# Simulations of supersymmetric and near conformal gauge theories on the lattice

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# Supersymmetric and conformal theories

Two different approaches to BSM physics...

1. supersymmetry: extensions by symmetry
2. compositeness: new strong scale by new strong dynamics

...lead to similar problems

- tuning on the lattice and the running coupling
- realization of conformal symmetry on the lattice
- non-standard strongly coupled theories with fermions in higher/mixed representations

# Alternative solutions to the Hierarchy problem

Compositeness:

- new strong dynamics beyond the standard model
- Higgs generated as bound state, natural due to scale of additional strong interactions

Symmetry:

- natural explanation by symmetry
- Higgs mass corrections canceled by fermionic partners

Both approaches lead also to interesting theoretical concepts that go beyond the phenomenological applications.

1. Supersymmetry on the lattice
2. Supersymmetric Yang-Mills theory on the lattice
3. The bound state spectrum of SU(3) supersymmetric Yang-Mills theory
4. Towards supersymmetric QCD: near conformal strong dynamics and SUSY theories

in collaboration with S. Ali, H. Gerber, P. Giudice, S. Kuberski, C. Lopez, G. Münster, I. Montvay, S. Piemonte, P. Scior

# Why study SUSY on the lattice?

1. BSM physics: Supersymmetric particle physics requires breaking terms based on an unknown non-perturbative mechanism.
   $\Rightarrow$ need to understand non-perturbative SUSY
2. Supersymmetry is a general beautiful theoretical concept: (Extended) SUSY simplifies theoretical analysis and leads to new non-perturbative approaches.
   $\Rightarrow$ need to bridge the gap between "beauty" and **"reality"**

# Lattice simulations of SUSY theories

Lattice simulations would be the ideal method to investigate non-perturbative sector of SUSY theories . . .

Theory: → next part

- Can we define a lattice SUSY?
- Can we control SUSY breaking?

Practical Simulations: → example SYM

- SUSY theories have nice properties, but require to rework numerical methods

. . . but are challenging from theoretical and practical point of view.

[G.B., S. Catterall, arXiv:1603.04478]

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# SUSY breaking and the Leibniz rule on the lattice

Like Nielsen-Ninomiya theorem: locality contradicts with SUSY

**On the lattice:**

There is no Leibniz rule for a discrete derivative operator. The action can only be invariant with a non-local derivative and non-local product rule. [GB],[Kato,Sakamoto,So],[Nicolai,Dondi]

Further problems:

- fermonic doubling problem, Wilson mass term
- gauge fields represented as link variables

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“The lattice is the only valid non-perturbative definition of a QFT and it can not be combined with SUSY. Therefore SUSY can not exist!” (Lattice theorist)

# General solution by generalized Ginsparg-Wilson relation?

"Mrs. RG, the good physics teacher…"
(Peter Hasenfratz)

Symmetry in the continuum ($S[(1+\varepsilon\tilde{M})\varphi]=S[\varphi]$) implies relation for lattice action $S_L$:

**Generalized Ginsparg-Wilson relation**

$$M^{ij}_{nm}\phi^j_m\frac{\delta S_L}{\delta\phi^i_n}=(M\alpha^{-1})^{ij}_{nm}\left(\frac{\delta S_L}{\delta\phi^j_m}\frac{\delta S_L}{\delta\phi^i_n}-\frac{\delta^2 S_L}{\delta\phi^j_m\delta\phi^i_n}\right)$$

$$\Phi[\tilde{M}\varphi]=M_{nm}\Phi_m[\varphi]$$

Still open problem how to find solutions. [GB, Bruckmann, Pawlowski]

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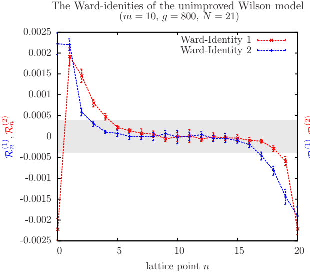
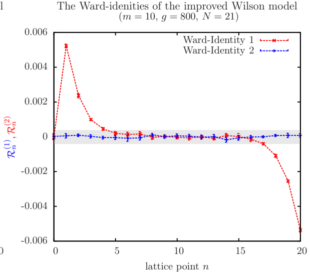
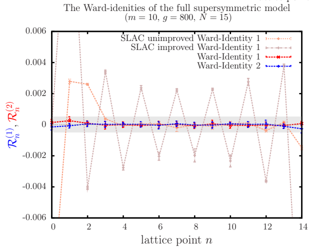
…but we still don't completely understand her lesson.

# Sketch of solutions




- only model dependent solutions
- partial realization of extended SUSY
- non-local actions
- otherwise: fine tuning.

# Super Yang-Mills theory

**Supersymmetric Yang-Mills theory:**

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \frac{i}{2}\bar{\lambda}\not{D}\lambda - \frac{m_g}{2}\bar{\lambda}\lambda\right]$$

- supersymmetric counterpart of Yang-Mills theory; but in several respects similar to QCD
- $\lambda$ Majorana fermion in the adjoint representation
- SUSY transformations: $\delta A_\mu = -2i\bar{\lambda}\gamma_\mu\varepsilon$, $\delta\lambda = -\sigma_{\mu\nu}F_{\mu\nu}\varepsilon$

# Why study supersymmetric Yang-Mills theory on the lattice ?

1. extension of the standard model
   - gauge part of SUSY models
   - understand non-perturbative sector: check effective actions etc.
2. controlled confinement [Ünsal,Yaffe, Poppitz] :
   - compactified SYM: continuity expected
   - small $R$ regime: semiclassical confinement
3. connection to QCD [Armoni,Shifman]:
   - orientifold planar equivalence: SYM $\leftrightarrow$ QCD
   - Remnants of SYM in QCD ?
   - comparison with one flavor QCD

# Supersymmetric Yang-Mills theory: Symmetries

SUSY

- gluino mass term $m_g \Rightarrow$ soft SUSY breaking

$U_R(1)$ symmetry, "chiral symmetry": $\lambda \to e^{-i\theta\gamma_5}\lambda$

- $U_R(1)$ anomaly: $\theta = \frac{k\pi}{N_c}$, $U_R(1) \to \mathbb{Z}_{2N_c}$
- $U_R(1)$ spontaneous breaking: $\mathbb{Z}_{2N_c} \overset{\langle\bar{\lambda}\lambda\rangle \neq 0}{\to} \mathbb{Z}_2$

# Supersymmetric Yang-Mills theory on the lattice

**Lattice action:**

$$\mathcal{S}_L = \beta \sum_P \left(1 - \frac{1}{N_c} \Re U_P\right) + \frac{1}{2} \sum_{xy} \bar{\lambda}_x \left(\mathrm{D}_w(m_g)\right)_{xy} \lambda_y$$

- Wilson fermions:

$$\mathrm{D}_w = 1 - \kappa \sum_{\mu=1}^{4} \left[(1-\gamma_\mu)_{\alpha,\beta} T_\mu + (1+\gamma_\mu)_{\alpha,\beta} T_\mu^\dagger\right] + \text{clover}$$

gauge invariant transport: $T_\mu \lambda(x) = V_\mu \lambda(x+\hat{\mu})$;

$$\kappa = \frac{1}{2(m_g+4)}$$

- links in adjoint representation: $(V_\mu)_{ab} = 2\mathrm{Tr}[U_\mu^\dagger T^a U_\mu T^b]$ of SU(2), SU(3)

# Lattice SYM: Symmetries

Wilson fermions:

- explicit breaking of symmetries: ~~chiral Sym. ($U_R(1)$)~~, ~~SUSY~~

fine tuning:

- add counterterms to action
- tune coefficients to obtain signal of restored symmetry

special case of SYM:

- tuning of $m_g$ enough to recover chiral symmetry [1]
- same tuning enough to recover supersymmetry [2]

[1] [Bochicchio et al., Nucl.Phys.B262 (1985)]

[2] [Veneziano, Curci, Nucl.Phys.B292 (1987)]

# Recovering symmetry

**Fine-tuning:**

chiral limit = SUSY limit $+O(a)$, obtained at critical $\kappa(m_g)$

- no fine tuning with Ginsparg-Wilson fermions (overlap/domainwall) fermions[3]; but too expensive

practical determination of critical $\kappa$:

- limit of zero mass of adjoint pion $(a-\pi)$

$\Rightarrow$ definition of gluino mass: $\propto (m_{a-\pi})^2$

- cross checked with SUSY Ward identities

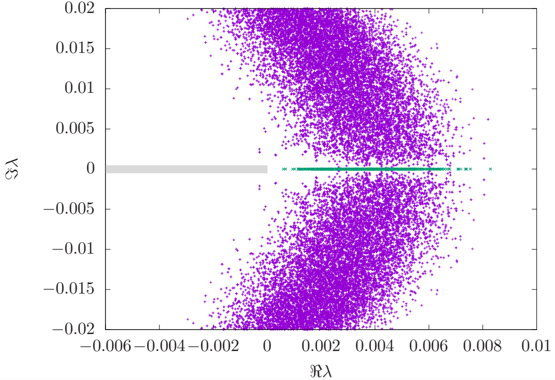
[3] [Fleming, Kogut, Vranas, Phys. Rev. D 64 (2001)], [Endres, Phys. Rev. D 79 (2009)], [JLQCD, PoS Lattice 2011]

# The sign problem in supersymmetric Yang-Mills

Majorana fermions:

$$\int \mathcal{D}\lambda e^{-\frac{1}{2}\int \bar{\lambda} D \lambda} = \mathrm{Pf}(CD) = (-1)^n \sqrt{\det D}$$

$n$ = number of degenerate real negative eigenvalue pairs




no sign problem
@ current
parameters

# Low energy effective theory

| | multiplet[1] | multiplet[2] |
|---|---|---|
| scalar | meson $a - f_0$ | glueball $0^{++}$ |
| pseudoscalar | meson $a - \eta'$ | glueball $0^{-+}$ |
| fermion | gluino-glue | gluino-glue |

- confinement: colourless bound states
- symmetries + confinement → low energy effective theory
- glueballs, gluino-glueballs, gluinoballs (mesons)
- build from chiral multiplet type

[1] [Veneziano, Yankielowicz, Phys.Lett.B113 (1982)]

[2] [Farrar, Gabadadze, Schwetz, Phys.Rev. D58 (1998)]

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**Supersymmetry**

Particles must have same mass.

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- build from chiral multiplet type

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# Bound states in supersymmetric Yang-Mills theory

- like in YM and QCD: glueball bound states of gluons
- meson states (like flavour singlet mesons in QCD)

$$\text{a–}f_0 \ : \quad \bar{\lambda}\lambda \ ; \qquad \text{a–}\eta' \ : \quad \bar{\lambda}\gamma_5\lambda$$

- gluino-glue spin-1/2 state

$$\sum_{\mu,\nu} \sigma_{\mu\nu} \mathrm{tr}\left[F^{\mu\nu}\lambda\right]$$

Quite challenging to get good signal for the correlators of these operators. Mass determined from exponential decay of the correlator.

# The status of the project

Advanced methods of lattice QCD required:

- disconnected contributions [LATTICE2011]
- eigenvalue measurements [GB,Wuilloud]
- variational methods (including mixing of glueball and meson operators) [LATTICE2017]

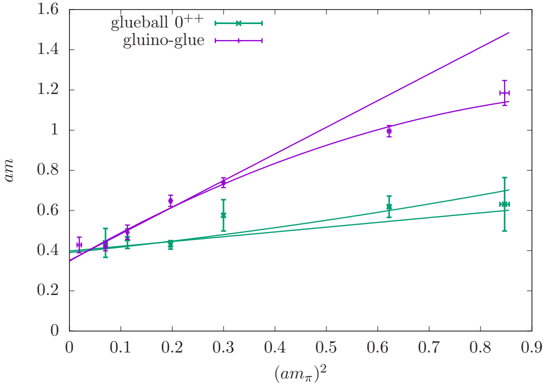
SU(2) SYM:

- multiplet formation found in the continuum limit of SU(2) SYM [JHEP 1603, 080 (2016)]

SU(3) SYM:

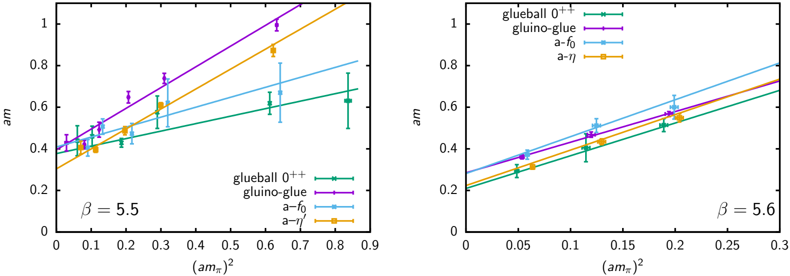
- adjoint representation much more demanding than fundamental one (limited to small lattice sizes)
- first SU(3) simulations [LATTICE99,LATTICE2016,LATTICE2017]
- results presented here: [arXiv:1801.08062]

# The fermion-boson mass degeneracy



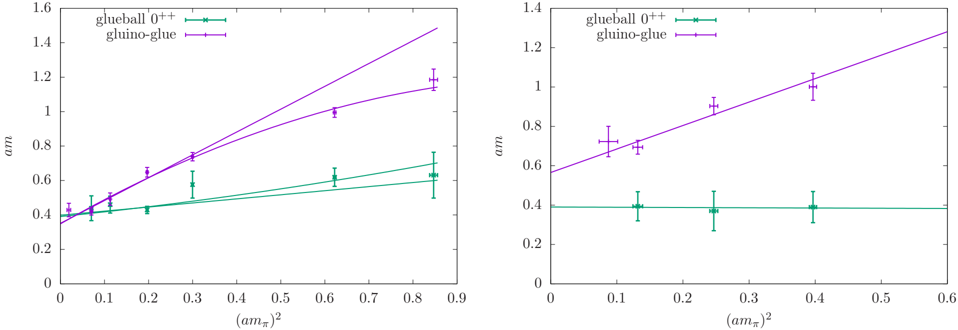

- gluino-glue and glueballs become degenerate in the chiral limit (lattice size $16^3 \times 32$, $\beta = 5.5$)

# The mesonic states and complete multiplet




- within errors: degeneracy of SUSY multiplet at two different lattice spacings

# Coarser lattices: gap in the particle spectrum




- complete continuum extrapolations: coming soon

# Going beyond $\mathcal{N} = 1$ SYM

General tuning approach:

- $O(a)$ SUSY breaking on the lattice
- radiative corrections lead to relevant breaking, compensated by counterterms
- required tuning: all operators with dimension less than four
- simplified approach: assume Ginsparg-Wilson fermions that preserve the R-symmetry

[J. Giedt,Int.J.Mod.Phys. A24 (2009)]

$\Rightarrow$ important additional problem: conformal theories (S-duality, $\mathcal{N} = 4$ SYM ...)

# Some history of composite Higgs and Technicolour

The attractive idea of Technicolour

- natural introduction of a new dynamical generated scale by additional strong interactions

the failure and the recovery

- plain Technicolour can not explain the large difference between the suppressed FCNC and fermion mass generating operators
- cure might be due to a walking behaviour of the running coupling

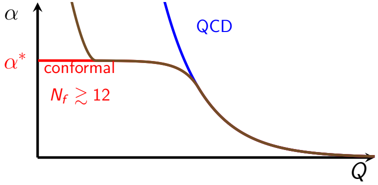
# Walking Technicolour and the conformal window

Walking Technicolour scenario:

- near conformal running of the gauge coupling to accommodate fermion mass generation and absence of FCNC
- approximate conformal symmetry might also lead to natural light scalar particle (Higgs)

Interesting general question:

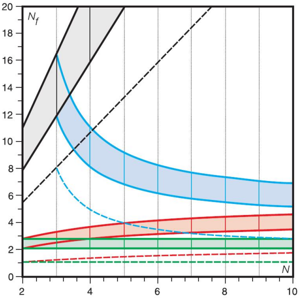
- conformal window, strong dynamics different from QCD
- conformal mass spectrum: $M \sim m^{1/(1+\gamma_m)}$ characterised by constant mass ratios



# Conformal window for adjoint QCD

Gauge theories in higher representation:

- smaller number of fermions needed
- here: conformal window for adjoint representation
- mass anomalous dimension $\gamma_*(N_f)$




[Dietrich, Sannino, hep-ph/0611341]

# Adjoint QCD

adjoint $N_f$ flavour QCD:

$$\mathcal{L} = \mathrm{Tr}\left[-\frac{1}{4}F_{\mu\nu}F^{\mu\nu} + \sum_i^{N_f} \bar{\psi}_i(\not{D} + m)\psi_i\right]$$

$$D_\mu\psi = \partial_\mu\psi + ig[A_\mu, \psi]$$

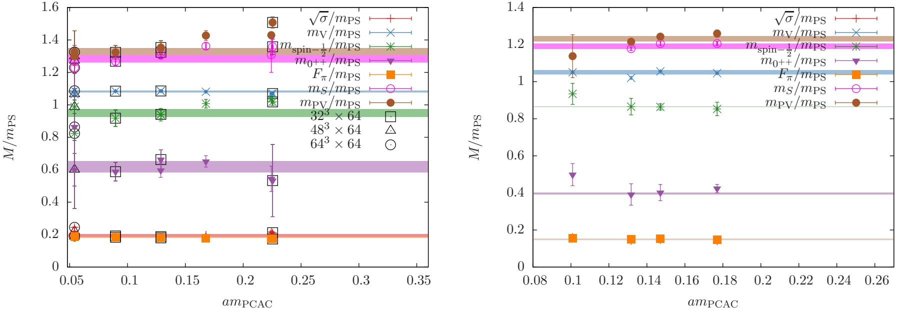
- $\psi$ Dirac-Fermion in the adjoint representation
- adjoint representation allows Majorana condition $\psi = C\bar{\psi}^T$

$\Rightarrow$ half integer values of $N_f$: $2N_f$ Majorana flavours

Chiral symmetry breaking:

$$Z_{2N_c} \times \mathrm{SU}(2N_f) \to Z_2 \times \mathrm{SO}(2N_f)$$

# $N_f = 2$ AdjQCD, Minimal Walking Technicolour



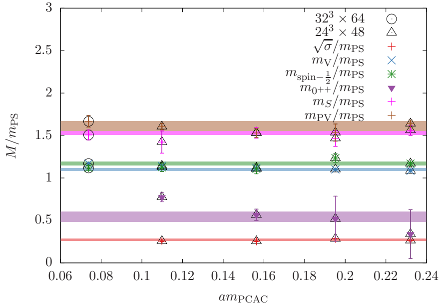

Expected behaviour of a (near) conformal theory:

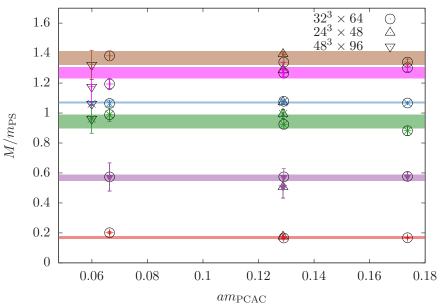
- constant mass ratios
- light scalar ($0^{++}$)
- no light Goldstone ($m_{PS}$)

Well established results: [Debbio, Lucini, Patella, Pica, 2016],[Catterall,Sannino,2007]

[Catterall,Del Debbio,Giedt, Keegan,2012],[GB, Giudice, Münster, Montvay, Piemonte, 2017]

# Results for $N_f = 3/2$ adjoint QCD

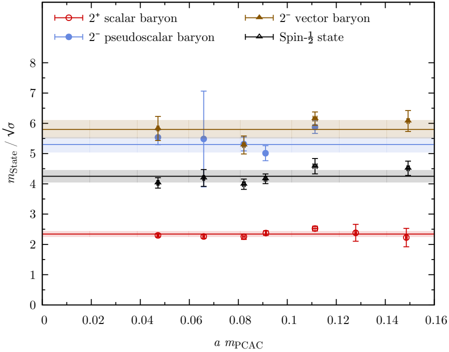


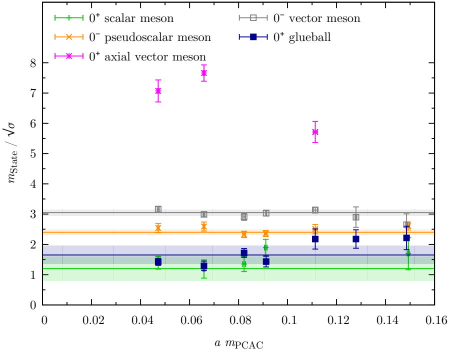




$N_f$ Dirac fermions $\rightarrow 2N_f$ Majorana fermions

- with our experience of supersymmetric Yang-Mills theory, we can simulate half integer fermion numbers
- requires the Pfaffian sign

# Results for $N_f = 1$ adjoint QCD




[Athenodorou, Bennett, GB, Lucini, arXiv:1412.5994]

# Comparison with of adjoint QCD with different $N_f$

| **Theory** | **scalar particle** | $\gamma_*$ small $\beta$ | $\gamma_*$ larger $\beta$ |
|---|---|---|---|
| $N_f = 1/2$ SYM | part of multiplet | – | – |
| $N_f = 1$ adj QCD | light | 0.92(1) | $0.75(4)^*$ |
| $N_f = 3/2$ adj QCD | light | $0.50(5)^*$ | $0.38(2)^*$ |
| $N_f = 2$ adj QCD | light | 0.376(3) | 0.274(10) |

($*$ preliminary)

- remnant $\beta$ dependence: $\gamma_*$ not real IR fixed point values
- final results require inclusion of scaling corrections
- investigation of (near) conformal theory requires careful consideration of lattice artefacts and finite size effects

# Towards more realistic theories: Ultra Minimal Walking Technicolour

- mass anomalous dimension too small in MWT to be a realistic candidate
- $N_f = 1$ has large mass anomalous dimension, but not the required particle content
- $N_f = 1$ in adjoint + $N_f = 2$ in fundamental representation of SU(2) has been conjectured to be ideal candidate (UMWT)
- $N_f = 1/2$ adjoint + $N_f = 2$ in fundamental might also be close enough to conformality

# Extensions towards SUSY theories

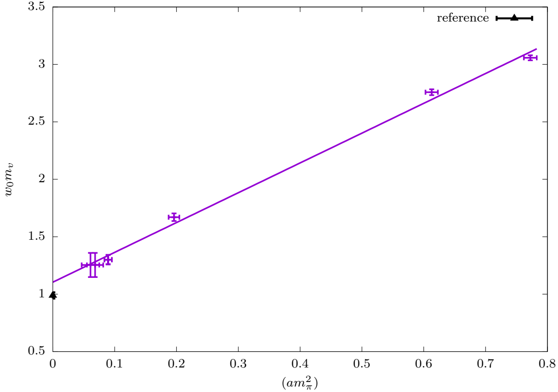
Relation to SUSY theories:

- $N_f = 1$ adjoint QCD corresponds to $\mathcal{N} = 2$ SYM without scalars
- $N_f = 1/2$ adjoint + $N_f = 2$ fundamental corresponds to SQCD without scalars

Ongoing work:

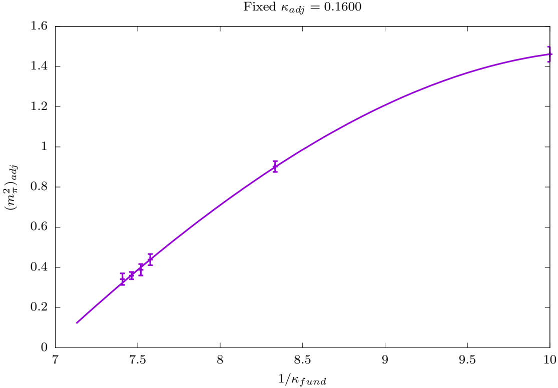
- test mixed representation setup with UMWT
- extend studies with scalars towards SUSY theories

# UMWT: Cross check in pure $N_f = 2$ SU(2) fundamental theory




- reasonable agreement with recent (continuum extrapolated) results [Arthur,Drach,Hansen,Hietanen,Pica,Sannino]
- larger $\beta$ to avoid possible bulk transition

# UMWT: First investigations in mixed representation setup: tuning




- one-loop improved Wilson clover fermions: tuning of fundamental and adjoint not independent

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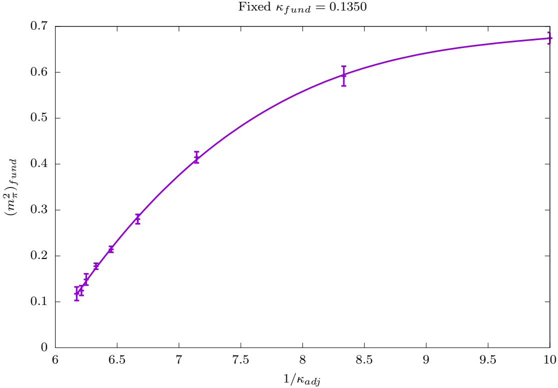


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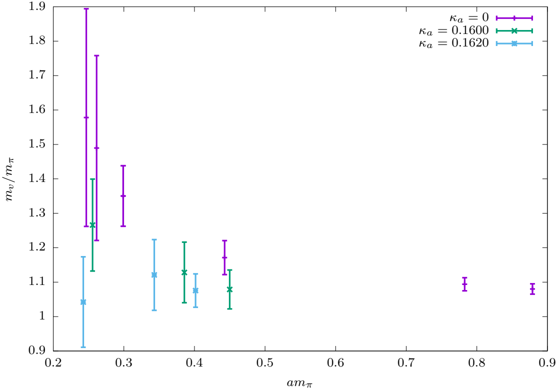


- adjoint flavour drives theory towards near conformal behaviour

# $\mathcal{N} = 1$ SYM and mixed representations: supersymmetric QCD

- add $N_c \oplus \bar{N}_c$ chiral matter superfield to supersymmetric Yang-Mills theory
- SYM + quarks $\psi$ and squarks $\Phi_i$ with covariant derivatives, mass terms and

$$i\sqrt{2}g\bar{\lambda}^a\left(\Phi_1^\dagger T^a P_+ + \Phi_2 T^a P_-\right)\psi$$
$$-i\sqrt{2}g\bar{\psi}\left(P_- T^a \Phi_1 + P_+ T^a \Phi_2^\dagger\right)\lambda^a$$
$$\frac{g^2}{2}\left(\Phi_1^\dagger T^a \Phi_1 - \Phi_2^\dagger T^a \Phi_2\right)^2 .$$

# Why we consider SQCD

- natural extension of supersymmetric Yang-Mills theory
- relation to possible extensions of the standard model
- earlier studies of lattice formulation: perturbative [Costa, Panagopoulos], tuning [Giedt, Veneziano]

SQCD analysis of Seiberg et al.:

- $N_f < N_c$ No vacuum
- $N_f = N_c$ confinement and chiral symmetry breaking
- $\frac{3}{2}N_c < N_f < 3N_c$ infrared fixed point (duality)

Like other SUSY theories beyond $\mathcal{N} = 1$ SYM: conformal or near conformal behaviour

# Why we should better not consider SQCD

- large space of tuning parameters [Giedt] ($O(10)$ parameters)
- just test the mismatch
- might need formulation with Ginsparg-Wilson fermions
- still test it with Wilson fermions
- complex Pfaffian
- related to bosonic symmetry transforming Pf $\rightarrow$ Pf*
- not well behaved chiral limit:
  - either near conformal
  - test near conformal scenario in a related theory
  - or unstable vacuum
  - test with $N_f = 1$ SQCD

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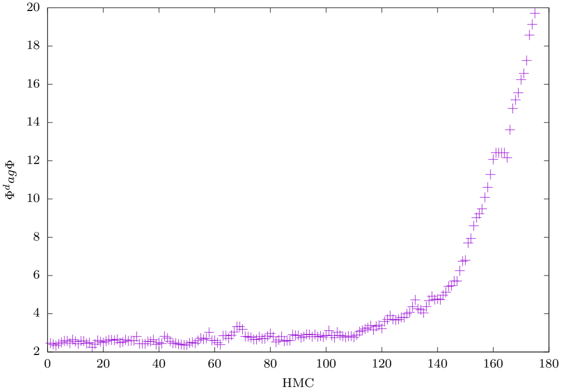
# $N_f = 1$ SU(2) SQCD vacuum




- the expected instability when going chiral

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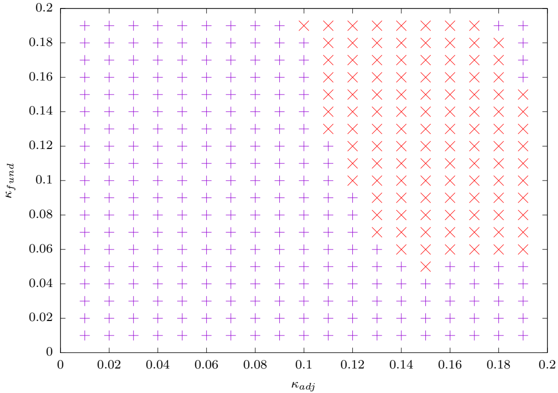



- constraint phase diagram for the parameter tuning
- simulations with an $O(g^0)$ SUSY action

# Conclusions

- simulation of supersymmetric theories on the lattice is still in some aspects an open theoretical problem

Supersymmetric Yang-Mills theory:

- theoretical problem is solvable, practical challenges
- SUSY breaking is under control and formation of chiral multiplet observed for the gauge groups SU(2) and SU(3)
- interesting non-perturbative physics like the phase diagram can be investigated on the lattice
- further aspects of the spectrum are currently investigated: mixing of glueballs and gluinoballs, excited states, further bound states

# Conclusions

Beyond SYM:

- challenging tuning problem
- at the same time challenging additional problem: non QCD-like behaviour
- learn from the investigations of conformal window
- already interesting to study non SUSY related versions of SQCD and $\mathcal{N} = 2$ SYM
- ongoing work: Yukawa couplings + Scalar potential
- Requires analysis in a regime where SUSY is restored in SYM (at least $24^3 \times 48$ lattice with unimproved action)

# SU(2) supersymmetric Yang-Mills theory at finite temperature

Deconfinement:

- above $T_c^{\text{deconf.}}$ plasma of gluons and gluinos
- Order parameter: Polyakov loop

Chiral phase transitions:

- above $T_c^{\text{chiral}}$ fermion condensate melts and chiral symmetry gets restored
- order parameter: $\langle\bar{\lambda}\lambda\rangle$

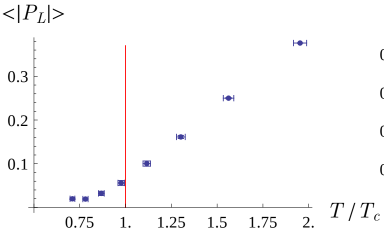
In QCD:

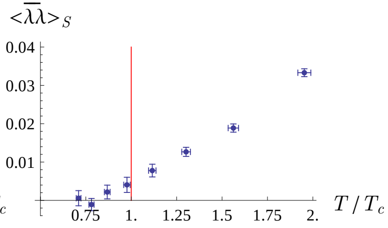
- quarks add screening effects
- explicit chiral symmetry breaking

$\rightarrow$ both transitions become crossover

In SYM: two independent transitions (at $m_g = 0$)

# Lattice results SYM at finite $T$



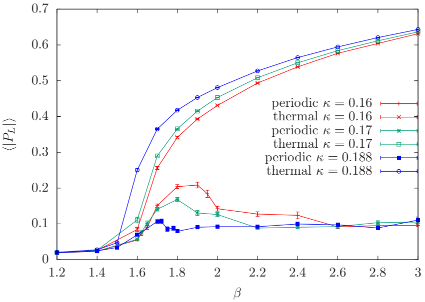



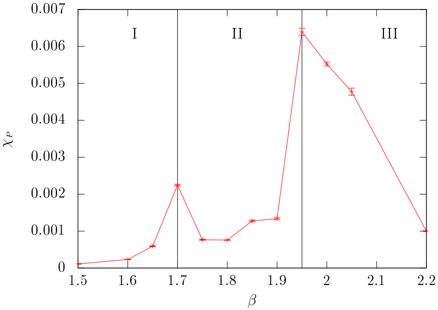

- second order deconfinement transition

$$\frac{T_c(\mathrm{SYM})}{T_c(\text{pure Yang-Mills})} = 0.826(18).$$

- coincidence of deconfinement and chiral transition
  $T_c^{\mathrm{chiral}} = T_c^{\mathrm{deconf.}}$ (within current precision) [JHEP 1411 (2014) 049]

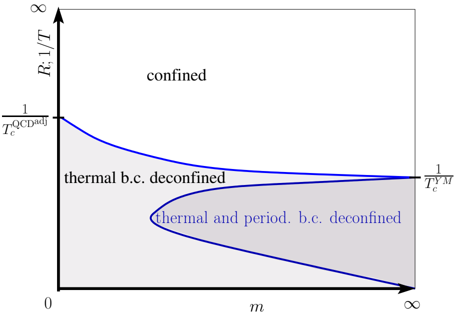
# Compactified SYM with periodic boundary conditions



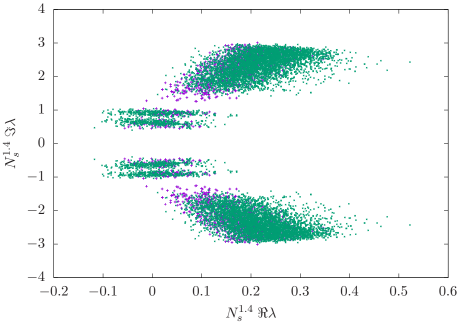

- fermion boundary conditions: thermal $\rightarrow$ periodic
- at small $m$ (large $\kappa$) no signal of deconfinement
- intermediate masses: two phase transitions (deconfinement + reconfinement) [GB,Piemonte]

# Phase diagram at finite temperature/compactification




- change of boundary conditions in compact direction $Z(\beta_B) \to \tilde{Z}(\beta_B)$ (Witten index)
- Witten index can not have $\beta_B$ dependence: states can only be lifted pairwise ⇒ continuity in SYM

# Pfaffian in $N_f = 3/2$ adjoint QCD




- even at the critical parameters: no sign fluctuations of Pfaffian

# What can we learn for phenomenological models?

Next to Minimal Walking Technicolour: $N_f = 2$ SU(3) in sextet representation [Bergner, Ryttov, Sannino, 1510.01763]

- conformal window for adjoint fermions approximately independent of $N_c$
- large $N_c$ up to factor 2 equivalence between symmetric, adjoint, and antisymmetric representation
- small $N_c = 2$ equivalence between symmetric and adjoint representation
- conformal behaviour of $N_f = 1$ indicates conformality of NMWT