathcal{T}^3 + \mathcal{T}^4 \right].$$

$$C_{abcd} = R_{abcd} - \frac{1}{8}(g_{ac}R_{db} - g_{ad}R_{cb} - g_{bc}R_{da} + g_{bd}R_{ca}) + \frac{1}{72} \times \\ \times \left(R g_{ac} g_{db} - R g_{ad} g_{cb}\right),$$

$$\mathcal{T}_{abcdef} = i\nabla_a F^{+}_{bcdef} + \frac{1}{16}\left(F^{+}_{abcmn}F^{+mn}_{def} - 3F^{+}_{abfmn}F^{+mn}_{dec}\right),$$

with two sets of antisymmetrized indices a, b, c and d, e, f. In addition the right hand side is symmetrized with respect to the interchange of $(a, b, c) \leftrightarrow (d, e, f)$ (Paulos, 2008).

Backup Slides

As in the coupling correction free case we introduce gauge fields by twisting the 5-sphere.

$$ds_{10}^2 = -r_h^2 U(u)dt^2 + \tilde{U}(u)du^2 + r_h^2 e^{2V(u)}(dx^2 + dy^2 + dz^2) + L(u)^2 \frac{4A_x(u,t,z)^2}{3} dx^2 + L(u)^2 \frac{4A_x(u,t,z)}{\sqrt{3}} dx \left(dy_3 \sin(y_1)^2 + dy_4 \cos(y_1)^2 \sin(y_2)^2 + dy_5 \cos(y_1)^2 \cos(y_2)^2\right) + L(u)^2 \left(dy_1^2 + \cos(y_1)^2 dy_2^2 + \sin(y_1)^2 dy_3^2 + \cos(y_1)^2 \sin(y_2)^2 dy_4 + \cos(y_1)^2 \cos(y_2)^2 dy_5^2\right).$$

We consider transverse fields $A_\mu(u,z,t)$. Without loss of generality we may assume that only the x-component is non-vanishing. Here U^2, $\tilde{U}^2$, e^{2V}, and L^2 are the α'-corrected metric components, whose $\lambda \to \infty$ limit gives the AdS-Schartzschild metric.