

IAS, 26 March, 2018  
CQuest, Sogang u., 29 March, 2018  
MIT, CTP, 4 Apr, 2018  
MPI, AEI, 13 Apr, 2018  
HET group, Osaka, 30 May, 2018  
DLAP2018 workshop, Osaka, 1 June, 2018  
Paris QCD workshop, 11 June, 2018  
Machine learning workshop, TSIMF, China, 15 June, 2018  
Workshop "Strings and Fields", YITP, Kyoto, 31 July, 2018  
APCTP focus program, Hanyang u. Seoul, 15 Aug, 2018  
Workshop "QG meets lattice QCD", ECT\*, Trento, 3 Sep, 2018

# Deep Learning and AdS/QCD

Koji Hashimoto (Osaka u)

ArXiv:1802.08313 + 1809.?????

w/ S. Sugishita (Osaka), A. Tanaka (RIKEN AIP),  
A. Tomiya (CCNU)

0. Bulk emergence?

1. Formulation of AdS/DL

2. Deeply learning QCD

0-1

## Emergent geometry?

### Emergence of AdS radial direction?

Bulk reconstruction and locality.

[Heemskerk, Penedones, Polchinski, Sully 09]

Entanglement entropy reconstruction.

[Balasubramanian, Chowdhury, Czech, de Boer, Heller 13]

[Myers, Rao, Sugishita 14]

Optimization of boundary path integral.

[Caputa, Kundu, Miyaji, Takayanagi, Watanabe 17]

Renormalization and effective LG theory.

[Ki-Seok Kim, Chanyong Park 16]

AdS/MERA. [Swingle 12]

### Emergence of smooth neural network space?

Statistical neural network. [Amari et al.]

# Solving inverse problem

AdS/CFT

(No proof, no derivation)

Classical gravity  
in  $d+1$  dim. spacetime

||

Quantum field theory  
in  $d$  dim. spacetime  
(Strong coupling limit,  
large DoF limit)

Conventional

holographic modeling

Model  
Metric  $g_{\mu\nu}$

Prediction

Prediction

↕

↕

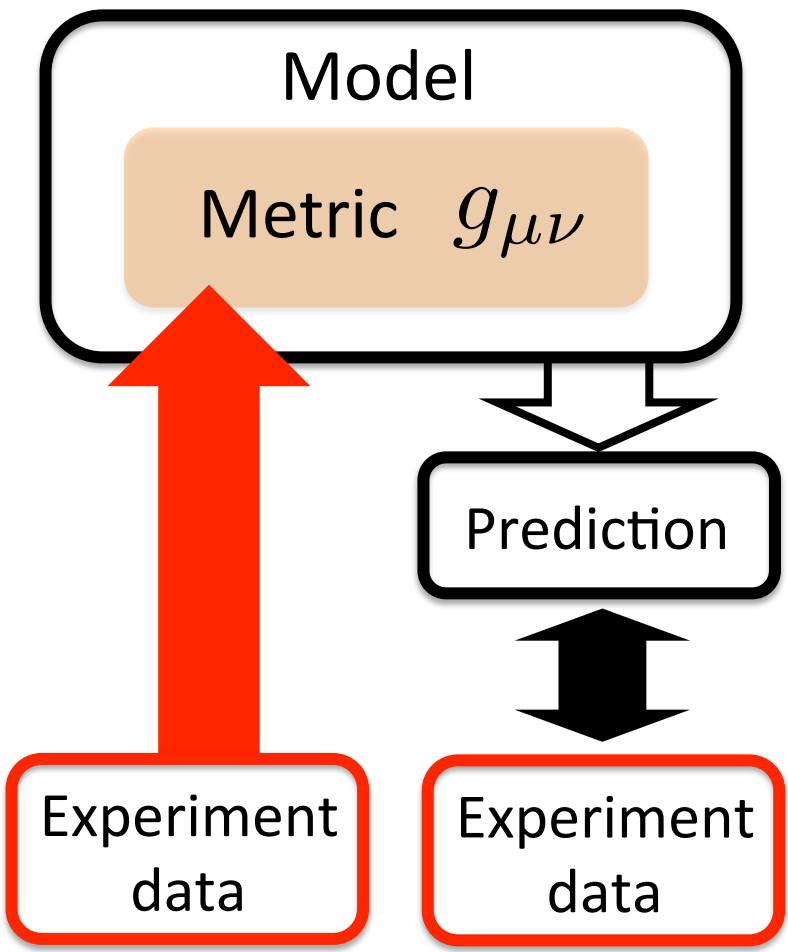
Comparison

Experiment  
data

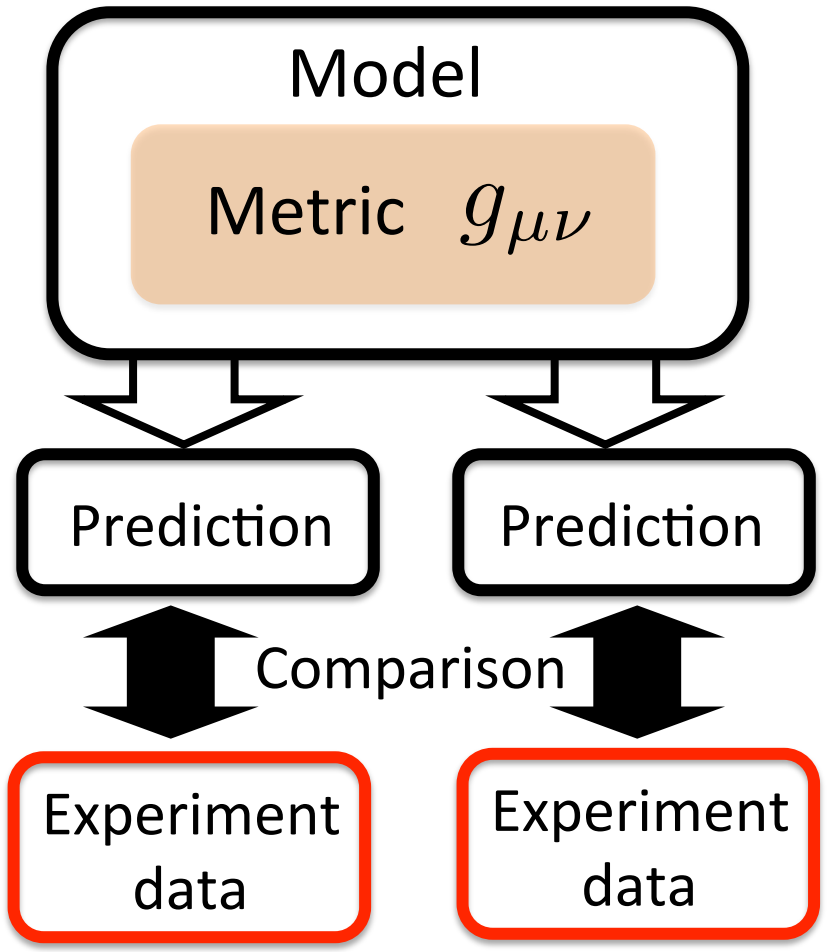
Experiment  
data

# Solving inverse problem

Our deep learning  
holographic modeling



Conventional  
holographic modeling



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# 1. Formulation of AdS/DL

review

**AdS/CFT: quantum response from geometry**

review

**Deep learning: optimized sequential map**

1-1

**From AdS to DL**

1-2

**Dictionary of AdS/DL correspondence**

# AdS/CFT: quantum response from geometry

[Klebanov, Witten]

Classical scalar field theory in  $(d+1)$  dim. geometry

$$S = \int d^{d+1}x \sqrt{-\det g} [(\partial_\eta \phi)^2 - V(\phi)]$$

$$ds^2 = -f(\eta)dt^2 + d\eta^2 + g(\eta)(dx_1^2 + \dots + dx_{d-1}^2)$$

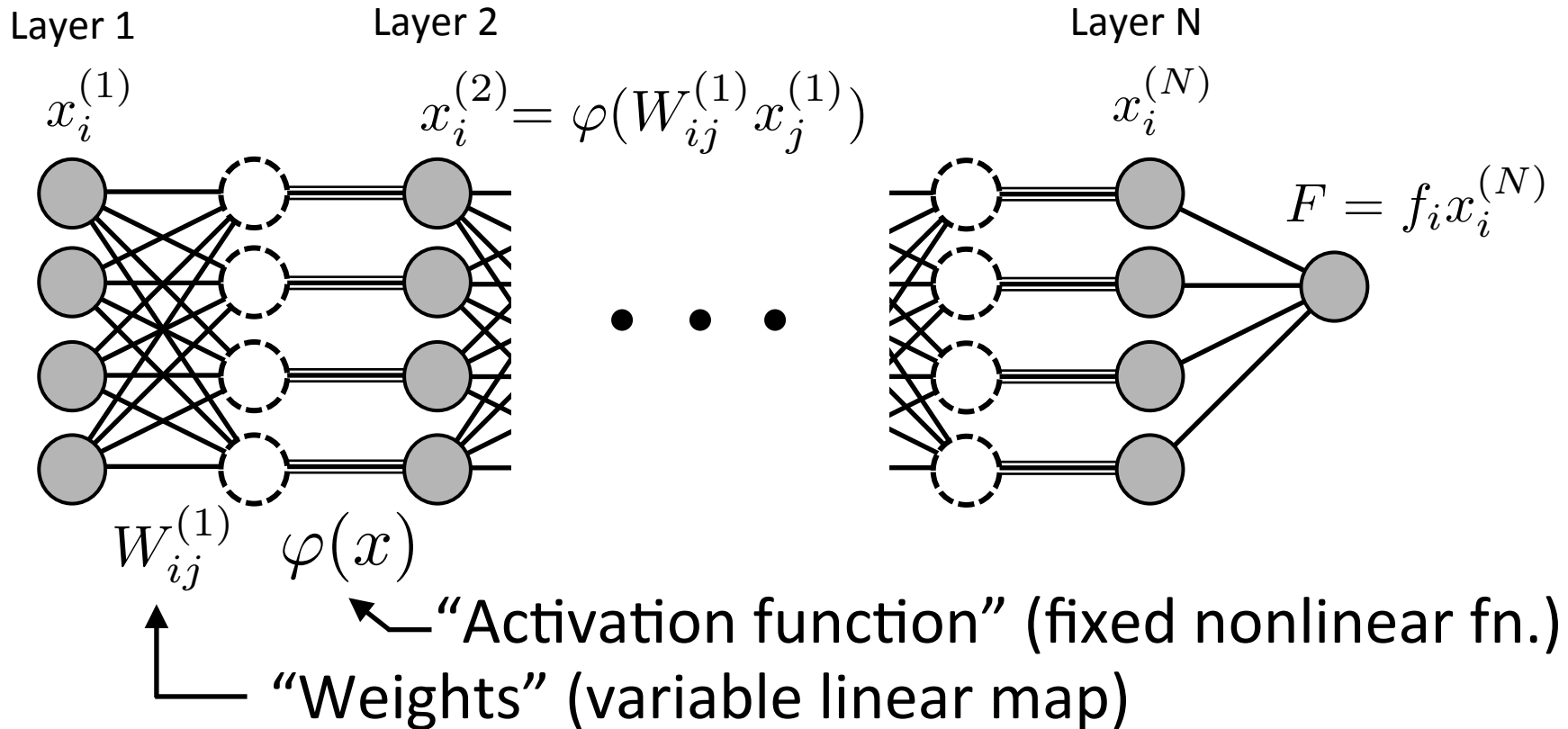
$$\left\{ \begin{array}{l} \text{AdS boundary ( } \eta \sim \infty \text{ ) : } f \sim g \sim \exp[2\eta/L] \\ \text{Black hole horizon ( } \eta \sim 0 \text{ ) : } f \sim \eta^2, g \sim \text{const.} \end{array} \right.$$

Solve EoM, get response  $\langle \mathcal{O} \rangle_J$ . Boundary conditions:

$$\left\{ \begin{array}{l} \text{AdS boundary ( } \eta \sim \infty \text{ ) :} \\ \phi = J e^{-\Delta_- \eta} + \frac{1}{\Delta_+ - \Delta_-} \langle \mathcal{O} \rangle e^{-\Delta_+ \eta} \\ \text{Black hole horizon ( } \eta \sim 0 \text{ ) : } \partial_\eta \phi \big|_{\eta=0} = 0 \end{array} \right.$$



# Deep learning : optimized sequential map



- 1) Prepare many sets  $\{x_i^{(1)}, F\}$  : input + output
- 2) Train the network (adjust  $W_{ij}$ ) by lowering

“Loss function”  $E \equiv \sum_{\text{data}} \left| f_i(\varphi(W_{ij}^{(N-1)} \varphi(\dots \varphi(W_{lm}^{(1)} x_m^{(1)}))) - F \right|$

# 1-1

## From AdS to DL

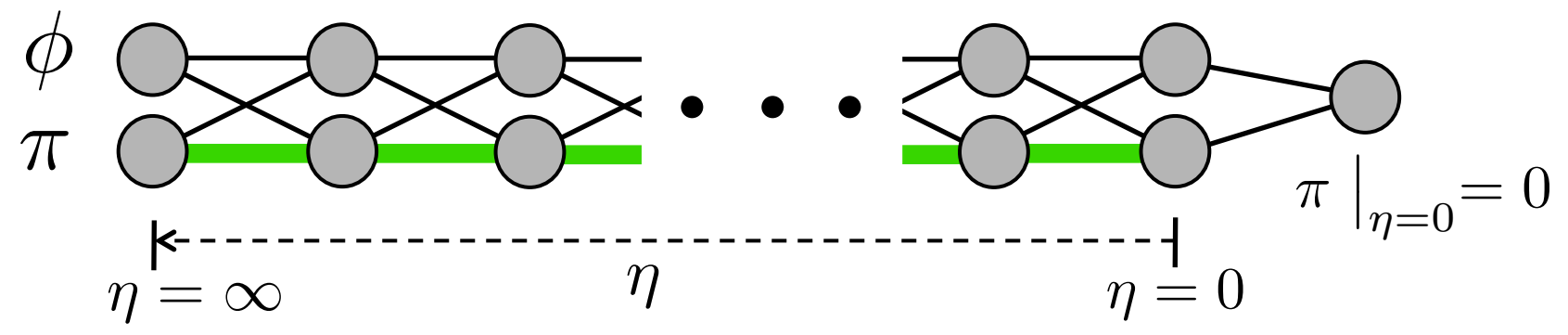
Bulk EoM  $\partial_\eta^2 \phi + \underbrace{h(\eta)}_{\text{metric}} \partial_\eta \phi - \frac{\delta V[\phi]}{\delta \phi} = 0$

$h(\eta) \equiv \partial_\eta \left[ \log \sqrt{f(\eta)g(\eta)^{d-1}} \right]$

Discretization, Hamilton form

$\begin{cases} \phi(\eta + \Delta\eta) = \phi(\eta) + \Delta\eta \pi(\eta) \\ \pi(\eta + \Delta\eta) = \pi(\eta) + \Delta\eta \left( h(\eta)\pi(\eta) - \frac{\delta V(\phi(\eta))}{\delta \phi(\eta)} \right) \end{cases}$

Neural-Network representation



1-2

# Dictionary of AdS/DL correspondence

AdS/CFT	Deep learning
Emergent space $\infty > \eta \geq 0$	Depth of layers $i = 1, 2, \dots, N$
Bulk gravity metric $h(\eta)$	Network weights $W_{ij}^{(a)}$
Nonlinear response $\langle \mathcal{O} \rangle_J$	Input data $x_i^{(1)}$
Horizon condition $\partial_\eta \phi \big _{\eta=0} = 0$	Output data $F$
Interaction $V(\phi)$	Activation function $\varphi(x)$

# 1. Formulation of AdS/DL

review

**AdS/CFT: quantum response from geometry**

review

**Deep learning: optimized sequential map**

1-1

**From AdS to DL**

1-2

**Dictionary of AdS/DL correspondence**

0. Bulk emergence?

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## 2. Deeply learning QCD

2-1

Demonstration of holographic modeling

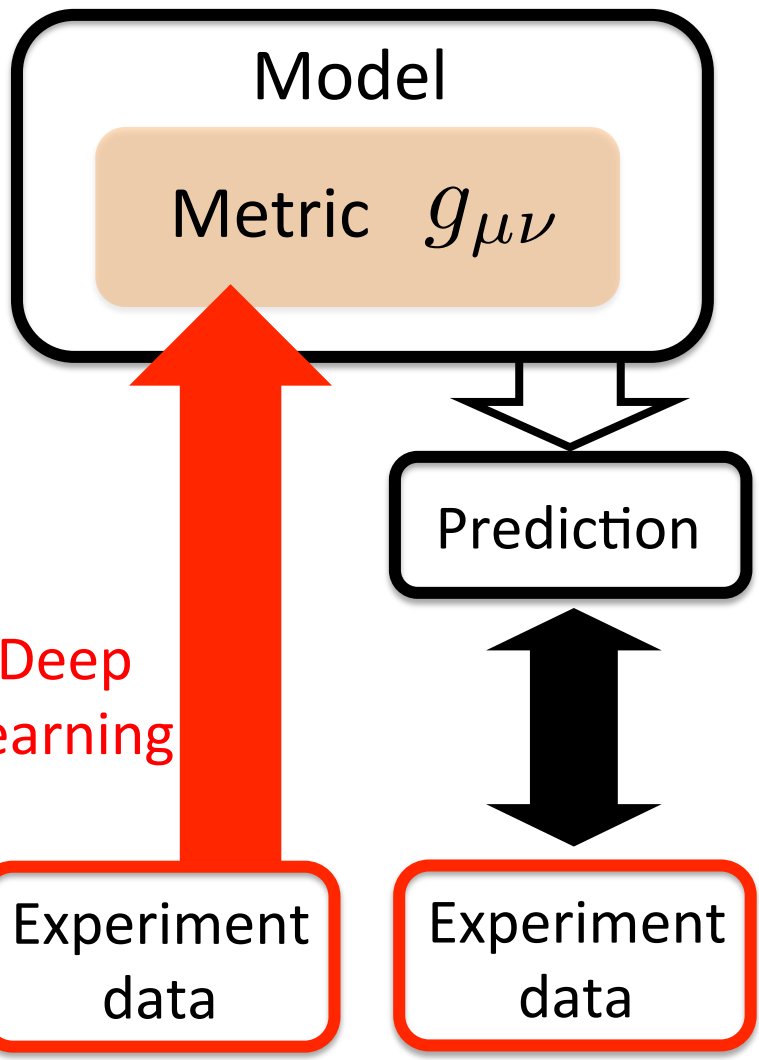
2-2

**Deeply learning QCD**

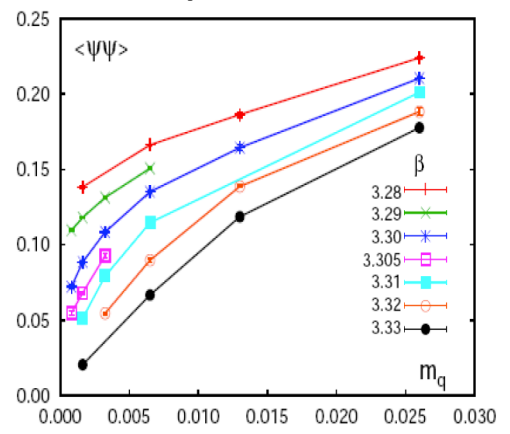
- 1) Use a QCD data.
- 2) Let the network learn the metric.
- 3) Calculate other physical quantities.

# 2-1

## Demonstration of holographic modeling

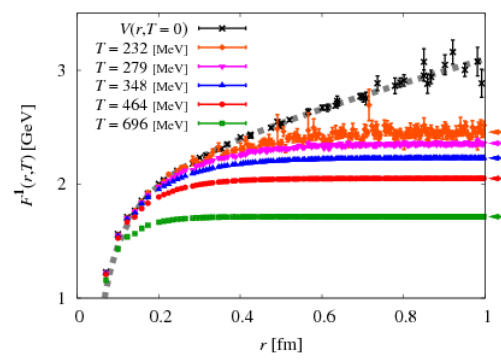
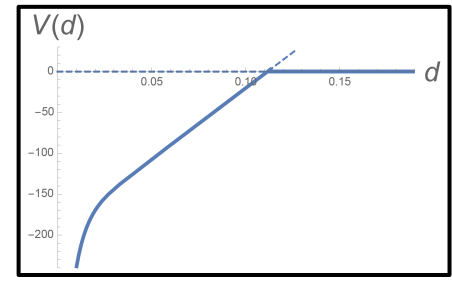


Lattice QCD data:  
chiral condensate  
VS quark mass



[RBC-Bielefeld collaboration, 2008]  
(Courtesy of W.Unger)

Q Qbar potential



[T.Ishikawa et al., 2008,  
CPPACS + JLQCD collaboration]

2-2

## Deeply learning QCD

- 1) Use a QCD data.
- 2) Let the network learn the metric.
- 3) Calculate other physical quantities.

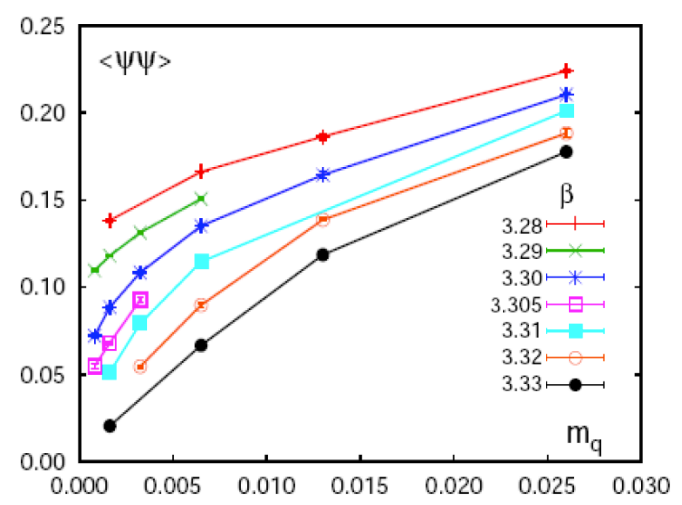


# 2-2

## Deeply learning QCD

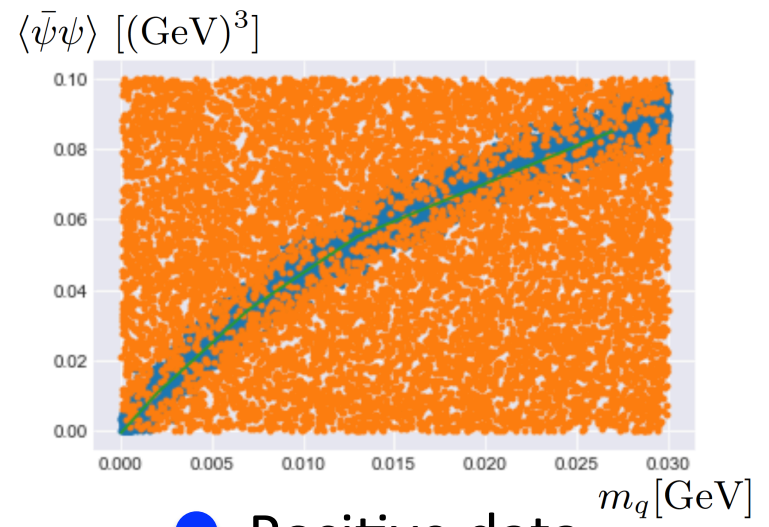
- 1) Use a QCD data.
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- 3) Calculate other physical quantities.

### Chiral condensate VS quark mass.



$\beta=3.30 \Leftrightarrow T=196[\text{MeV}]$   
[RBC-Bielefeld collaboration, 2008]  
(Courtesy of W.Unger)

Pick up  
➔  
 $\beta=3.33$   
data



● Positive data  
● Negative data

- 1) Use a QCD data.
- 2) Let the network learn the metric.
- 3) Calculate other physical quantities.

Map it to asymptotic scalar configuration. [Klebanov, Witten]  
 [DaRold,Pomarol][Karch,Katz,Son,Stephanov] [Cherman,Cohen,Werbos]

$$\phi = \frac{\sqrt{N_c}}{4\pi} m_q e^{-\eta} + \frac{\pi}{2\sqrt{N_c}} \langle \bar{q}q \rangle e^{-3\eta} - \frac{\lambda}{2} \left( \frac{\sqrt{N_c}}{4\pi} m_q \right)^3 \eta e^{-3\eta}$$

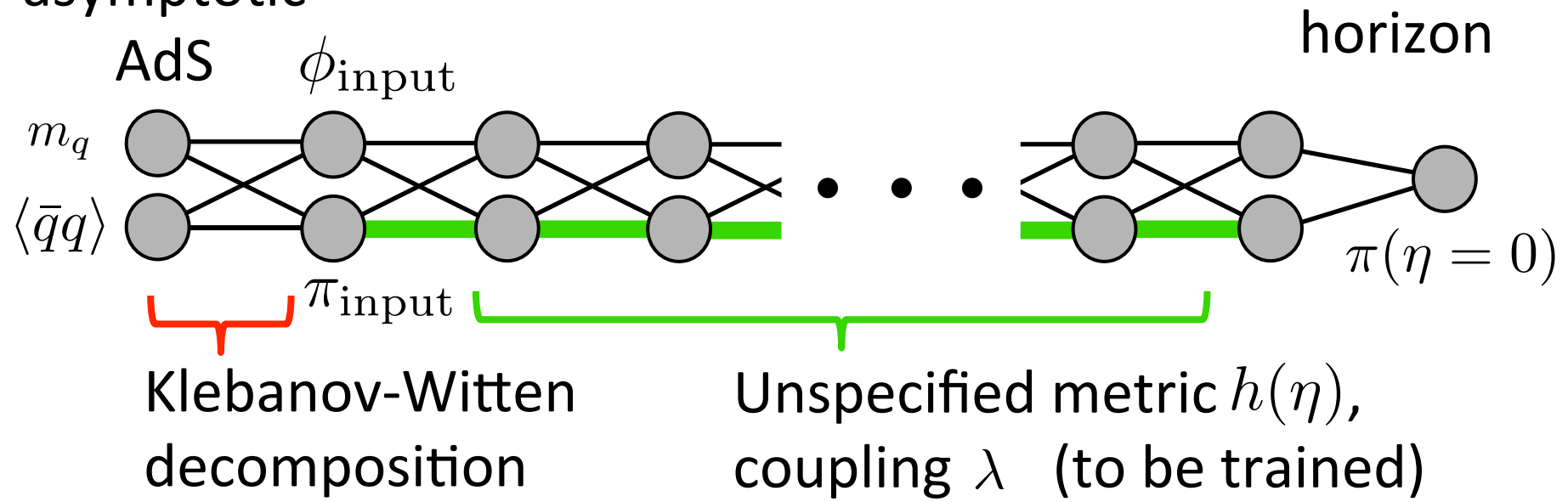
- Conformal dimension of  $\langle \bar{q}q \rangle$  is 3.
- Sub-leading contribution, present.
- Everything measured in unit of AdS radius.

# 2-2

## Deeply learning QCD

- 1) Use a QCD data.
- 2) Let the network learn the metric.
- 3) Calculate other physical quantities.

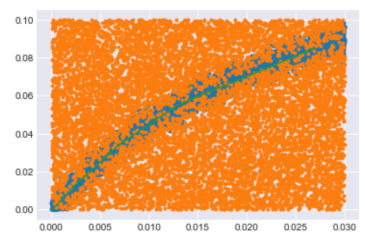
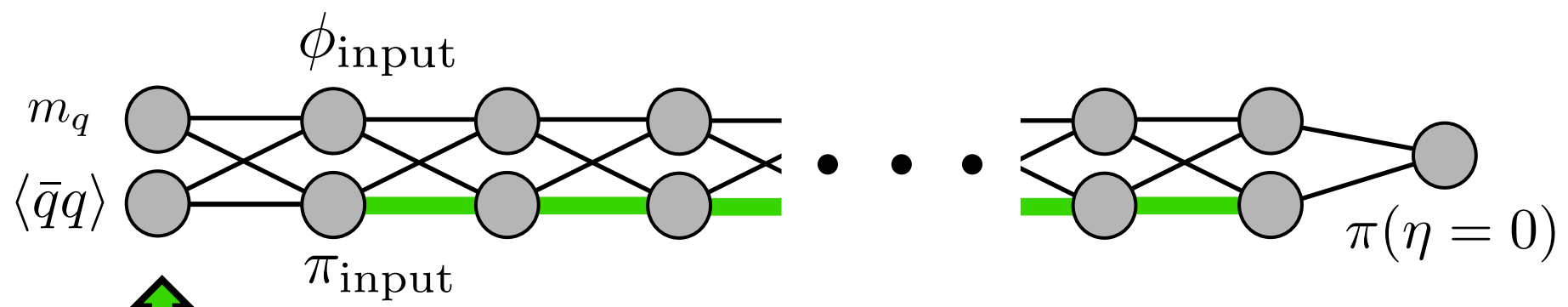
asymptotic



# 2-2

## Deeply learning QCD

- 1) Use a QCD data.
- 2) Let the network learn the metric.
- 3) Calculate other physical quantities.

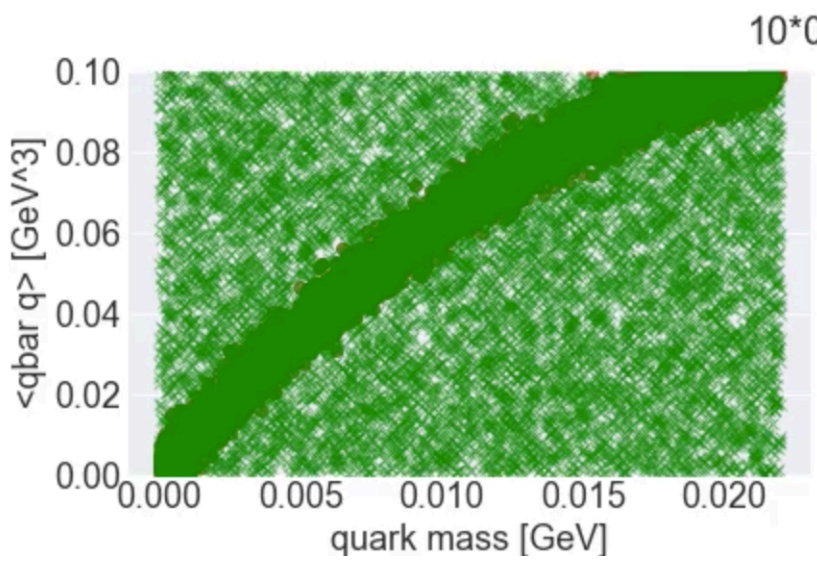


QCD lattice data

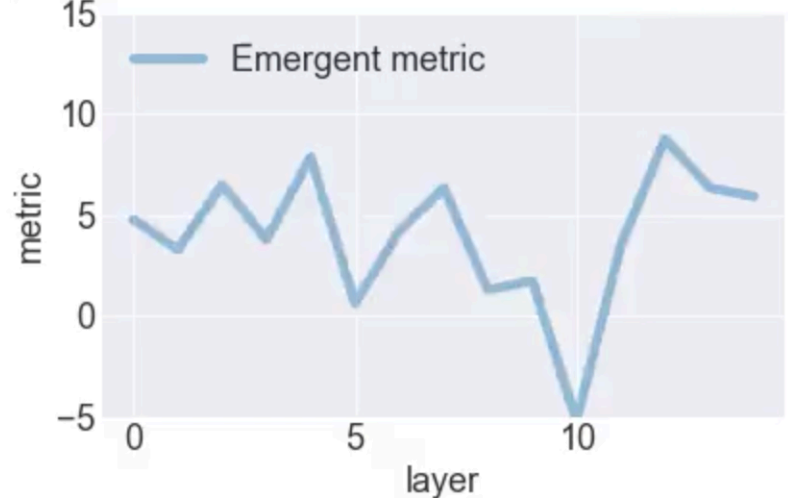
# 2-2

## Deeply learning QCD

- 1) Use a QCD data.
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$$h(\eta) \equiv \partial_\eta \left[ \log \sqrt{f(\eta)g(\eta)^{d-1}} \right]$$



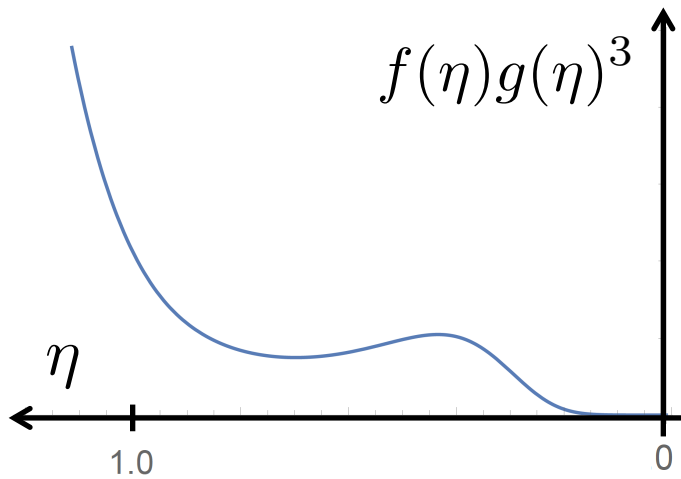
Learned value of (AdS radius)<sup>-1</sup> : 1/L = 237(3)[MeV]  
 bulk coupling : λ/L = 0.0127(6)

# 2-2

## Deeply learning QCD

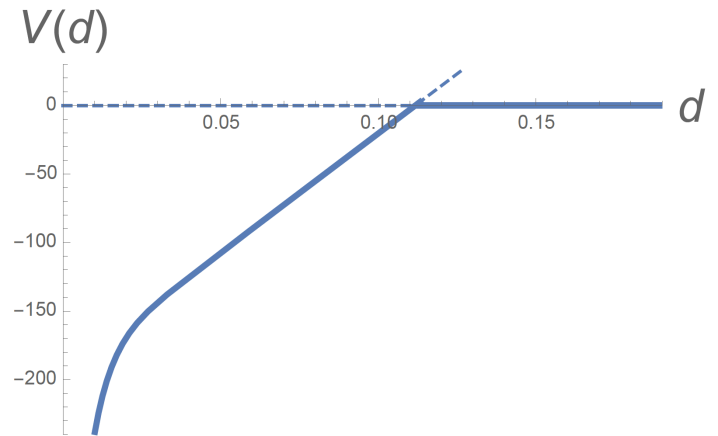
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Learned metric



Procedures based on [Maldacena] [Rey,Theisen,Yee]

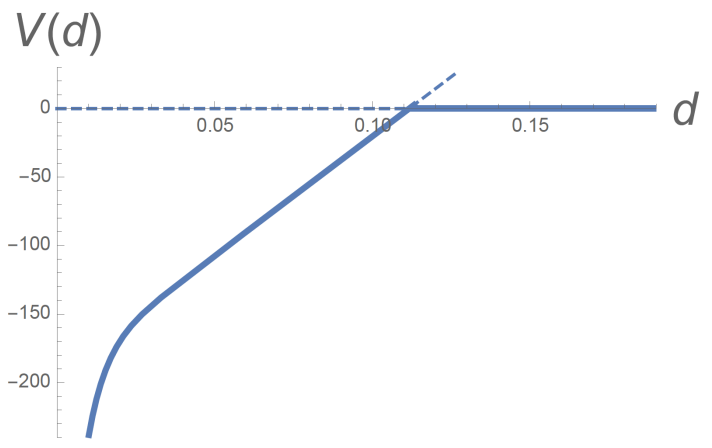
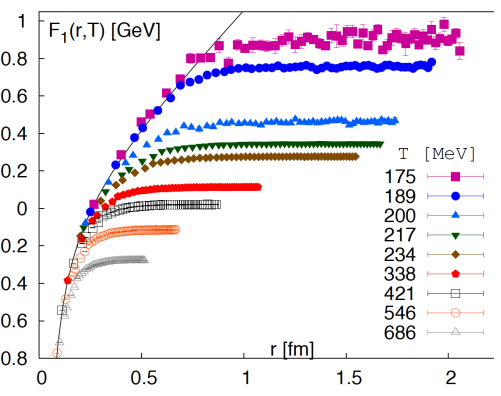
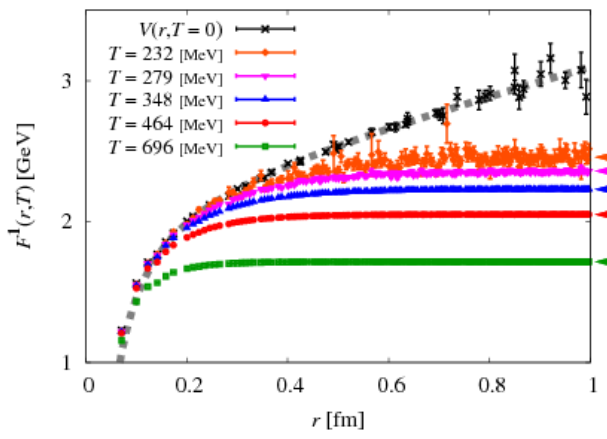
Q Qbar potential



# Deeply learning QCD

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## Q Qbar potential

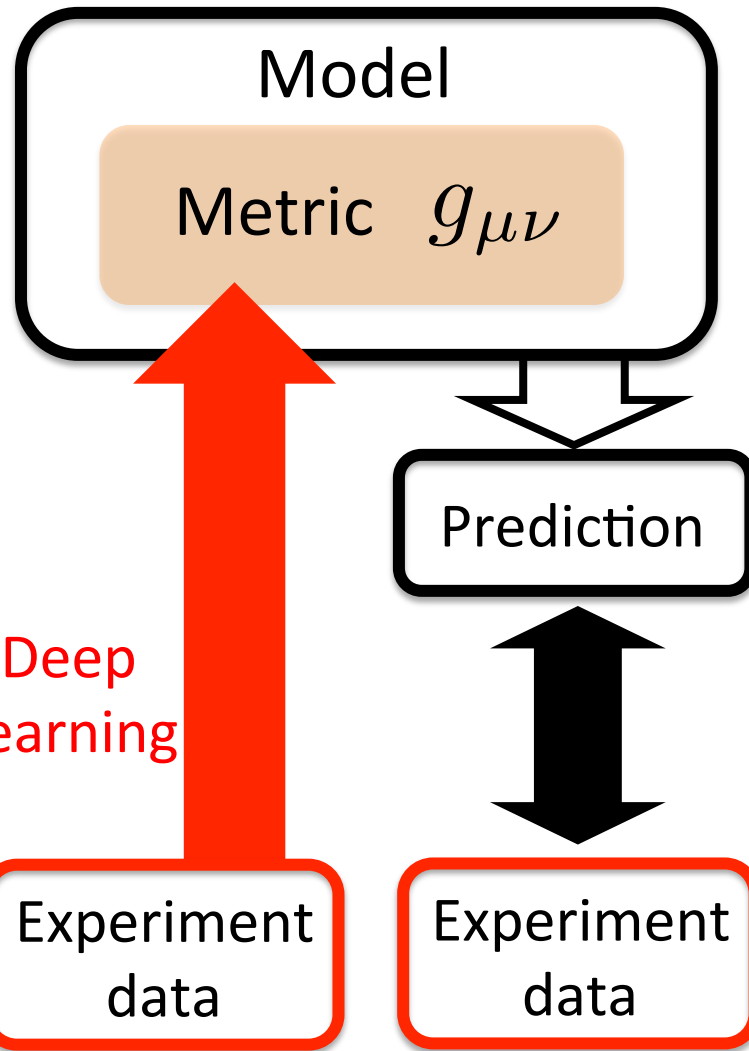


[T.Ishikawa et al., 2008, CPPACS + JLQCD collaboration]

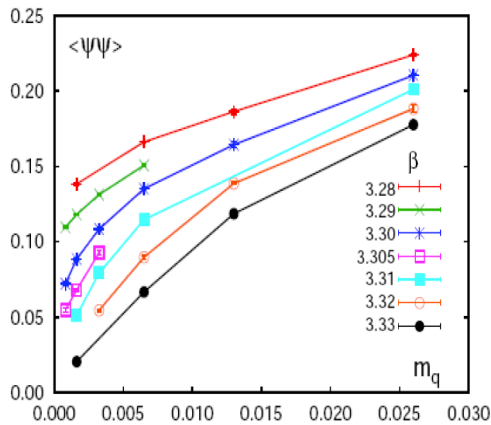
[Petreczky, 2010]

# 2-1

## Demonstration of holographic modeling



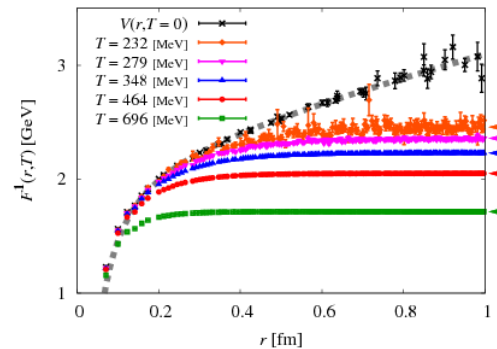
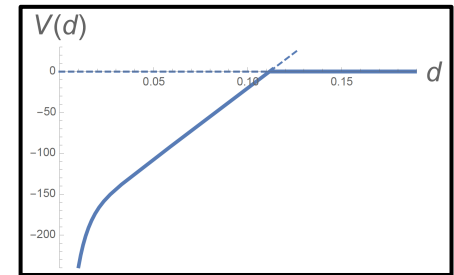
Lattice QCD data:  
chiral condensate  
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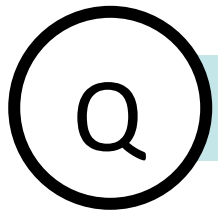
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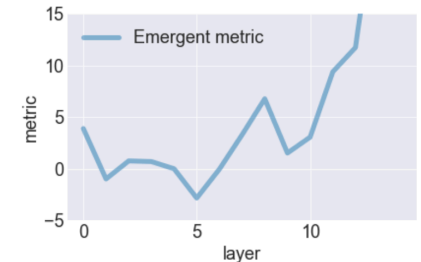
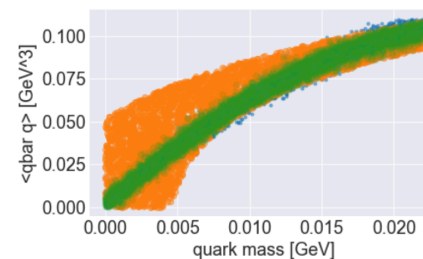
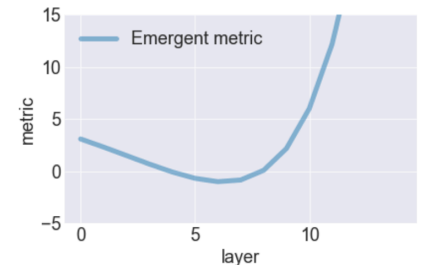
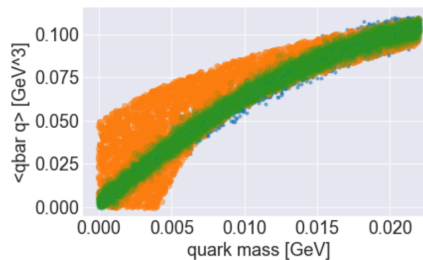


# Machines learn..., what do we learn?

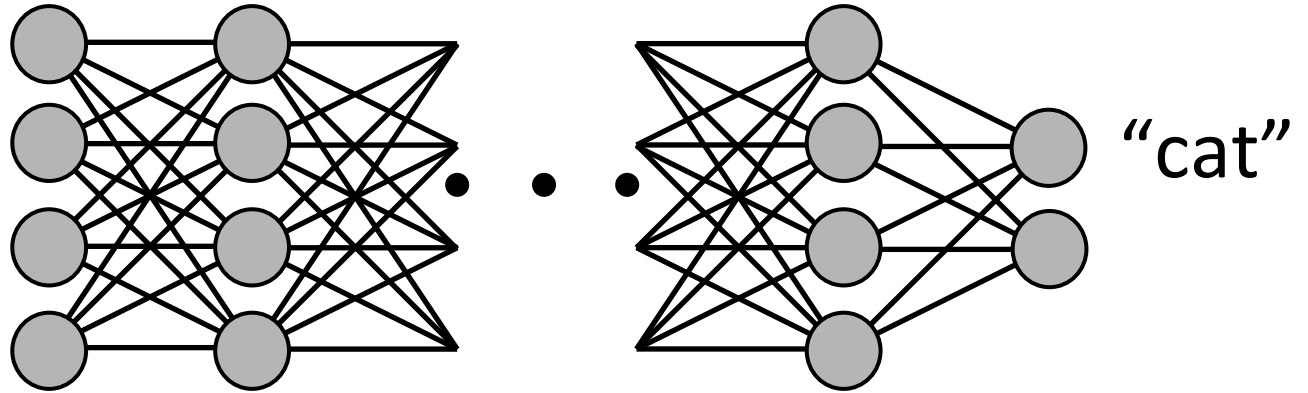
- Physics message :  
Emergent metric has a bump.  
→ No chiral symmetry breaking while “confinement”
- Toward a quantum gravity :  
Finite  $N_c$ , finite coupling  
→ Need of path integral of many emergent metrics

Emergent metric  
-with regularization

-without regularization



# Deep Learning



# AdS/CFT

[Maldacena '97]

