THE GAUGE/GRAVITY CORRESPONDENCE FOR THE BFSS MATRIX MODEL

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Precision lattice test of the gauge/gravity duality at large-N

PRD 94, 094501 (2016), arXiv: 1606.04951 [hep-lat],

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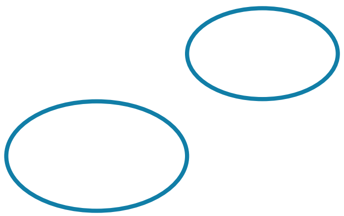
Shinji Shimasaki (Keio University),
Pavlos Vranas (LLNL)

Main contribution (Evan Berkowitz, Enrico Rinaldi)

Introduction

- Fundamental objects in string theory

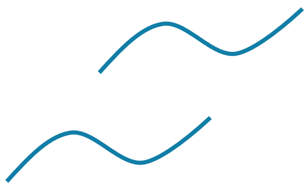
Closed strings



Massless modes
include gravitons

$$g_{\mu\nu}, \phi, \cdots$$

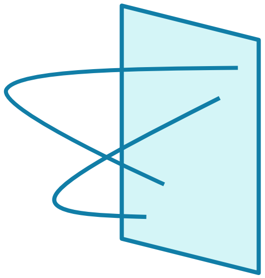
Open strings



Massless modes
include gauge fields

$$A_\mu, \phi^i, \cdots$$

D-branes



Originally introduced
as hypersurfaces on which
strings can have endpoints

Massive modes
(oscillating modes) $m \sim \dfrac{1}{l_s}$

- String theory is quantizable without any UV divergence and gives a consistent theory of quantum gravity

Problems of string theory

- String theory can be quantized only on simple background geometries (flat, pp-wave). Furthermore, the known formulation is perturbative and does not include nonperturbative information like the dynamics of D-branes.

- Nonperturbative formulation is needed (like the lattice theory for field theories)

The gauge/gravity correspondence

- A conjecture between gravitational and non-gravitational theories [Maldacena]

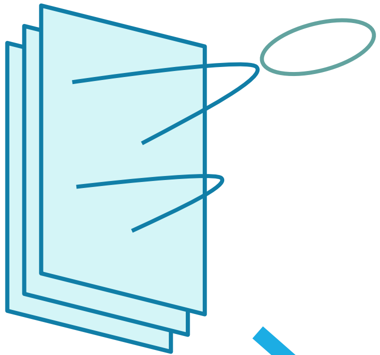
Closed string theory (gravity) = Open string theory (gauge theory)

Equivalent ?

- If this is true, the (lattice) gauge field theory may provide a nonperturbative formulation of the closed string theory, which is a theory of quantum gravity!

The gauge/gravity correspondence from D-branes

Let us consider N coincident D-branes with very large N
(⇒ total mass is very heavy)
Living in 10D space time



They interact with bulk gravity (closed strings) but we can take the decoupling limit

Newton constant ~ $(\text{Length})^8$

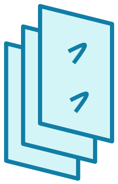
At sufficient large distance scale (low energy) the bulk interaction becomes negligible

D-branes can be described by the gauge theory on D-branes in the low energy limit

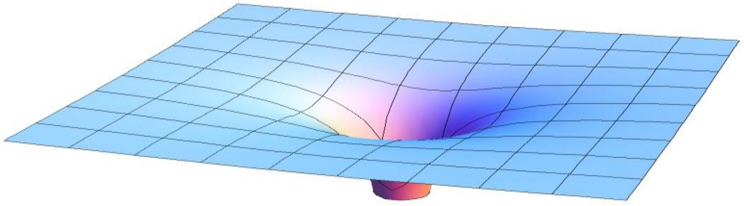
=

?

The heavy D-branes can also be described as sources of closed strings (gravity)

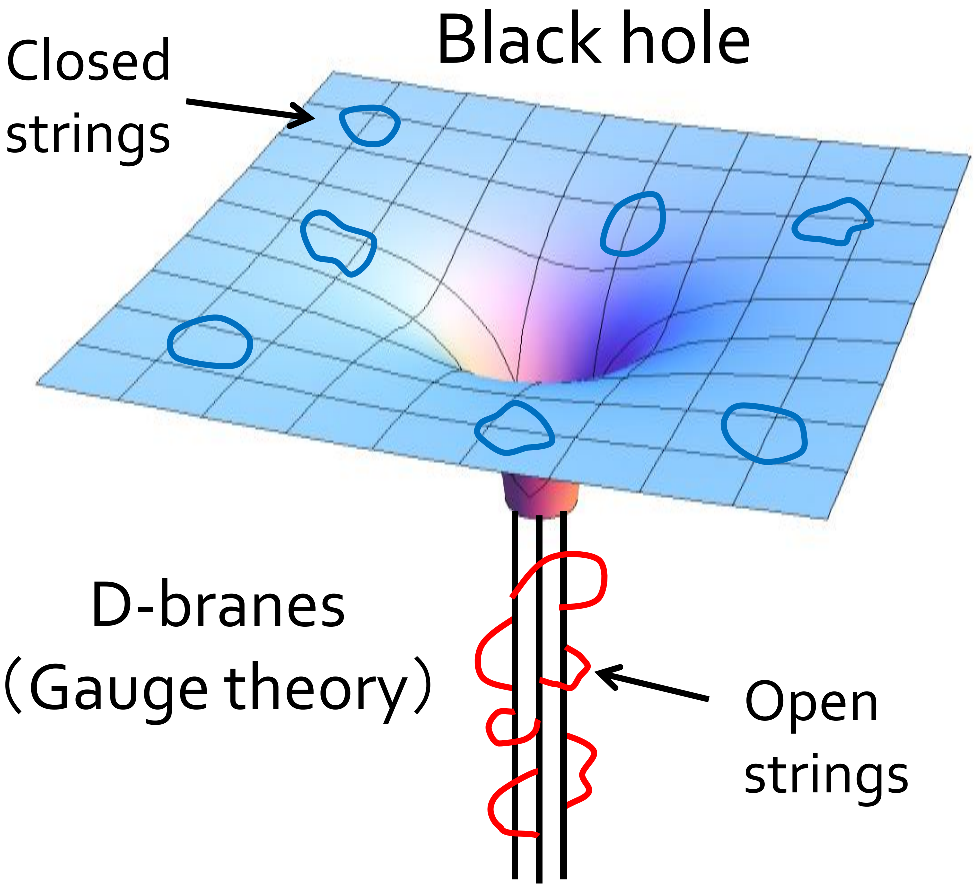


Open string (gauge theory) picture



Closed string (gravity) picture

Schematic picture:



Closed string theory on the near horizon geometry is conjectured to be equivalent to the gauge theory on D-branes

Various gauge/gravity correspondences

- Dp-branes are (p+1)-dimensional BPS objects

	Dimension of World volume	Low energy Effective theory	Corresponding String theory	
D0	$0+1$	BFSS matrix model ((0+1)d SYM)	IIA superstring Black 0-brane	We consider this case
D1	$1+1$	(1+1)d SYM with 16SUSY	IIB superstring Black 1-brane	
D2	$2+1$	(2+1)d SYM with 16SUSY	IIA superstring Black 2-brane	
D3	$3+1$	(3+1)d N=4 SYM	IIB superstring Black 3-brane	AdS/CFT
⋮				

Number of D-branes N = The rank of the gauge group U(N)

Can we prove the conjecture?

- At this moment, it seems too difficult to prove (or disprove) the correspondence….

  ∵ Strong/Weak duality (string th. at weak coupling ⇔ gauge th. at strong coupling)

- Toward this goal, having many evidences (or counter evidences) should be important.

- So far, a lot of positive evidences have been found mainly for the supersymmetric operators for which one can use some analytical methods (integrability, localization etc).

- However, the nonsupersymmetric sector seems very difficult

  - General non SUSY (non BPS) operators
  - Thermodynamics at finite temperature (antiperiodic b.c. for fermions)

  ⇒ the known analytic methods do not work

- The numerical method is promising for studying the gauge/gravity correspondence. It can also be applied to non SUSY problems. Today, we will consider:

Thermodynamics of black holes ⇔ Thermodynamics of gauge theory

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In this paper, we showed that the internal energy of the blackhole on the gravity side is reproduced from the numerical calculation of the gauge theory side

Plan of my talk

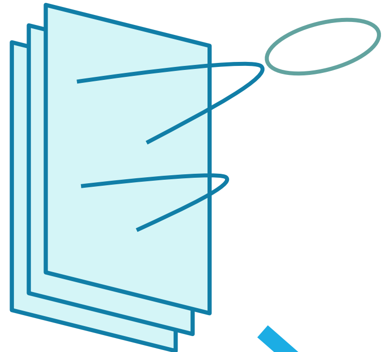
1. Introduction
2. Gravity picture of D0-branes
3. Gauge theory picture of D0-branes

    Results from our numerical simulations

4. Summary

2. Gravity picture of D0-branes

Let us consider N coincident D-branes with N very large (total mass is very heavy)



They interact with bulk gravity (closed strings) but we can take the decoupling limit

Newton constant $\sim$ (Length)$^{D-2}$

D-branes can be described by **the gauge theory** on D-branes in the low energy limit

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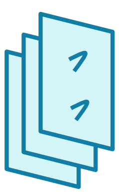
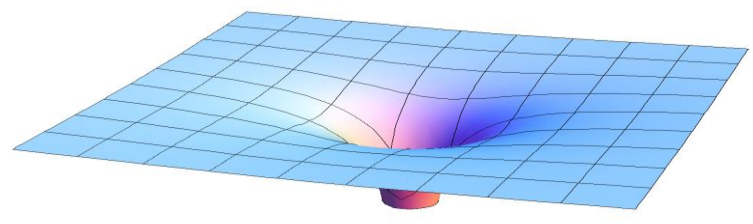
The heavy D-branes can also be described as **sources of closed strings (gravity)**

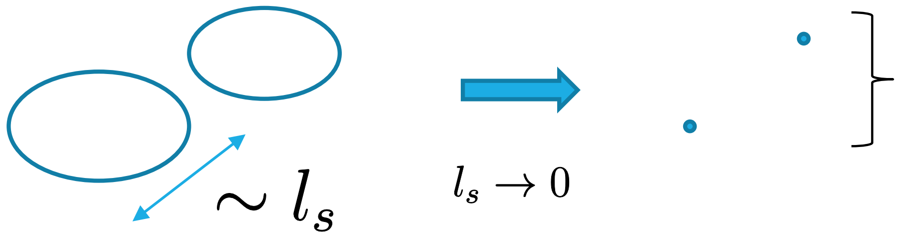


Open string (gauge theory) picture



Closed string (gravity) picture

◆ Low energy limit of the type IIA superstring theory = IIA supergravity (SUGRA)



Massive modes decouple

$$m \sim \frac{1}{l_s} \to \infty$$

$$\mathcal{S}_{\rm IIAsugra} = \frac{1}{2\kappa_{10}^2} \int d^{10}x \sqrt{-g} \left\{ e^{-2\phi} \left(R + 4\partial_\mu \phi \partial^\mu \phi \right) - \frac{1}{4} G_{\mu\nu} G^{\mu\nu} \right\} + \cdots$$

- $2\kappa_{10}^2 = (2\pi)^7 \ell_s^8 g_s^2$: 10D Newton constant
- g_s : string coupling
- l_s : string length $(\alpha' = l_s^2)$

◆ D0-branes are realized as the black 0-brane solution in IIA SUGRA

$$ds^2 = -H^{-\frac{1}{2}} dt^2 + H^{\frac{1}{2}} (dr^2 + r^2 d\Omega_8^2), \quad e^{\phi} = H^{\frac{3}{4}}, \quad G_{tr} = H^{-2} H'$$

$$H = 1 + \frac{60\pi^3 g_s N \ell_s^7}{r^7}$$ mass = charge = $\frac{N}{\ell_s g_s}$ (extremal)

The geometry has the same symmetry & charge as the D0-branes.

[Itzhaki-Maldacena-Sonnenshein-Yankielowicz]

Relation to the parameters in the gauge theory

$$g_{YM}^2 = \frac{g_s}{(2\pi)^2 l_s^3} \qquad U = \frac{r}{l_s^2}$$: energy scale of YM

Fixing these parameters, we take the low energy limit $l_s \to 0$

$$H = 1 + \frac{60\pi^3 g_s N l_s^7}{r^7} = 1 + \frac{2^4 15\pi^5 g_{YM}^2 N}{l_s^4 U^7} \to \frac{2^4 15\pi^5 \lambda}{l_s^4 U^7}$$

$$\lambda = g_{YM}^2 N$$

't Hooft coupling

Thus, we obtain the near horizon geometry of the black 0-brane

$$ds^2 = \alpha' \left(-H^{-\frac{1}{2}} dt^2 + H^{\frac{1}{2}} (dU^2 + U^2 d\Omega_8^2) \right), \quad e^{\phi} = \alpha'^{-\frac{3}{2}} H^{\frac{3}{4}}$$

$$H = \frac{2^4 15\pi^5 \lambda}{U^7}$$

SUGRA description is valid for $1 \gg l_s^2 R \sim \sqrt{\frac{U^3}{\lambda}}$

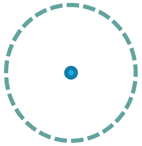
Gauge theory side has to be strongly coupled

◆ Finite temperature : Non-extremal black 0-branes

$$ds^2 = \alpha'\Big(-H^{-\frac{1}{2}} f dt^2 + H^{\frac{1}{2}}(f^{-1}dU^2 + U^2 d\Omega_8^2)\Big), \quad e^{\phi} = \alpha'^{-\frac{3}{2}} H^{\frac{3}{4}}$$

$$H = \frac{2^4 15 \pi^5 \lambda}{U^7}, \quad f = 1 - \frac{U_0^7}{U^7}$$

Horizon at $U = U_0$



- Blackhole temperature
  (Inverse of the Euclidean time period)

$$T = \frac{1}{4\pi} H^{-\frac{1}{2}} f' \Big|_{U=U_0} = c_2 \lambda^{\frac{1}{3}} \left(\frac{U_0}{\lambda^{\frac{1}{3}}}\right)^{\frac{5}{2}}$$

$$c_2 = 7/(2^4 15^{\frac{1}{2}} \pi^{\frac{7}{2}})$$

- Entropy (Hawking-Beckenstein)

$$\frac{S}{N^2} = \frac{1}{N^2} \frac{\mathcal{A}}{4G_N} = c_3 \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{9}{5}}$$

$$c_3 = 4^{\frac{13}{5}} 15^{\frac{2}{5}} (\pi/7)^{\frac{14}{5}}$$

- Internal energy ($dE = TdS$)

$$\frac{1}{N^2} \frac{E}{\lambda^{\frac{1}{3}}} = c_1 \left(\frac{T}{\lambda^{\frac{1}{3}}}\right)^{\frac{14}{5}}$$

$$c_1 = \frac{9}{14} c_3 = 7.407 \cdots$$

$\lambda = 1$ →

$$\frac{E}{N^2} = 7.41\, T^{2.8}$$

($\lambda = g_{YM}^2 N$ has mass dim 3. We fix it to 1)

String loop & α′ corrections

So far, we discussed classical solutions (valid at $g_s \to 0$) of the supergravity, where the supergravity is the low energy limit ($\alpha' \to 0$) of the string theory.

In general parameter region, there should be g_s and α' corrections.
This two parameter expansion is expressed as

$$
\begin{array}{ll}
\qquad\qquad\qquad\qquad\quad \alpha'^3 \qquad\quad \alpha'^5 \qquad \cdots & \\
\dfrac{E}{N^2} = T^{14/5}(a_0 + a_1 T^{9/5} + a_2 T^3 + \cdots) & g_s^0 \\
\qquad + \dfrac{T^{-12/5}}{N^2}(b_0 + b_1 T^{9/5} + \cdots) & g_s^1 \\
\qquad + \dfrac{T^{-27/5}}{N^4}(c_0 + c_1 T^{9/5} + \cdots) & g_s^2 \\
\qquad + \cdots & \vdots
\end{array}
$$

From the leading non-extremal solution, we have $a_0 = 7.41$

The first string loop correction

- From the supersymmetry and studies of string amplitueds, one can find the first g_s correction to SUGRA

$$S_{\mathrm{SUGRA}} \to S_{\mathrm{SUGRA}} + \gamma \Delta S \qquad \gamma = \frac{1}{2^8 4!}\frac{\pi^2}{3}\frac{g_s^2}{\ell_s^6} = \frac{\pi^6}{2^7 3^2}\frac{\lambda^2}{N^2} \equiv \epsilon \lambda^2$$

$$= \frac{1}{2\kappa_{11}^2}\int d^{11}x\sqrt{g}(R + \gamma l_s^{12}[R^4]) + \cdots$$

- Ansatz with SO(9) rotational symmetry

$$ds_{11}^2 = \ell_s^4\Big(-H_1^{-1}F_1 dt^2 + F_1^{-1}U_0^2 dx^2 + U_0^2 x^2 d\Omega_8^2 + \big(\ell_s^{-4} H_2^{\frac{1}{2}} dz - H_3^{-\frac{1}{2}} dt\big)^2\Big),$$

$$H_i = \frac{(2\pi)^4 15\pi\lambda}{U_0^7}\Big(x^{-7} + \frac{\gamma}{U_0^6}h_i\Big), \qquad F_1 = 1 - x^{-7} + \frac{\gamma}{U_0^6}f_1$$

When $\gamma \to 0$, this reduces to the form of the black 0-brane (after the compactification to 10 D).

- One can solve EOM perturbatively in γ and determine h_i, f_1

[Hyakutake]

$$ds_{11}^2 = \ell_s^4\Big(- H_1^{-1}F_1 dt^2 + F_1^{-1}U_0^2 dx^2 + U_0^2 x^2 d\Omega_8^2 + \big(\ell_s^{-4}H_2^{\frac{1}{2}}dz - H_3^{-\frac{1}{2}}dt\big)^2\Big),$$

$$H_i = \frac{(2\pi)^2 15\pi\lambda}{U_0^7}\Big(\frac{1}{x^7} + \epsilon\frac{\lambda^2}{U_0^6}h_i\Big),\quad (i=1,2,3),\qquad F_1 = 1 - \frac{1}{x^7} + \epsilon\frac{\lambda^2}{U_0^6}f_1,\qquad \epsilon = \frac{\pi^6}{2^7 3^2 N^2},$$

$$\left\{\begin{aligned}
h_1 =& \frac{1302501760}{9x^{34}} - \frac{57462496}{x^{27}} + \frac{12051648}{13x^{20}} - \frac{4782400}{13x^{13}} \\
&- \frac{3747840}{x^7} + \frac{4099200}{x^6} - \frac{1639680(x-1)}{(x^7-1)} + 117120\Big(18 - \frac{23}{x^7}\Big)I(x), \\
h_2 =& \frac{19160960}{x^{34}} - \frac{58528288}{x^{27}} + \frac{2213568}{13x^{20}} - \frac{1229760}{13x^{13}} \\
&- \frac{2108160}{x^7} + \frac{2459520}{x^6} + 1054080\Big(2 - \frac{1}{x^7}\Big)I(x), \\
h_3 =& \frac{361110400}{9x^{34}} - \frac{59840032}{x^{27}} - \frac{24021312}{13x^{20}} + \frac{3747840}{x^{14}} - \frac{58072000}{13x^{13}} \\
&- \frac{2108160}{x^7} + \frac{2459520}{x^6} + 117120\Big(18 - \frac{41}{x^7}\Big)I(x), \\
f_1 =& -\frac{1208170880}{9x^{34}} + \frac{161405664}{x^{27}} + \frac{5738880}{13x^{20}} + \frac{956480}{x^{13}} + \frac{819840}{x^7}I(x),
\end{aligned}\right.$$

$$I(x) = \log(x-1) - \log(1-x^{-7}) - \sum_{n=1,3,5}\cos\tfrac{n\pi}{7}\log\left(x^2 + 2x\cos\tfrac{n\pi}{7} + 1\right) - 2\sum_{n=1,3,5}\sin\tfrac{n\pi}{7}\left\{\tan^{-1}\left(\frac{x+\cos\frac{n\pi}{7}}{\sin\frac{n\pi}{7}}\right) - \frac{\pi}{2}\right\}.$$

From this solution, one can compute the correction to the internal energy. $b_0 = -5.77$

Summary: What is known on the gravity side

- The internal energy of the black 0 -brane is given as

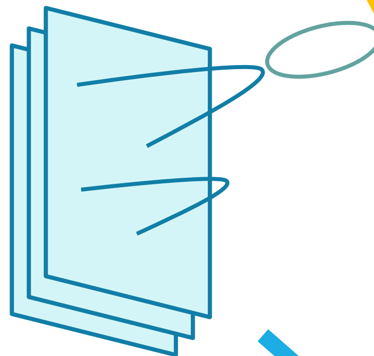
$$\begin{array}{lll} & \qquad\qquad\qquad \alpha'^3 \qquad \alpha'^5 \quad \cdots & \\ \frac{E}{N^2} = & T^{14/5}(a_0 + a_1 T^{9/5} + a_2 T^3 + \cdots) & g_s^0 \\ & +\frac{T^{-12/5}}{N^2}(b_0 + b_1 T^{9/5} + \cdots) & g_s^1 \\ & +\frac{T^{-27/5}}{N^4}(c_0 + c_1 T^{9/5} + \cdots) & g_s^2 \\ & + \cdots & \vdots \end{array}$$

$$\begin{cases} a_0 = 7.41 \\ b_0 = -5.77 \end{cases}$$

The other $\{a_i, b_i, c_i, \cdots\}$ are unknown

3. Gauge theory picture of D0-branes

Let us consider N coincident D-branes with N very large (total mass is very heavy)



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Newton constant ∼ $(\text{Length})^{D-2}$

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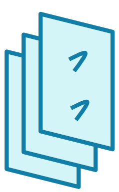
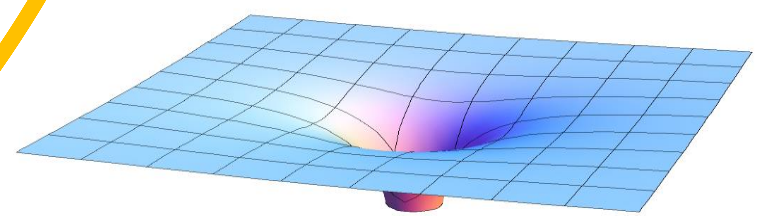
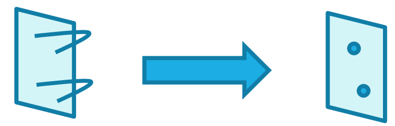




◆ Note that we fixed the gauge theory parameters in taking the decoupling limit

⇒ There remains nontrivial dynamics on the D-branes

BFSS matrix model



◆ The low energy effective theory of N Dp-branes = (p+1)d U(N) Super Yang-Mills theory

◆ The effective theory of N D0-branes = (0+1)d U(N) SYM theory = BFSS matrix model

$$S = \frac{N}{\lambda}\int dt \mathrm{Tr}\left\{\frac{1}{2}(D_t X_i)^2 - \frac{1}{4}[X_i, X_j]^2 + \frac{1}{2}\psi_\alpha D_t \psi_\alpha - \frac{1}{2}\psi_\alpha \gamma^i_{\alpha\beta}[X_i, \psi_\beta]\right\}$$

$$D_t = \partial_t - i[A_t,\ ]$$

SUSY matrix quantum mechanics

$X_i(t)$: N × N Hermitian matrices (i=1,2, ..., 9)

$\psi_\alpha(t)$: N × N Grasmannian matrices

◆ We consider this theory at finite temperature, compactifying the Euclidean time with the circumference $\beta = 1/T$

$X_i(t)$: Periodic b.c.

$\psi_\alpha(t)$: Anti periodic b.c.

⇒ SUSY is broken at finite T

Numerical computation

- Static diagonal gauge

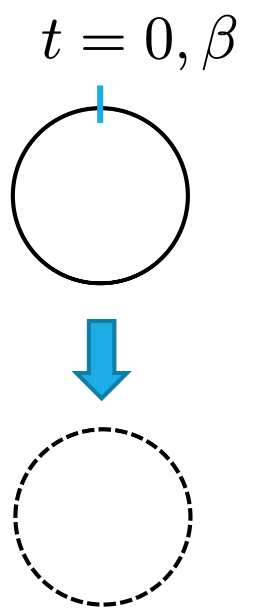
$$A_t = \frac{1}{\beta}\mathrm{diag}(\alpha_1, \cdots, \alpha_N), \qquad -\pi < \alpha_i \leq \pi$$

- Lattice action

$$S_b = \frac{N}{2a}\sum_{t,M}\mathrm{Tr}\left\{(D_+X_M(t))^2\right\} - \frac{Na}{4}\sum_{t,M,N}\mathrm{Tr}\left\{[X_M(t), X_N(t)]^2\right\},$$

$$S_f = \sum_t \mathrm{Tr}\left\{iN\bar{\psi}(t)\begin{pmatrix} 0 & D_+ \\ D_- & 0\end{pmatrix}\psi(t) - aN\sum_{t,M}\bar{\psi}(t)\gamma^M[X_M(t), \psi(t)]\right\},$$

$$S_{F.P.} = -\sum_{i<j}^{N} 2\log\left|\sin\left(\frac{\alpha_i - \alpha_j}{2}\right)\right|,$$



L lattice points

- We apply the usual rational Hybrid Monte Carlo algorithm.

- Here, we replaced the Pfaffian of the fermions with its absolute value. (quenching)
  See also [Filev, O'Connor, 1506.01366, Sec. 4.1, The Pfaffian phase is not a problem!]

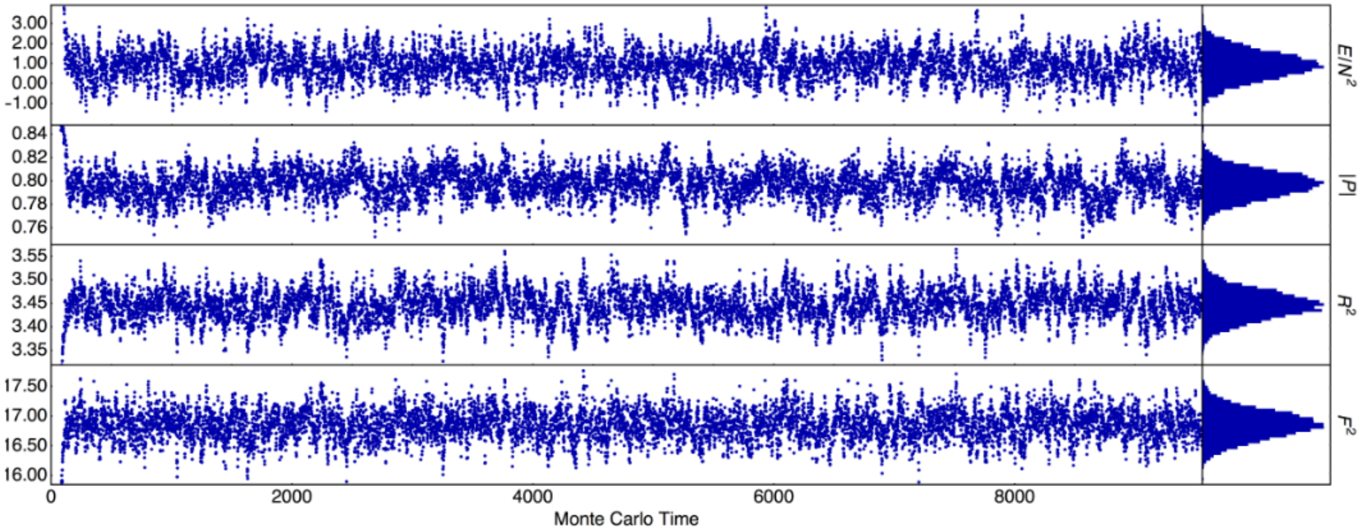
Internal energy of BFSS matrix model

◆ By using the Ward identity, we can write the internal energy of BFSS model as the VEV of the bosonic action.

$$\frac{E}{N^2} = \frac{1}{N^2}\frac{\partial}{\partial\beta}(\beta \log Z)$$
$$= \frac{3}{2N^2\beta}(9(N^2L-1) - 2\langle S_b \rangle)$$

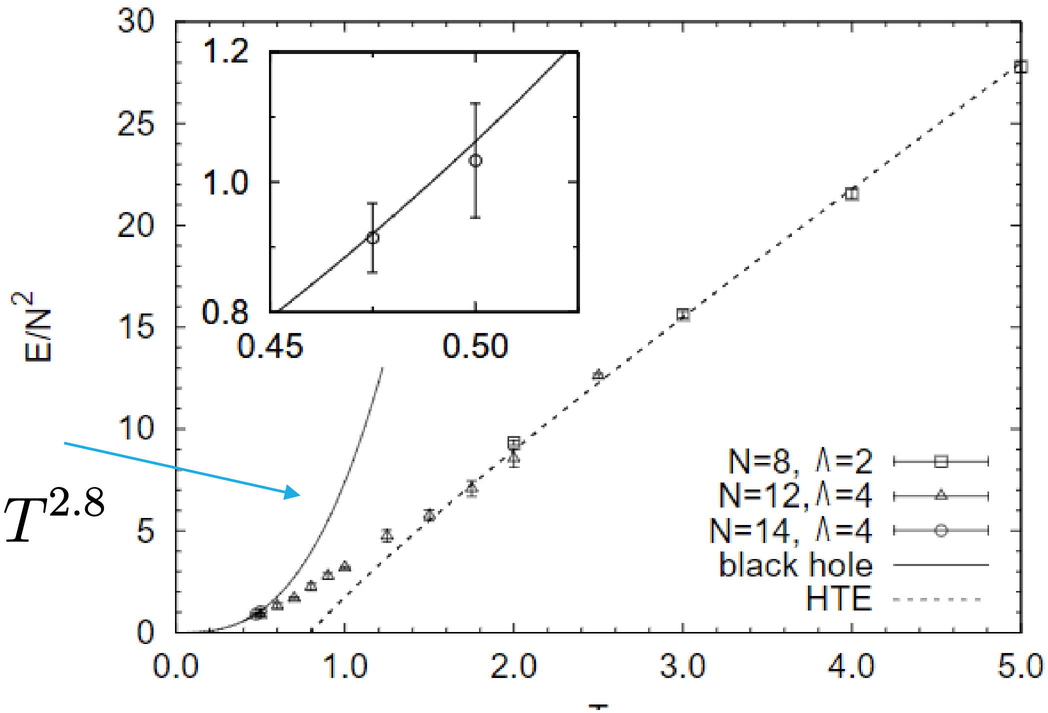
Bosonic part of the action

◆ We compute this VEV using the RHMC method and compare it with the gravity side.



Previous results

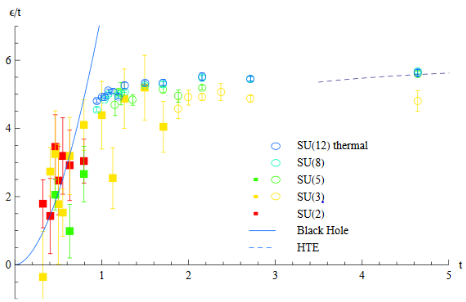
- Anagnostopoulos-Hanada-Nishimura-Takeuchi 2007 (Momentum cutoff reg.)



Leading SUGRA

$$E/N^2 = 7.41T^{2.8}$$

- Catterall-Wisoman 2008



Good agreements but qualitative

Large-N, continuum limit are not taken

Previous work

2007: Anagnostopoulos-Hanada-Nishimura-Takeuchi
2008: Catterall-Wiseman
} Large but finite N & L

2008: Hanada-Hyakutake-Nishimura-Takeuchi
2015: Kadoh-Kamata
} Large but finite N & L
α' correction

2013: Hanada-Hyakutake-Ishiki-Nishimura — Smaller N in the continuum L $\longrightarrow \infty$
g_s correction

2015: Filev-O' Connor — Large but finite N in the continuum L $\longrightarrow \infty$

This talk

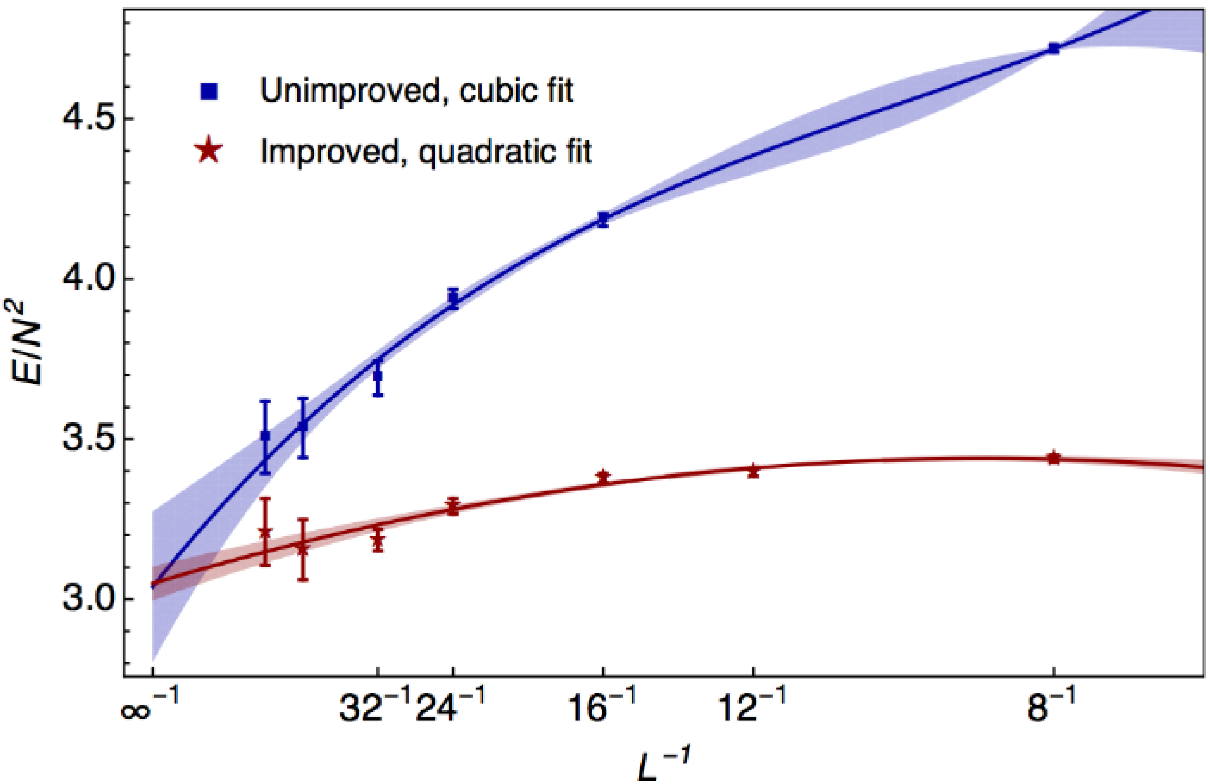
2016: Berkowitz-Rinaldi-Hanada-Ishiki-Shimasaki-Vranas

We took both N $\longrightarrow \infty$ and L $\longrightarrow \infty$ for the first time.
We reproduced $a_0 = 7.41$ from the gauge theory with a good accuracy.

Taking the continuum limit

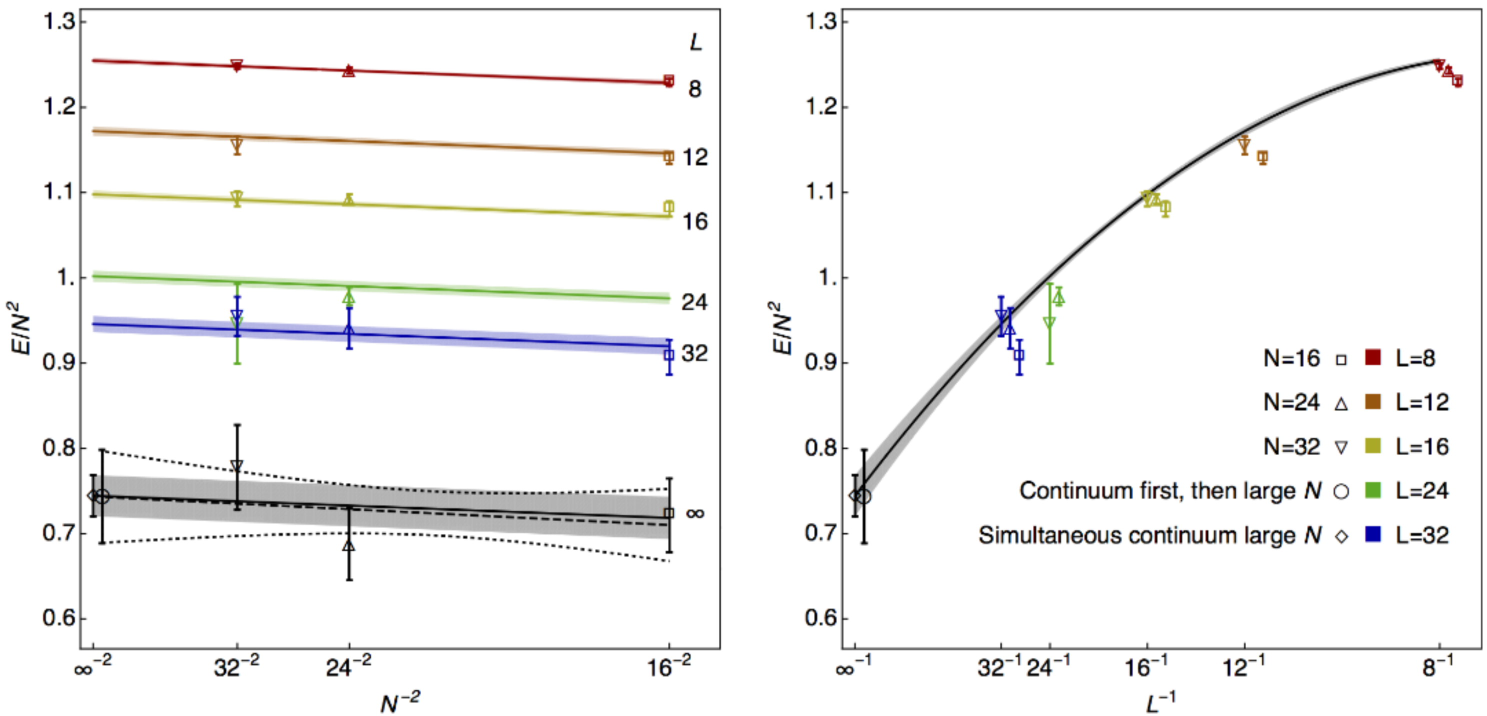
$$\frac{E}{N^2} = e_0 + \frac{e_1}{L} + \frac{e_2}{L^2} + \mathcal{O}\left(L^{-3}\right)$$

T=1, N=16



Taking the Large-N limit and the continuum limit simultaneousely

$$\frac{E}{N^2} = \sum_{i,j\geq 0} \frac{e_{ij}}{N^{2i} L^j}$$



Our target

$$\alpha'^3 \qquad \alpha'^5 \qquad \cdots$$

$$\frac{E}{N^2} = T^{14/5}(a_0 + a_1 T^{9/5} + a_2 T^3 + \cdots) \qquad g_s^0$$

Vanish In the large-N limit

$$+\frac{T^{-12/5}}{N^2}(b_0 + b_1 T^{9/5} + \cdots) \qquad g_s^1$$

$$+\frac{T^{-27/5}}{N^4}(c_0 + c_1 T^{9/5} + \cdots) \qquad g_s^2$$

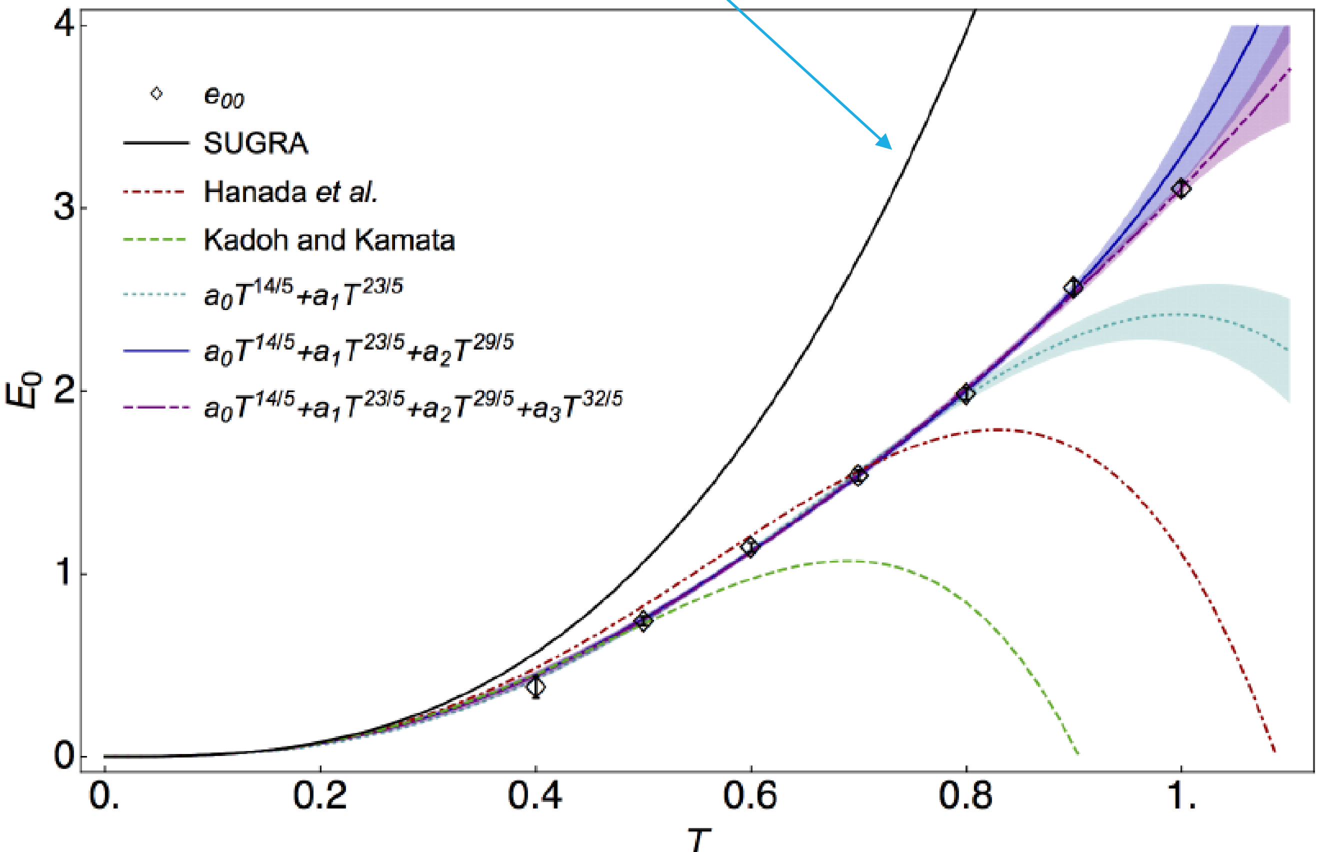
$$+\cdots \qquad \vdots$$

- From the gravity side, $a_0 = 7.41$
- On the gauge theory side, we fit our data by $\frac{E}{N^2} = T^{14/5}(a_0 + a_1 T^{9/5} + a_2 T^3)$ and determine a_0 numerically.

Our result

The leading term on the gravity side

$$E/N^2 = 7.41T^{2.8}$$



$$a_0 = 7.4 \pm 0.5, \quad a_1 = -9.7 \pm 2.2, \quad a_2 = 5.6 \pm 1.8 \qquad \chi^2/DOF = 0.87$$

The result agrees with the gravity side!!

4． Summary

- BFSS matrix model ⇔ IIA superstring on the near horizon geometry of black D0-branes
- On the gravity side, one can compute the thermodynamical quantities of the black zero branes.
- We studied the gauge theory side numerically.
  We took both the large-N and continuum limit for the first time.
- The internal energy in the large-N & continuum limits shows a very good agreement with the gravity side.
- When we fit the data, we used the T-dependence derived from the gravity side

$$\frac{E}{N^2} = T^{14/5}(a_0 + a_1 T^{9/5} + a_2 T^3)$$

More data ⇒ Determine T-dependence as well?